UNIT 8

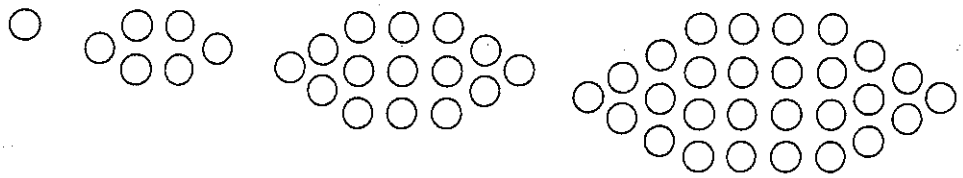
Geometry

Just for Fun

Penny Patterns

This is a pattern of pennies.

Set 1 Set 2 Set 3 Set 4



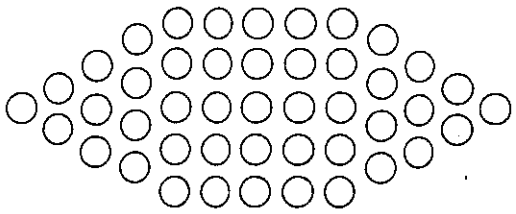
Complete the pattern for Set 5.

How many pennies do you need? _____

How many pennies do you need for Set *n*? _____

Test out your rule for Set 5.

Set 5



Mental Squares

Here is a method for squaring 2-digit numbers ending in 5.
Impress your friends by squaring their 2-digit numbers ending in 5 by mental math.

Follow these steps:

Start with a 2-digit number ending in 5. _____

Multiply the first digit by the next higher digit. _____

This product forms the first part of the square number. The last 2 digits are always 25.

Combine the 2 parts to get the square number. _____

Try this method with 3-digit numbers ending in 5. Multiply the first 2 digits by the next higher number. Can you use the method to mentally square 3-digit numbers ending in 5?

_____

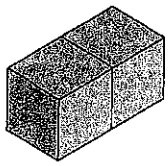
Activating Prior Knowledge

Using Isometric Dot Paper

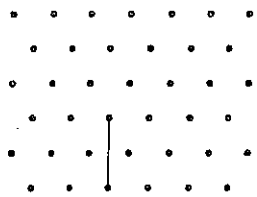
You can use isometric dot paper to represent a 3-dimensional object on a 2-dimensional drawing. Draw the parallel edges as parallel line segments on the isometric dot paper.

Example 1

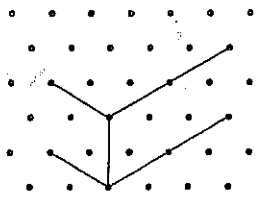
Draw this rectangular prism on isometric dot paper.



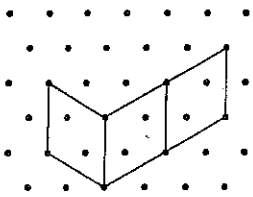
i) Start with one vertical edge.



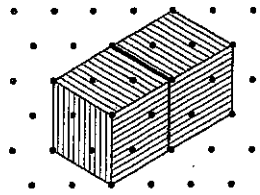
ii) Draw the adjacent horizontal edges that slant up to the left and to the right.



iii) Draw other vertical edges.



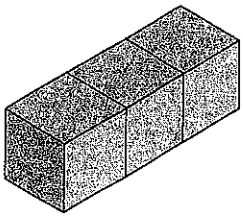
iv) Complete all edges and shade the visible faces to get a 3-D look.

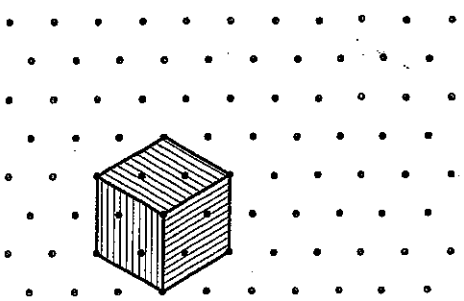


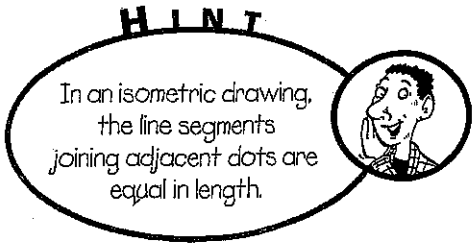
Check

1. Complete the drawing of each object on isometric dot paper.

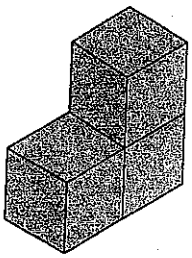
a)

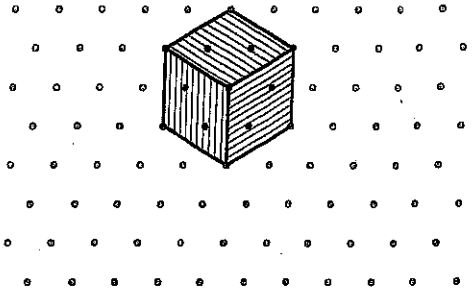




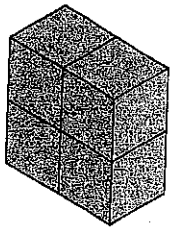


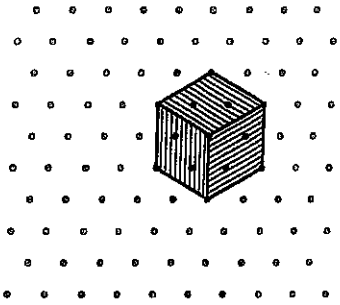
b)





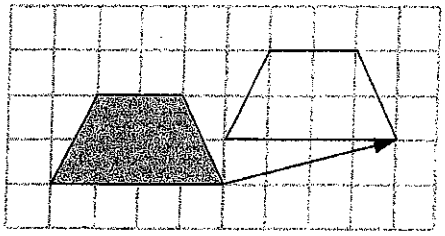
c)



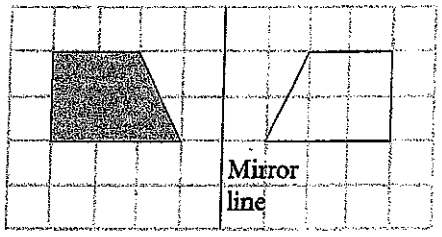


Transformations

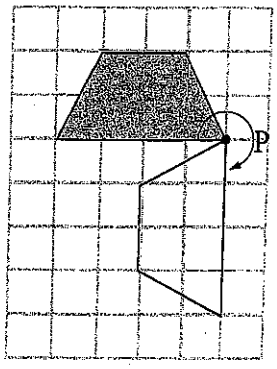
In this **translation**, the shaded shape is moved 4 units right and 1 unit up.



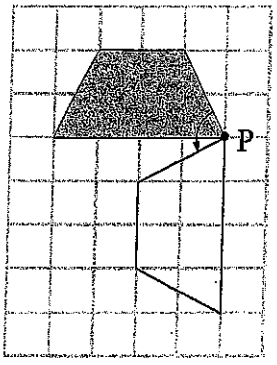
In this **reflection**, the shaded shape is reflected in a vertical line 1 unit to the right of the shape.



In this **rotation**, the shaded shape is rotated 270° clockwise about point P.

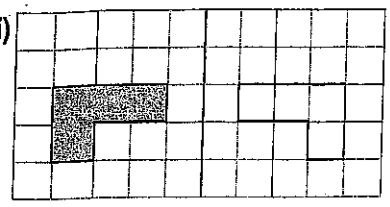


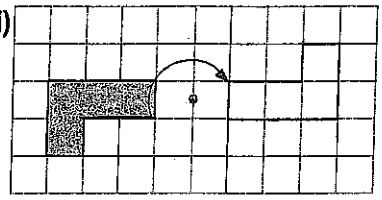
You get the same image if you rotate the shaded shape 90° counterclockwise about point P.

