

Integers

Just for Fun

Modified Sudoku

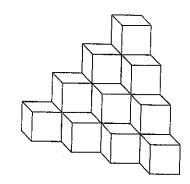
This is a modified version of a Sudoku puzzle, which originated in Japan.

Complete this grid so that every row, column, and 2×3 box contains every digit from 1 to 6.

	1	6			
5					
1		5	4		
6		3	2		5
					4
			1	3	

Cube Count

How many cubes are in this figure? Look for a pattern to find the answer.



There are _____ cubes

Add 'Em Up

Find the value of this expression without using a calculator. Explain your work.

$$1-2+3-4+5-6+\ldots+99-100=$$

Activating Prior Knowledge

Using Models to Add Integers

>> You can use coloured tiles to model integers.

A black tile models –1.

A white tile models +1.

-1

+1

A black tile and a white tile combine to model 0.

They form a zero pair: (+1) + (-1) = 0

+1

-

To add: (-4) + (-2)

Model -4 with 4 black tiles.

Model -2 with 2 black tiles.



-2:

There are 6 black tiles altogether.

They model -6.

So, (-4) + (-2) = -6

To add: (+5) + (-2)

Model +5 with 5 white tiles.

Model -2 with 2 black tiles.

Circle zero pairs.

3 white tiles remain.

They model +3.

So,
$$(+5) + (-2) = +3$$

> You can also use a number line to add integers.

Find the first integer on the number line.

- To add a positive integer, move right on the number line.
- To add a negative integer, move left on the number line.

To add: (-4) + (+2)

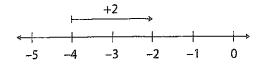
Start at -4.

Move 2 units right to add +2.

To add: (+5) + (-4)

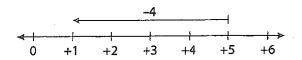
Start at +5.

Move 4 units left to add -4.



The arrow ends at -2.

So,
$$(-4) + (+2) = -2$$



The arrow ends at +1.

So,
$$(+5) + (-4) = +1$$



1. Use tiles to add.

a)
$$(+1) + (+3) =$$

b)
$$(-2) + (-3) =$$

2. Use a number line to add.

a)
$$(+2) + (-2) =$$

b)
$$(+11) + (-5) =$$

a)
$$(+2) + (-2) =$$
 b) $(+11) + (-5) =$ c) $(+9) + (+7) =$

d)
$$(-2) + (-9) =$$

e)
$$(-12) + (+7) =$$

d)
$$(-2) + (-9) =$$
 e) $(-12) + (+7) =$ f) $(+7) + (-15) =$

Using Models to Subtract Integers

To subtract, you take away tiles.

If there are not enough tiles to remove, add zero pairs.

To subtract: (-3) - (+2)

Model -3 with 3 black tiles.

To take away +2, 2 white tiles are needed.

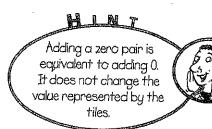
Add 2 zero pairs of tiles to provide 2 white tiles.





5 black tiles remain. They model -5.

So,
$$(-3) - (+2) = -5$$



> To subtract an integer on a number line, move in the opposite direction of adding the same integer.

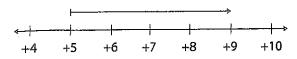
To subtract a positive integer, move left.

To subtract a negative integer, move right.

To subtract: (+5) - (-4)

Start at +5.

Move 4 units right to subtract -4.



The arrow ends at +9.

So,
$$(+5) - (-4) = +9$$



3. Subtract using a model of your choice

a)
$$(+2) - (-7) =$$

a)
$$(+2) - (-7) =$$
 b) $(-3) - (-4) =$ c) $(-5) - (-5) =$

e)
$$(-5) - (+6) =$$

d)
$$(+10) - (-4) =$$
 e) $(-5) - (+6) =$ f) $(-3) - (-5) =$

4. Match each description with the correct subtraction expression and answer.

Temperature Change	Expression	Answer
From 8°C to 3°C	(-3) - (-8)	-11
From 8°C to -3°C	(-3) - (+8)	-5
From –8°C to 3°C	(+3) - (+8)	+5
From -8°C to -3°C	(+3) - (-8)	+11

Quick Review



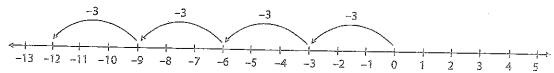
> You can think of multiplication as repeated addition.

 $4 \times (-3)$ is the same as adding -3 four times.

As a sum:
$$(-3) + (-3) + (-3) + (-3) = -12$$

As a product:
$$4 \times (-3) = -12$$

On a number line:



➤ You can use tiles to multiply integers.