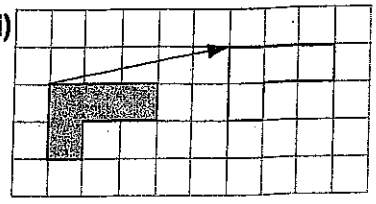


Check

2. Each drawing shows a transformation. Match the transformation to the drawing.

i) 

ii) 

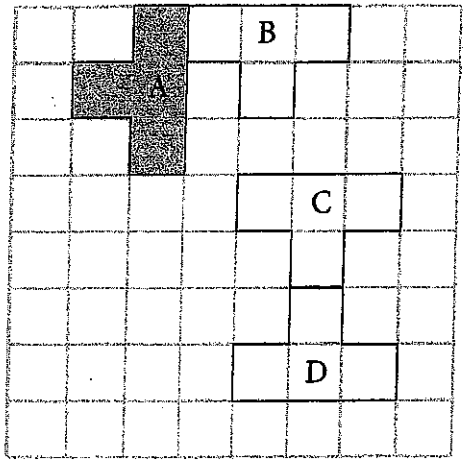
iii) 

a) translation _____

b) reflection _____

c) rotation _____

3. Identify each transformation.



a) Shape B is an image of Shape A.

_____

b) Shape C is an image of Shape B.

_____

c) Shape D is an image of Shape C.

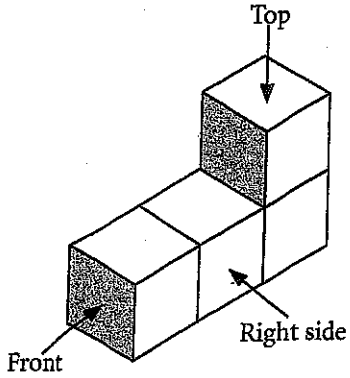
_____

8.1 Sketching Views of Objects

Quick Review

➤ The front, top, and side views of an object can be drawn by looking at a model or an **isometric drawing** of the object.

The front, top, and side views of this model can be drawn by rotating the model in order to look at the views directly.



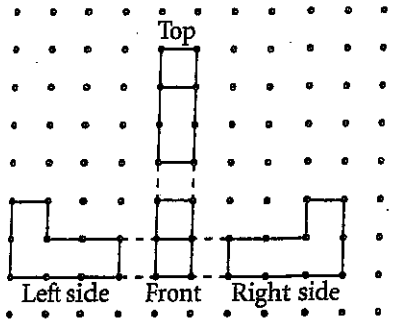
When you draw the different views of the object, draw the front view first.

Place the top view above the front view, and the side views beside the front view.

Broken lines show how the views align.

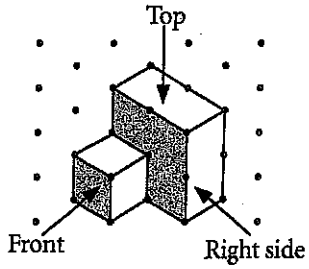
Internal line segments are used to show changes in depth or thickness.

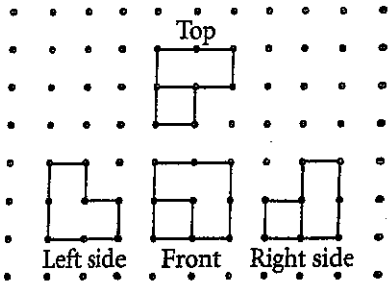
Notice the internal line segments on the front and top views that show the changes in depth. Since there are no changes in depth on the two side views, there are no internal line segments.



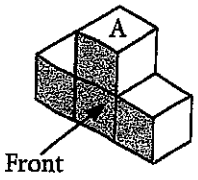
➤ When an isometric drawing of an object is given, you can build a model, and then draw the different views.

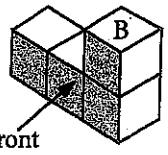
For example, the object with this isometric drawing has the front, top, and side views shown.

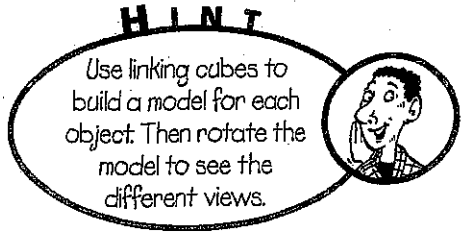


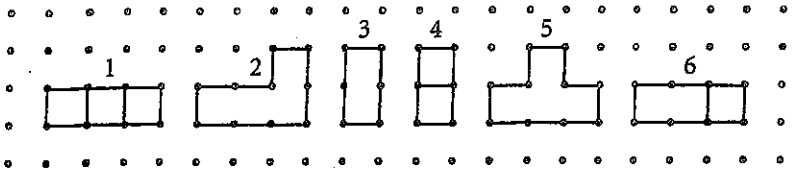


1. Figures 1 to 6 are views of objects A and B. Match each view (1 to 6) to each object A or B. A numbered view may be used more than once.



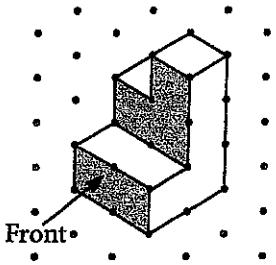


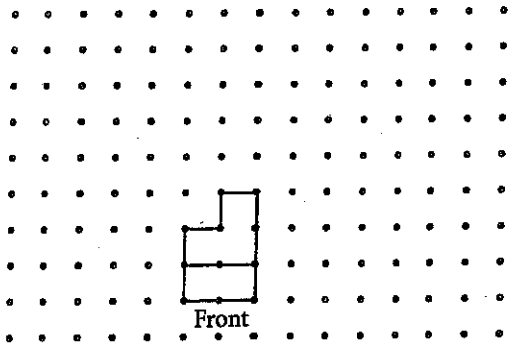




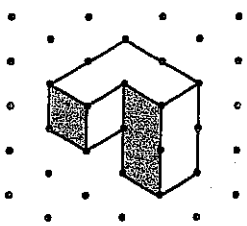
Object	Front View	Top View	Left Side View	Right Side View
A				
B				

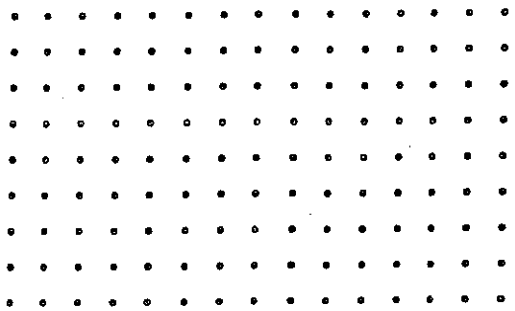
2. The front view of this object is given. Sketch the top and side views.



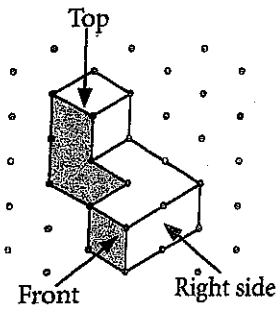


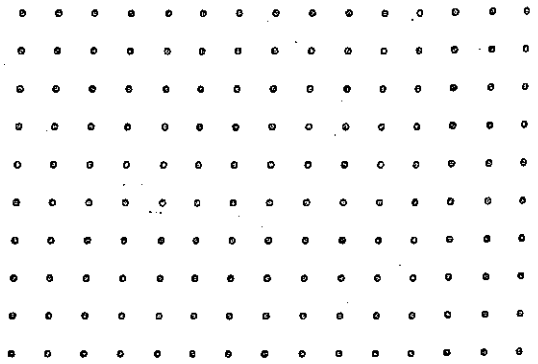
3. Sketch the front, top, and side views of this object.



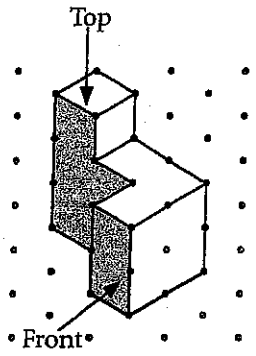


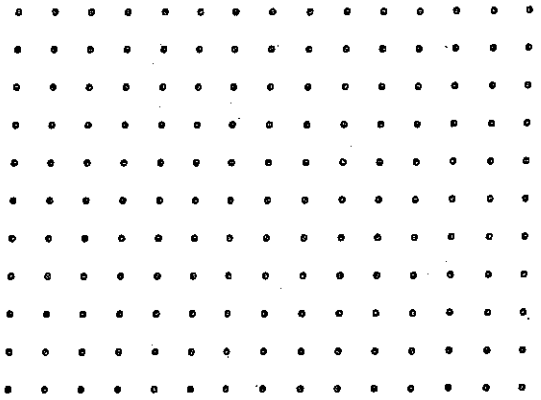
4. Sketch the front, top, and side views of this object.



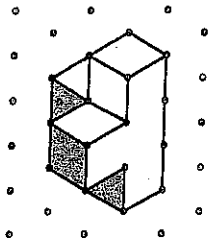


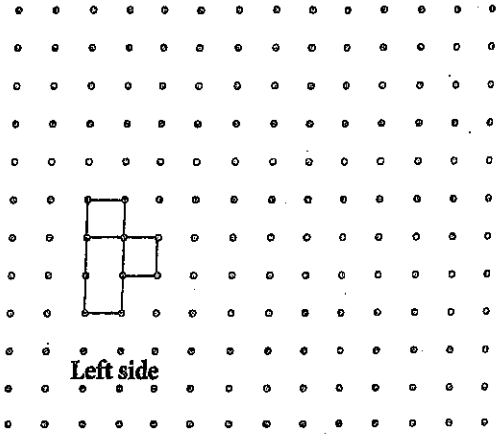
5. Use linking cubes to build a model of the object in the isometric drawing. Then draw the front, top, and side views of the object.





6. Use linking cubes to build a model of the object in the isometric drawing.
Then draw the front, top, and side views of the object. The left side is done for you.

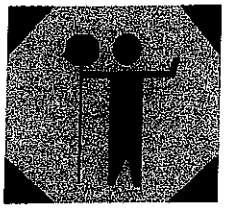




7. Many signs have views of objects.
Identify the view (front, top, or side) of the object on each sign.

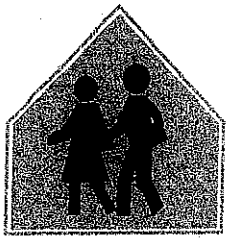
a) construction worker

_____



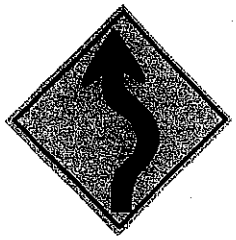
b) school children

_____



c) curved road

_____

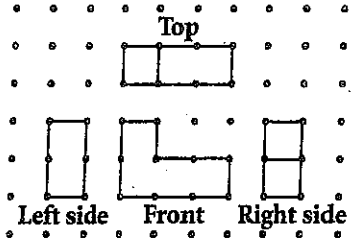


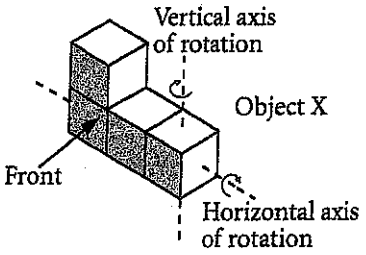
8.2 Drawing Views of Rotated Objects

Quick Review

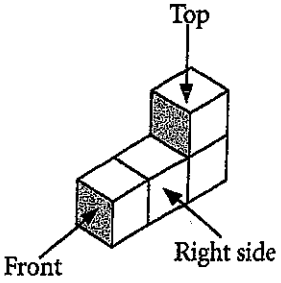
- An object can be rotated horizontally about a vertical **axis of rotation.** The rotation can be clockwise or counterclockwise.
- An object can also be rotated vertically about a horizontal axis of rotation. The rotation can be toward you or away from you.

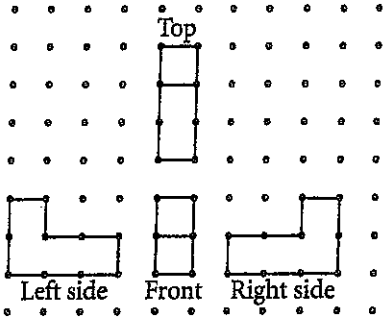
Object X has these views.



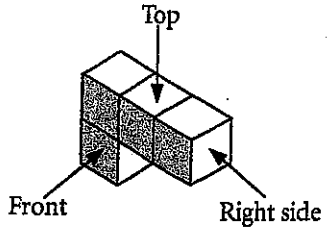


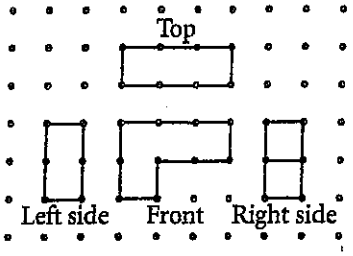
Object X is rotated horizontally 90° clockwise about a vertical axis. Here are the object's new orientation and views.





Object X is rotated vertically 180° about a horizontal axis away from you. Here are the object's new orientation and views.

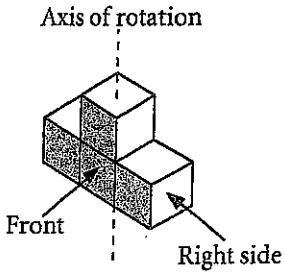




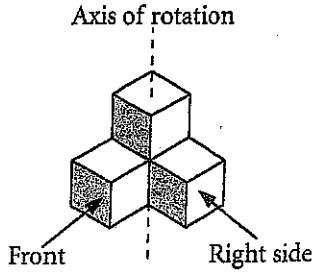
- A 180° clockwise rotation is the same as a 180° counterclockwise rotation.
  A 90° clockwise rotation is the same as a 270° counterclockwise rotation.
  A 270° clockwise rotation is the same as a 90° counterclockwise rotation.

Practice

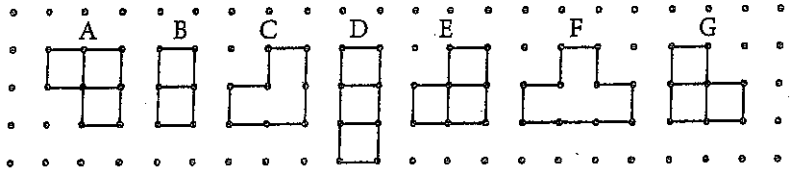
1. a) Build each object. Rotate each object horizontally 90° clockwise about the axis of rotation shown. Match each view (A to G) to the front, top, and side views of each rotated object. A lettered view may be used more than once.



Object P



Object Q



Object	Front View	Top View	Left Side View	Right Side View
P				
Q				

b) Which object, P or Q, has the left side view the same as the right side view after the rotation? _____

2. The objects P and Q in question 1 are rotated horizontally 270° counterclockwise about the axis of rotation shown.

a) Which view (A to G) is the top view of the rotated Object P? _____

b) Which view (A to G) is the front view of the rotated Object Q? _____

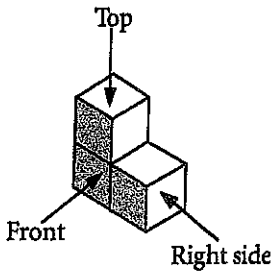
c) Explain your answers to parts a) and b).

_____

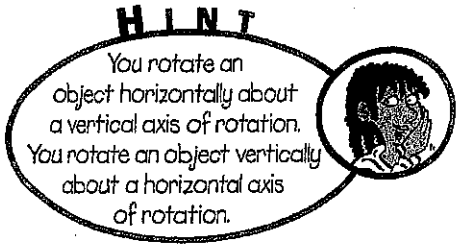
_____

_____

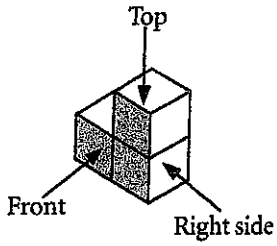
3. Use linking cubes to build Object Y. Match each rotation description i) to iv) to the drawing of the rotated Object Y in a) to d).