Let a circle represent the bank. The bank has zero value at the start.

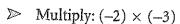
Multiply:
$$(+2) \times (-3)$$

- +2 is a positive integer.
- -3 is modelled with 3 black tiles.

So, put 2 sets of 3 black tiles into the circle.

The 6 black tiles in the circle represent -6.

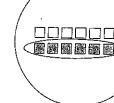
So,
$$(+2) \times (-3) = -6$$



- -2 is a negative integer.
- -3 is modelled with 3 black tiles.

So, we need to take 2 sets of 3 black tiles from the circle.

Add zero pairs until there are enough black tiles to remove. Take out 2 sets of 3 black tiles.



There now are 6 white tiles left in the circle.

So,
$$(-2) \times (-3) = 6$$

Practice

1. Write a multiplication expression for each repeated addition.

a)
$$(-2) + (-2) + (-2) + (-2) + (-2) = 5 \times$$

$$b) (+11) + (+11) + (+11) = \underline{\hspace{1cm}}$$

c)
$$(-5) + (-5) + (-5) =$$

2. Write each multiplication expression as a repeated addition. Then use a number line to find each sum.

a)
$$(+2) \times (-4) = (-4) + (-4)$$

=____

c)
$$(-3) \times (+2) = (+2) \times (-3)$$

3. Write a multiplication equation for each model. Find the product.

a) Deposit 3 sets of 2 black tiles.

$$3 \times (-2) =$$

b) Deposit 5 sets of 2 white tiles.

c) Withdraw 2 sets of 3 black tiles.

	,		
Х			

d) Withdraw 9 sets of 2 black tiles.

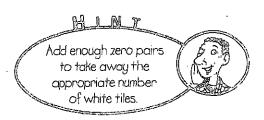
e)	Deposit 4	sets	of 3	black	tiles.

4. Use a tile model to find each product.

a)
$$(+7) \times (-2) =$$

c)
$$(+2) \times (-3) =$$

d)
$$(-4) \times (+5) =$$



5. Use a model to represent each product. Draw the model you used each time.

a)
$$(-3) \times (-4) =$$

(b)
$$(+2) \times (-5) =$$

d)
$$(-3) \times (+6) =$$

6. The temperature dropped 2°C each hour for 4 h. Use integers to find the total change in temperature.

Quick Review



- Integers have these properties of whole numbers.
 - Multiplying by 0: $4 \times 0 = 0$ and $0 \times 4 = 0$ So, $(-4) \times 0 = 0$ and $0 \times (-4) = 0$
 - Multiplying by 1: $4 \times 1 = 4$ and $1 \times 4 = 4$ So, $(-4) \times (+1) = -4$ and $(+1) \times (-4) = -4$
 - Commutative Property: $4 \times 2 = 8$ and $2 \times 4 = 8$ So, $(-4) \times (+2) = -8$ and $(+2) \times (-4) = -8$
 - Distributive Property: $4 \times (2 + 3) = 4 \times 2 + 4 \times 3 = 20$ So, $(-4) \times [(+2) + (+3)] = (-4) \times (+2) + (-4) \times (+3) = -20$
- You can write the product of integers without the use of the \times sign. $(-4) \times (+2)$ can simply be written as: (-4)(+2)
- When 2 integers with the same sign are multiplied, their product is positive. (-2)(-3) = +6(+2)(+3) = +6

When 2 integers with different signs are multiplied, their product is negative.

$$(+2)(-3) = -6$$

$$(-2)(+3) = -6$$

Find a pattern rule for each multiplication pattern. Extend the pattern for 3 more rows.

a)
$$(+3)(+3) = +9$$

b)
$$(-3)(+3) = -9$$

$$(+2)(+3) = +6$$

$$(-3)(+2) = -6$$

$$(+1)(+3) = +3$$

$$(-3)(+1) = -3$$

$$(0)(+3) = \underline{\hspace{1cm}} (-3)(0) = \underline{\hspace{1cm}}$$

$$(-3)(0) =$$

To find a pattern rule, look for a pattern in the integer factors and in the products.

2. In this chart, write the sign of each product of multiplying 2 integers.

×	positive integer	negative integer
positive integer		
negative integer		

- When 2 integer factors have the same sign, their product is ______.
- When 2 integer factors have different signs, their product is ______.