Object Y

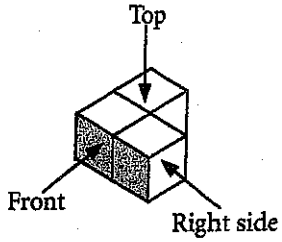


a)



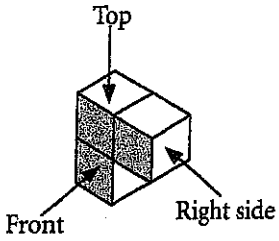
Rotation: _____

b)



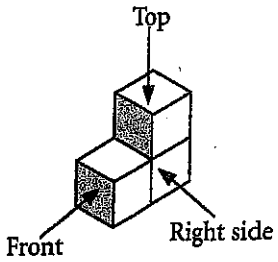
Rotation: _____

c)



Rotation: _____

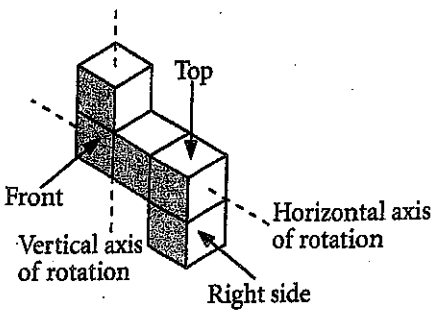
d)



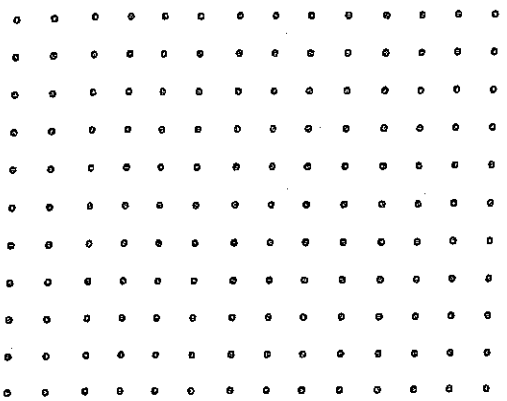
Rotation: _____

i) a rotation of 270° counterclockwise about a vertical axis
ii) a rotation of 180° about a horizontal axis
iii) a rotation of 180° about a vertical axis
iv) a rotation of 90° about a horizontal axis away from you

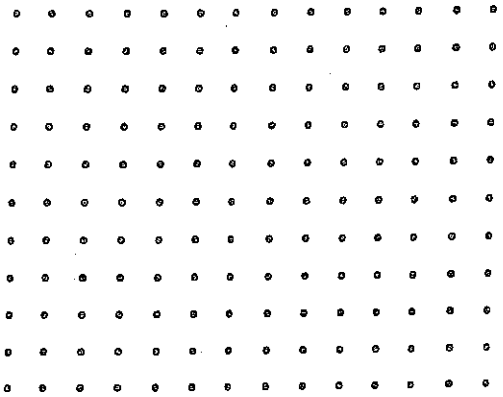
4. Use linking cubes to build this object. Draw the front, top, and side views after each horizontal rotation about the vertical axis shown.



a) 90° clockwise

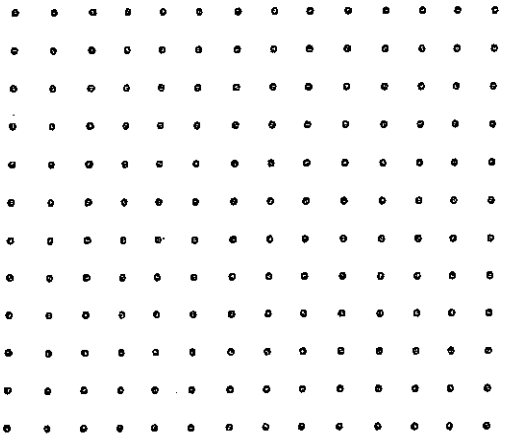


b) 180°

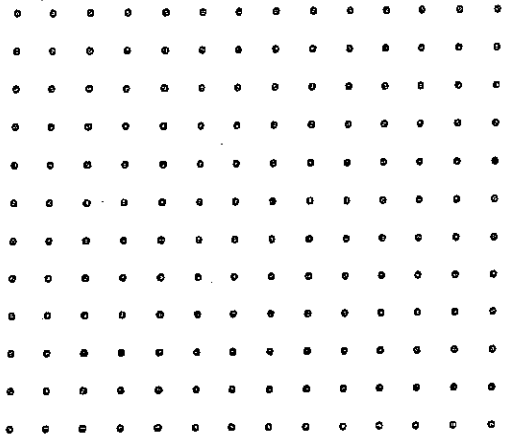


5. Draw the front, top, and side views of the object in question 4 after each vertical rotation about the horizontal axis shown.

a) 90° toward you



b) 180°



. Use linking cubes to build an object that has the same front, top, and side views after each of the horizontal or vertical rotations in this lesson.
Sketch or describe the object you built.

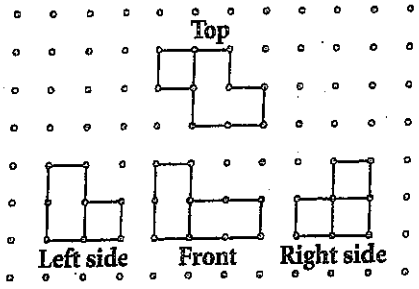
8.3 Building Objects from Their Views

At Home / At School

Quick Review

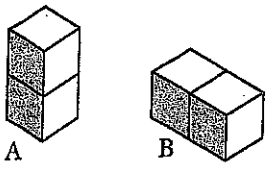
➤ You can often build an object given the front, top, and side views.
Note that internal line segments in these views show changes in depth.

The views of an object are shown.

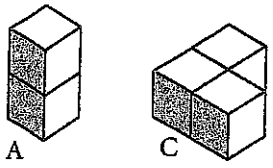


The object can be built using linking cubes:

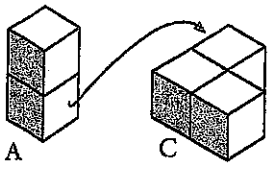
The front view shows that you need 2 cubes in a vertical column and 2 cubes in a horizontal row.



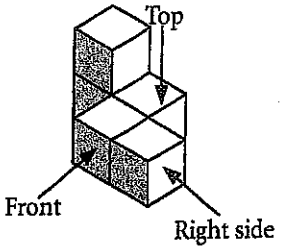
The top view shows that you need to add a cube onto B to make an L-shape C.



The top view also shows that A must be attached to C with a change in depth.



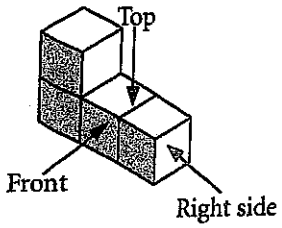
The resulting object looks like this:



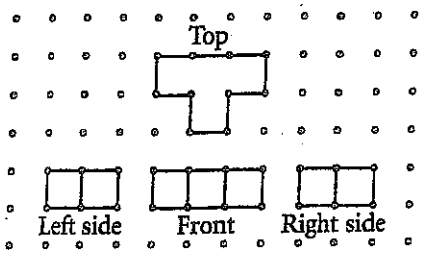
Then check the side views of the object to see if they match the given views.
If they do, the object is correct.

1. Match each set of views i) to iii) to the object (A to C).

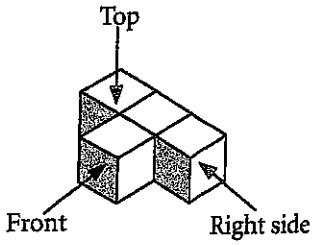
A: _____



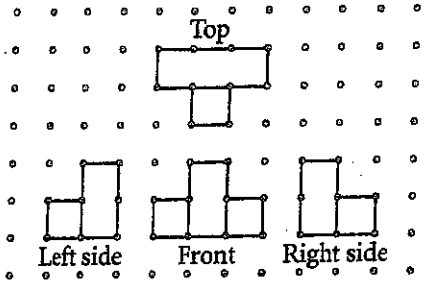
i)



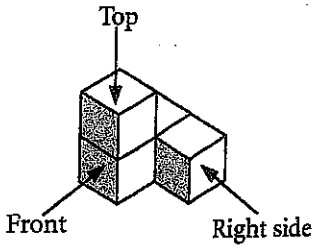
B: _____



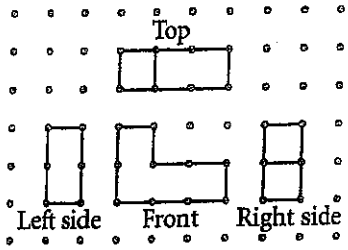
ii)



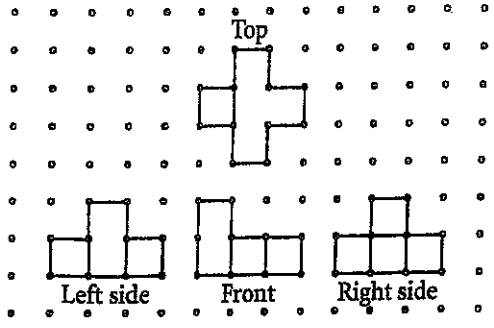
C: _____

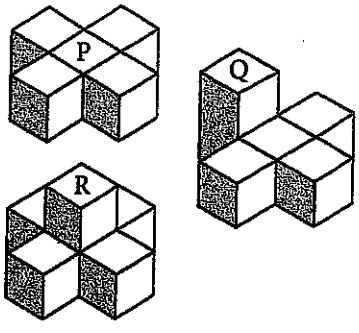


iii)

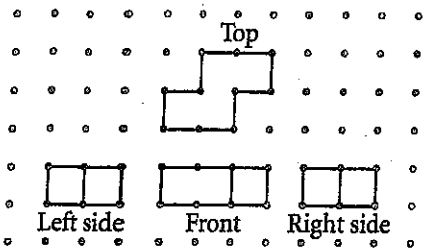


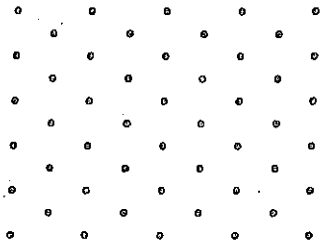
Which object, P, Q, or R, has these views? _____



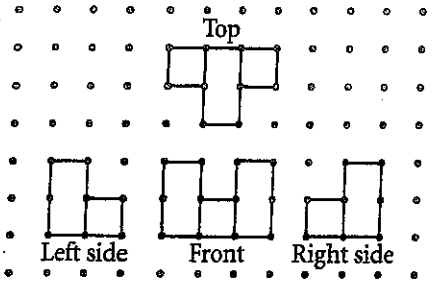


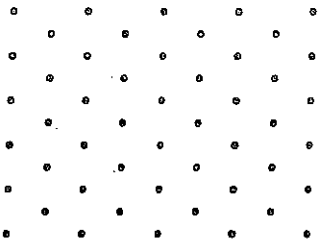
3. Use these views to build an object.



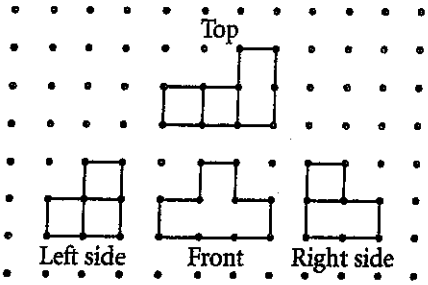


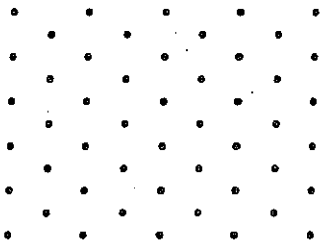
4. Use these views to build an object.



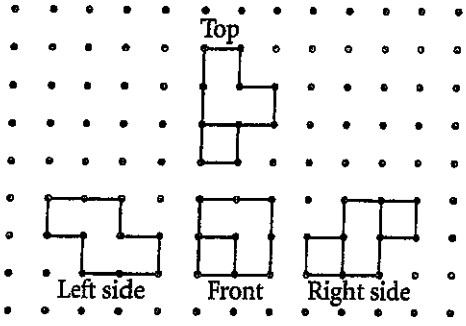


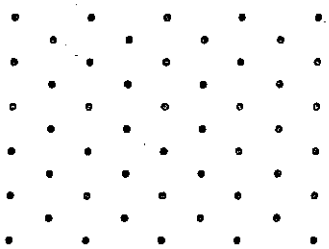
5. Use these views to build an object.





6. Use these views to build an object.





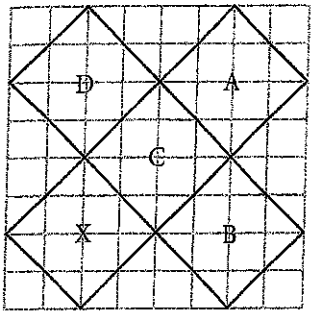
8.4 Identifying Transformations

Quick Review

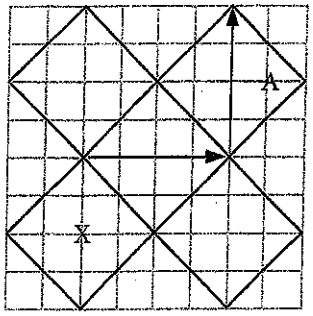
At Home / At School

➤ The 3 different transformations—translation, reflection, and rotation—have been used to create a design.

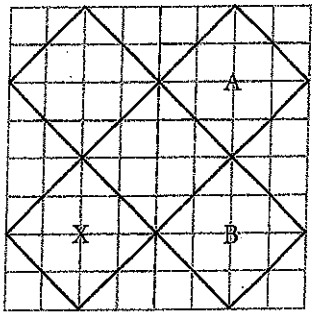
Here are some transformations that can be identified in this design.



Square A is the image of Square X after a translation 4 units right and 4 units up.



Square B is the image of Square X after a reflection in the broken line.

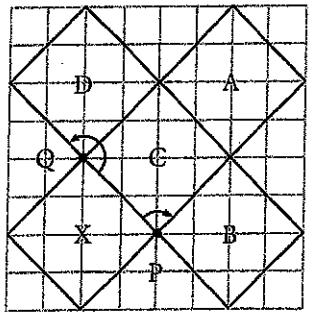


Square C is the image of Square X after a rotation of 90° clockwise about point P.

Square C is also the image of Square X after a rotation of 90° counterclockwise about point Q.

Square D is the image of Square X after a rotation of 180° about point Q.

You can make a tracing of square X and rotate it about points P and Q to check these results.



➤ A rotation of 180° clockwise about a point gives the same image as a rotation of 180° counterclockwise about the same point.

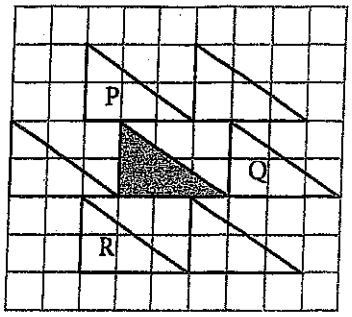
➤ Under any transformation, the original shape and its image are always congruent.