3. Find each product.

a)
$$(+7)(-2) =$$
 b) $(-4)(-3) =$ **c**

b)
$$(-4)(-3) =$$

c)
$$(-8)(+9) =$$

d)
$$(+10)(-5) =$$

e)
$$(+5)(-7) =$$

e)
$$(+5)(-7) =$$
 _____ f) $(-9)(-4) =$ _____

i)
$$(-7)(-1) =$$

$$\mathbf{j}$$
) $(+5)(0) =$ _____

k)
$$(+20)(-20) =$$

4. Fill in the blank to make each equation true.

b) _____
$$\times$$
 (-9) = +99

b)
$$\times$$
 (-9) = +99 **c)** (-10) \times = -320

d) _____
$$\times$$
 (-5) = +20

d) _____
$$\times$$
 (-5) = +20 e) (+7) \times ____ = -49 f) ____ \times (+13) = -65

g)
$$\times (-15) = -180$$

g)
$$\times (-15) = -180$$
 h) $(+14) \times = -140$ i) $\times (-7) = 56$

$$\times$$
 (-7) = 56

5. Match each pattern rule with the corresponding pattern. Complete each pattern and pattern rule.

Number Pattern

$$-3, +9, -27, +81, \dots$$

$$-1, -2, -4, -8, -16, \dots$$

Pattern Rule

Start at 2. Multiply by ______ each time.

Start at 1. Multiply by -10 each time.

Start at _____. Multiply by -3 each time.

Start at 3. Multiply by -1 each time.

Start at -1. Multiply by _____ each time.

2.3

Using Models to Divide Integers

Quick Review



Division is the inverse of multiplication.

So,
$$10 \div 5 = ?$$
 is the same as ? $\times 5 = 10$.

The product means, "how many sets of 5 produce 10?"

You can "walk" a number line to model the division of two integers.

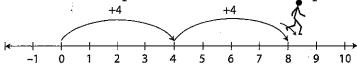
If the step size is positive, walk forward. If the step size is negative, walk backward.

The number of steps is the quotient and the direction you are facing at the end determines its sign.

➤ Positive ÷ Positive

Divide: $(+8) \div (+4)$

Start at 0. Take steps of size 4 forward to end up at +8.

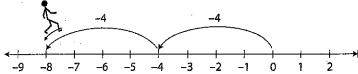


You took 2 steps and are facing the positive end of the line. So, $(+8) \div (+4) = +2$

➤ Negative ÷ Negative:

Divide: $(-8) \div (-4)$

Start at 0. Take steps of size 4 backward to end up at -8.

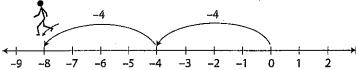


You took 2 steps and are facing the positive end of the line. So, $(-8) \div (-4) = +2$.

➤ Negative ÷ Positive:

Divide: $(-8) \div (+4)$

Start at 0. Take steps of size 4 forward to end up at -8.



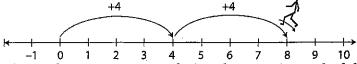
You took 2 steps and are facing the negative end of the line. So, $(-8) \div (+4) = -2$.

➤ Positive ÷ Negative:

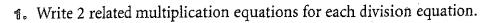
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Divide: $(+8) \div (-4)$

Start at 0. Take steps of size 4 backward to end up at +8.



You took 2 steps. You are facing the negative end of the line. So, $(-8) \div (+4) = -2$.



b)
$$(+36) \div (-9) = -4$$

c)
$$(-45) \div (-9) = +5$$

- 2. Suzanne wanted to model division using a number line. She started at zero and took steps backward of size 3. She ended up at -21.
 - a) Illustrate this problem using a number line.

3. Use a number line. Find each quotient.

a)
$$(+24) \div (-8) =$$

b)
$$(-20) \div (-5) =$$

c)
$$(+25) \div (+5) =$$

d)
$$(-18) \div (-9) =$$

Æ,	Find	each	quotient.

a)
$$(-12) \div (+4) =$$

a)
$$(-12) \div (+4) =$$
 b) $(-12) \div (-6) =$ c) $(-8) \div (+4) =$

c)
$$(-8) \div (+4) =$$

5. The water level in a well dropped 4 cm each hour. The total drop in the water level was 28 cm. Use an integer model to find out how long it took for the water level to change.

6. Use coloured tiles, a number line, or another model to clearly show your thinking. Find each quotient.

a)
$$(+10) \div (+2) =$$

b)
$$(-10) \div (-2) =$$

c)
$$(+10) \div (-2) =$$

d)
$$(-10) \div (+2) =$$

Compare the quotients. What do you notice?

7. The temperature dropped a total of 12°C over a 4-h period. The temperature dropped the same amount each hour. Using a model, show the hourly drop in temperature.