1. Match each translation of the shaded triangle to its image.

a) 3 units right _____

b) 1 unit left and 2 units down _____

c) 1 unit left and 2 units up _____



2. Match each reflection of the shaded octagon to its image.

a) reflection in Line 1

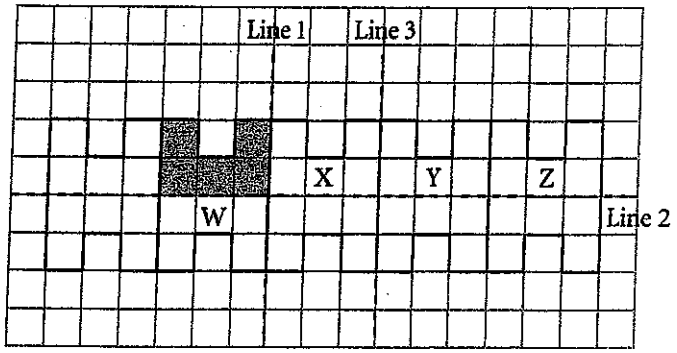
_____

b) reflection in Line 2

_____

c) reflection in Line 3

_____



3. Match each rotation of the shaded hexagon to its image.

a) 90° counterclockwise about point P

_____

b) 180° about point P

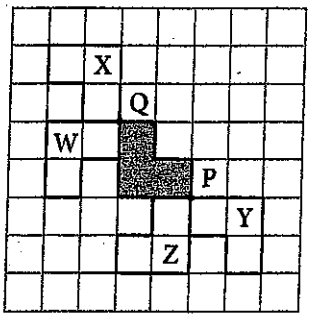
_____

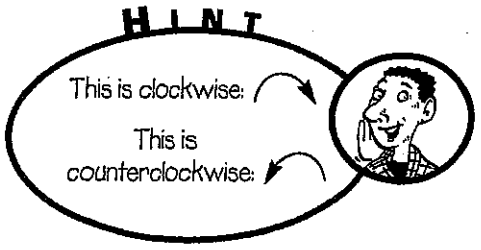
c) 90° clockwise about point Q

_____

d) 180° about point Q

_____





4. Match each transformation of the shaded polygon with its image.

a) a rotation of 180° about point P _____

b) a translation 3 units up _____

c) a reflection in Line x _____

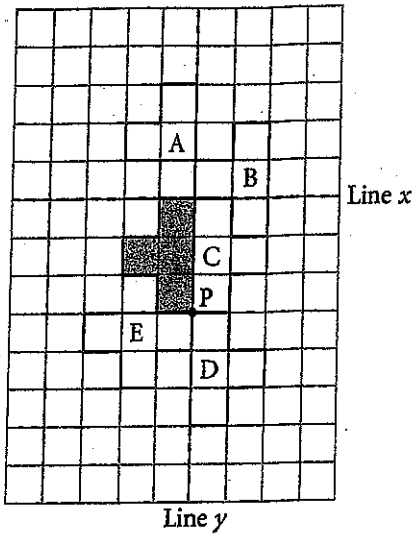
d) a reflection in Line y _____

e) a rotation of 90° counterclockwise about point P

_____

f) a translation 2 units right and 2 units up

_____



5. Identify each transformation of the shaded Shape X.
Describe each transformation in as many ways as you can.

a) Shape A is an image of Shape X.

_____

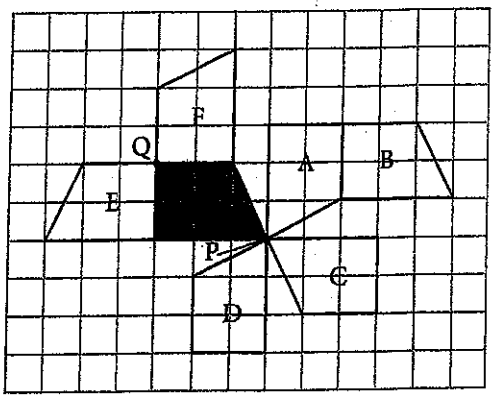
_____

b) Shape B is an image of Shape X.

_____

c) Shape C is an image of Shape X.

_____



d) Shape D is an image of Shape X.

_____

e) Shape E is an image of Shape X.

_____

f) Shape F is an image of Shape X.

_____

_____

Tip

Make a tracing of Shape X. Translate, reflect, or rotate the shape to check your results.

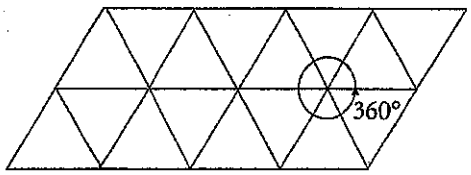
8.5 Constructing Tessellations

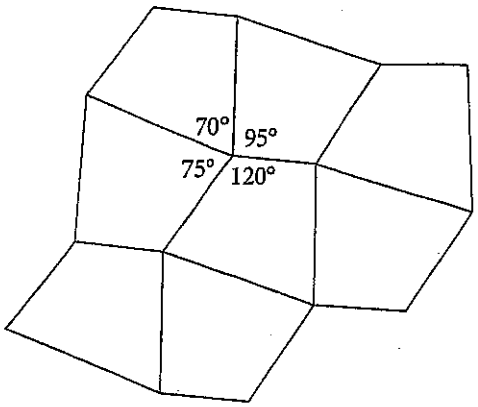
Quick Review

At Home / At School

➤ When you can cover a page using congruent copies of a shape with no overlaps and gaps, the shape **tessellates**, creating a design called a **tessellation**.

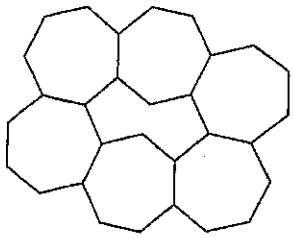
All triangles and all quadrilaterals tessellate.





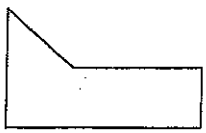
At any point where the vertices meet, the angles add up to 360°.

➤ There are some shapes that do not tessellate because they cover a page with overlap or gaps. For example, this heptagon does not tessellate.



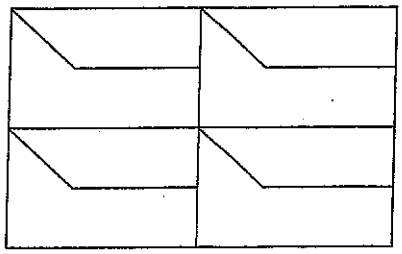
➤ You can combine shapes to tessellate.
These combined shapes are called **composite shapes.**

For example, Shape A combines with Shape B to form a quadrilateral that tessellates.



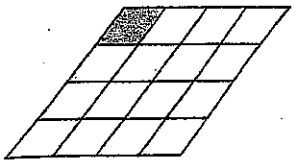
Shape A

Shape B

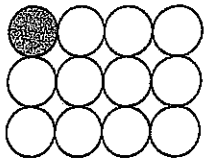


1. Which of these designs are tessellations? Justify your answer.

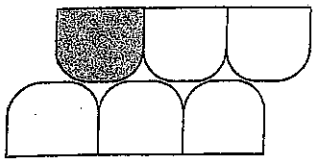
a)



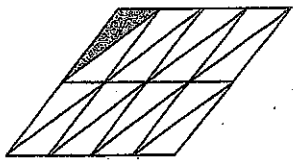
b)



c)



d)

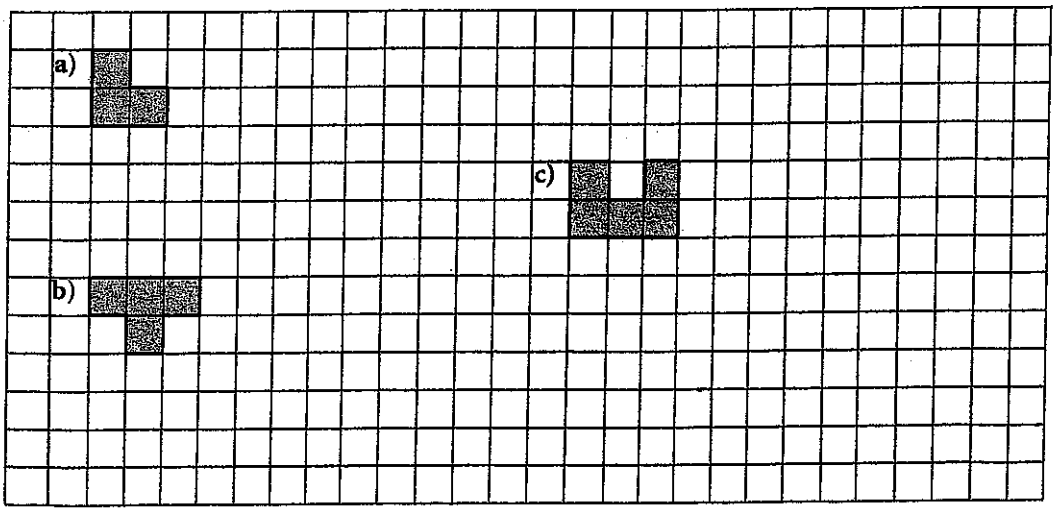


2. Which of these shapes tessellate? Use a drawing to justify your answer.

a) L-shape _____

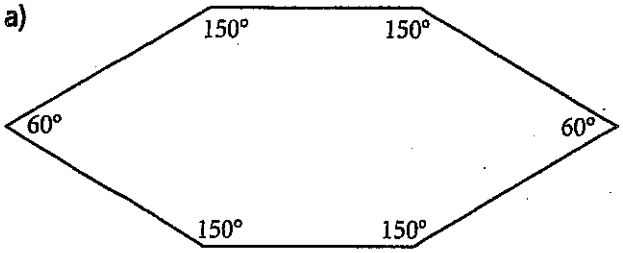
b) T-shape _____

c) U-shape _____

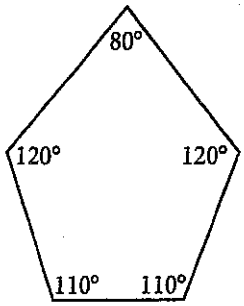


3. Which of the polygons can be used to create a tessellation?
Justify your answer by checking if copies of the polygon can surround a point.

a)



b)



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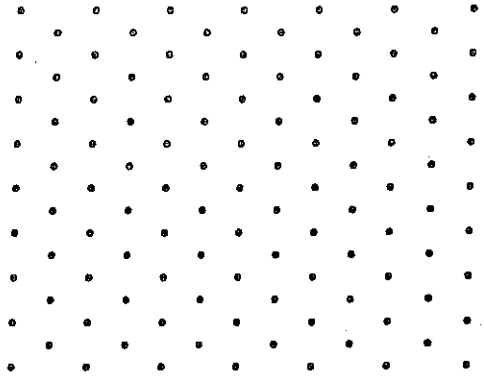
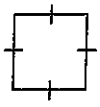
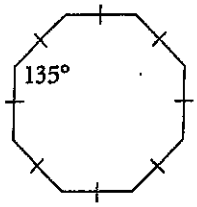
4. Create a composite shape that tessellates using a regular hexagon and one or more equilateral triangles. Show your tessellation on the isometric dot paper.



5. Arlene is planning to create a tessellating quilt pattern using one of these shapes.



square



regular octagon

a) Which shape can Arlene use? Why?

---

b) Can Arlene use a combination of these shapes to create a tessellating quilt pattern? Explain.

8.6 Identifying Transformations in Tessellations

Quick Review

At Home / At School

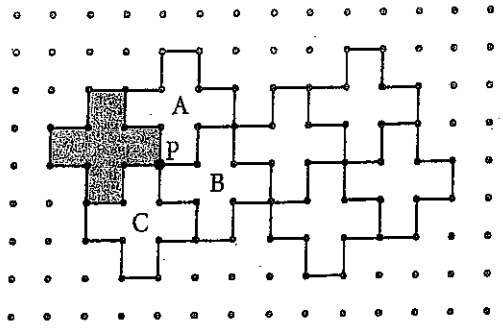
- A tessellation can be described using transformations of shapes.
- Under each transformation, the area of the shape does not change. This is known as **conservation of area.**
- A tessellation may be described by one or more than one type of transformation.

This tessellation can be described by translations or by rotations. Start with the shaded shape.

To get Shape A, translate the shaded shape 2 units right and 1 unit up.

To get Shape B, translate the shaded shape 3 units right and 1 unit down.

To get Shape C, translate the shaded shape 1 unit right and 2 units down.



Alternatively:

To get Shape A, rotate the shaded shape 90° clockwise about point P.

To get Shape B, rotate the shaded shape 180° about point P.

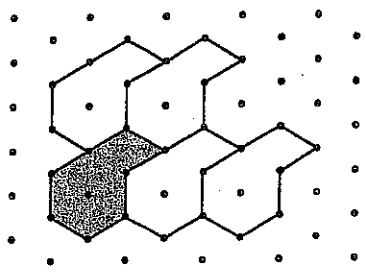
To get Shape C, rotate the shaded shape 90° counterclockwise about point P.

You can make a tracing of the shaded shape and translate it or rotate it about point P to check these results.

To complete the tessellation, repeat these translations or rotations on the shaded shape.

Practice

1. Identify the transformation in this tessellation. Circle your answer.



translation

reflection

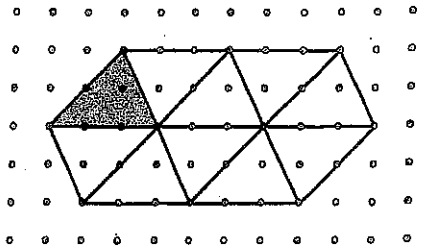
rotation

2. Identify the two transformations in this tessellation.
Circle your answer.

translation and reflection

translation and rotation

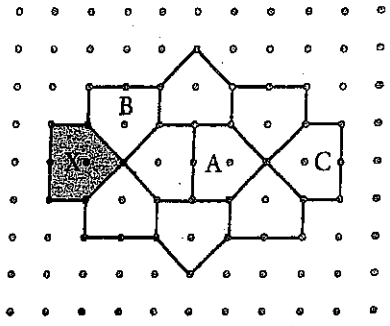
rotation and reflection



3. Identify the transformations in this tessellation.

Use these words or phrases to complete each sentence.

translation, reflection, rotation, vertical line, horizontal line, 4 units up, 4 units right, 90° , 180°, clockwise, counterclockwise

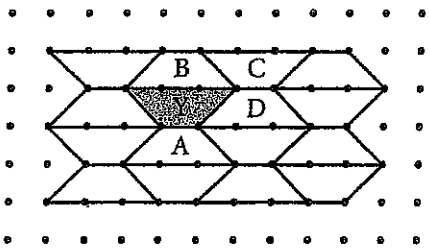


a) Shape A is a _____ of Shape X _____.

b) Shape B is a _____ of Shape X _____ about a point.

c) Shape C is a _____ of Shape X in a _____.

4. In the tessellation, Shape Y is the starting shape.



Describe the transformation needed to get to each of the lettered shapes.

A: _____

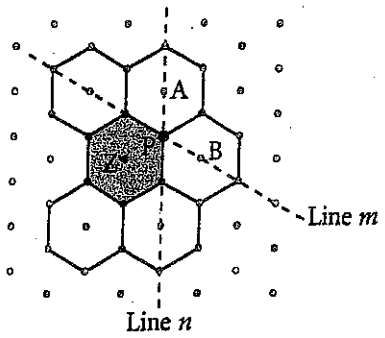
B: _____

C: _____

D: _____

_____

5. In the tessellation, Shape Z is the starting shape.



Describe as many different transformations as you can to get to each lettered shape.

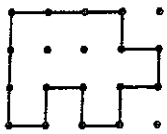
A: _____

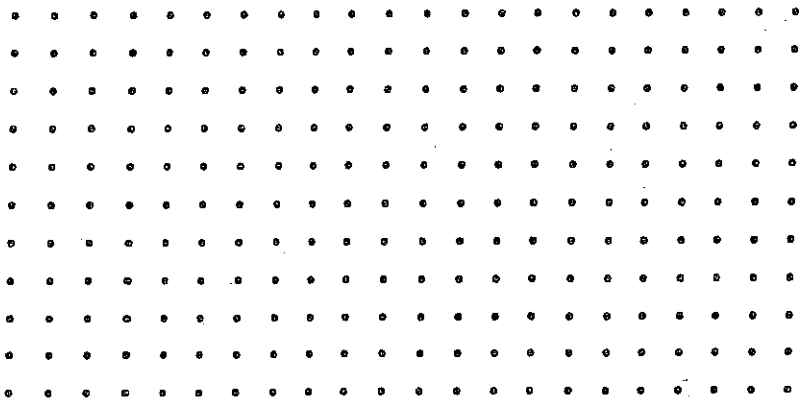
_____

B: _____

_____

6. Use this shape, or one of your own shapes, to create a tessellation on square dot paper. Identify the transformations you used.





_____

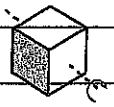
_____

_____

In Your Words

Here are some of the important mathematical words of this unit.
Build your own glossary by recording definitions and examples here. The first one is done for you.

axis of rotation *the straight line about which an object or a shape is rotated. For example, the broken line is an axis of rotation.*



isometric drawing

transformation

tessellation

composite shape

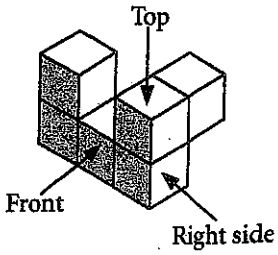
conservation of area

List other mathematical words you need to know.

Unit Review

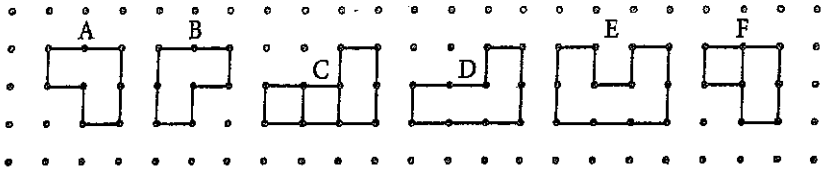
LESSON

8.1 **1.** Match each of the front, top, and side views of this object to the correct figure.

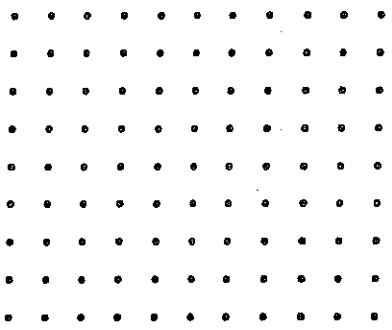


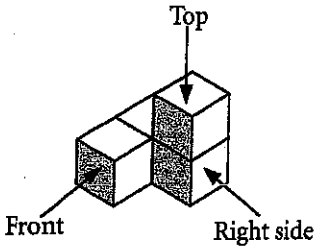
Front: _____ Top: _____

Left side: _____ Right side: _____

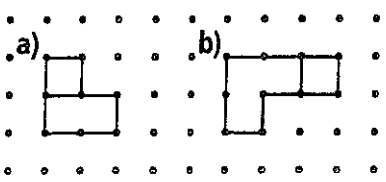


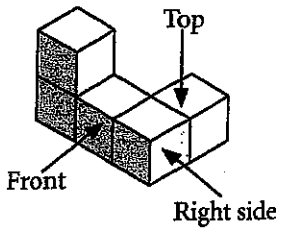
2. Sketch the front, top, and side views of this object.





8.2 **3.** This object is rotated horizontally. The two new views are shown.





Describe the rotation that produced each view.

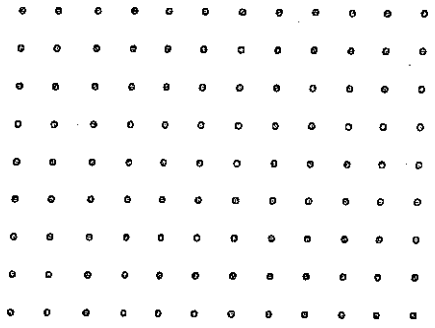
a) This is the front view after a rotation of _____

b) This is the top view after a rotation of _____

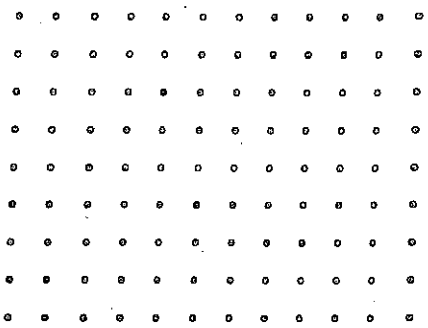
LESSON

4. Use linking cubes to build the object in question 3.
Draw the views of the object after each rotation.

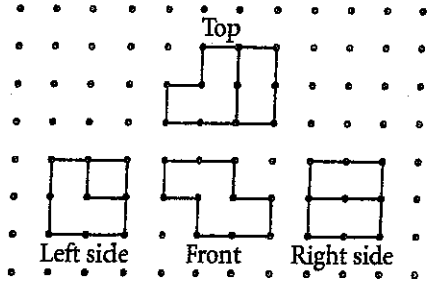
a) a vertical rotation of 90° toward you

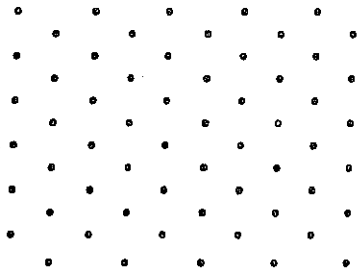


b) a vertical rotation of 180°



8.3 **5.** Use these views to build an object. How can you check that the object is correct?





8.4 **6.** Match each transformation of the shaded shape to its image.

a) a translation 1 unit right and 3 units up _____

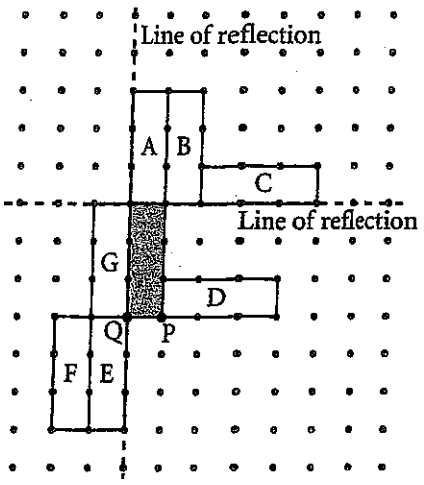
b) a translation 2 units left and 3 units down _____

c) a reflection in the vertical line _____

d) a reflection in the horizontal line _____

e) a rotation of 180° about point Q _____

f) a rotation of 90° clockwise about point P _____



LESSON

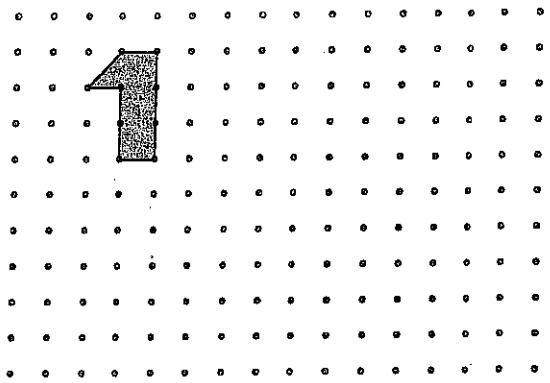
8.5 **7.** Which of these shapes tessellate? Justify your answer.

a) a triangle with angles 58°, 102°, and 20°

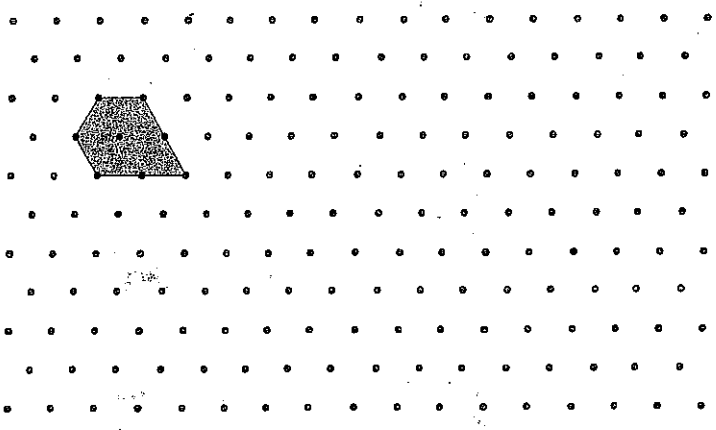
b) a square

c) a regular 12-sided polygon with each angle 150°

8. The shaded shape does not tessellate. Combine it with another shape to form a composite shape that tessellates. Show your tessellation.



8.6 **9.** a) Use the shaded shape to create a tessellation on isometric dot paper.



b) Use as many different transformations as you can, and describe the transformations you used.