Quick Review



For any multiplication of 2 different factors, there are 2 related division facts: For $4 \times 3 = 12$, the related division facts are: $12 \div 3 = 4$ and $12 \div 4 = 3$

The same rules apply to the product of 2 integers.

For (-2)(+5) = -10, the related division facts are:

$$(-10) \div (-2) = +5$$
 and $(-10) \div (+5) = -2$

dividend divisor quotient

 \triangleright The quotient of 2 integers with the same sign is positive.

$$(+10) \div (+2) = +5$$

$$(-10) \div (-2) = +5$$

> The quotient of 2 integers with different signs is negative.

$$(+10) \div (-2) = -5$$

$$(-10) \div (+2) = -5$$

A division expression can be written using a division sign, $(-24) \div (-6)$, or it can be written as a fraction, $\frac{(-24)}{(-6)}$.

1. For each product, complete the 2 related division facts and name the sign of the quotient.

Multiplication Fact	Related Division Facts	Sign of Quotient
(+2)(+3) = +6	(+6) ÷ (+2) =	
•	(+6) ÷ (+3) =	
(-2)(-3) = +6	$(+6) \div (-2) = $	
	$(+6) \div (-3) = $	
(+2)(-3) = -6	(-6) ÷ (+2) =	
	(-6) ÷ (-3) =	
(-2)(+3) = -6	(-6) ÷ (-2) =	
·.	(-6) ÷ (+3) =	

2. Use your results in question 1. Complete these 2 statements.

When 2 integers have the same sign, their quotient is ______.

When 2 integers have different signs, their quotient is _____

3. Find a pattern rule for each division pattern. Extend the pattern 3 more rows.



b)
$$(-12) \div (-4) = +3$$

$$(+4) \div (-2) = -2$$

$$(-8) \div (-4) = +2$$

$$(+2) \div (-2) = -1$$

$$(-4) \div (-4) = +1$$

$$(0) \div (-2) =$$

$$(0) \div (-4) =$$

Use the last 3 rows of each pattern. Complete these statements.

When both the dividend and divisor are negative, the quotient is _____.

When the dividend is positive and the divisor is negative, the quotient is _____

4. Find each quotient.

a)
$$(+15) \div (-3) =$$

b)
$$(-32) \div (+4) =$$

a)
$$(+15) \div (-3) =$$
 b) $(-32) \div (+4) =$ c) $(+72) \div (-8) =$

HINT

To find a pattern rule, look for a pattern in the

dividends and in the auotients.

d)
$$(-54) \div (-9) =$$

(e)
$$(-72) \div (+6) =$$

d)
$$(-54) \div (-9) =$$
 ______ e) $(-72) \div (+6) =$ _____ f) $(+88) \div (+11) =$ _____

g)
$$(-42) \div (-6) =$$

h)
$$(+108) \div (+9) =$$

g)
$$(-42) \div (-6) =$$
 h) $(+108) \div (+9) =$ i) $(-56) \div (+7) =$

Use 2 of these 5 integers. Write a division fact with each quotient.

-1

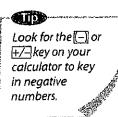
b) the greatest quotient

- d) a quotient between -5 and -10
- **6.** Use a calculator to divide.

a)
$$(+247) \div (-13) =$$
 b) $(-851) \div (-37) =$ _____

c)
$$\frac{(-748)}{(-68)} =$$

d)
$$\frac{(-1485)}{(+33)} =$$



Quick Review



- The order of operations with whole numbers also applies to integers.
 - ① Perform operations in brackets first.
 - ② Divide and multiply, in order, from left to right.
 - 3 Add and subtract, in order, from left to right.

	$(1+2)-3\times4$
=	$=3-3\times4$
=	= 3 - 12
_	- 0

The letters BDMAS can help you remember the order of operations. B-Brackets DM-Divide, Multiply AS-Add, Subtract

A fraction bar indicates division.

It also acts like brackets.

Evaluate the numerator and denominator separately before dividing.

For example, $\frac{12+8}{2-6} = \frac{20}{-4} = -5$

If an integer does not have a sign, it is assumed to be positive: 2 = +2

① B 2 DM 3 AS

1. Simplify.

a)
$$5 - 2 - 6$$

b)
$$3(8-12)$$

c)
$$-4 + 2 \times 3$$

e)
$$10 - [(5-3) + 9]$$

d)
$$21 \div (-7) \times 5$$

f)
$$-8 + 15 \div (-3) + 7$$

f)
$$-8 + 15 \div (-3) + 7$$
 g) $(-3)(-8) + 24 \div (-2)$

Brackets symbolize multiplication as

well as grouping. 3(8 - 12) means

 $3 \times (8 - 12)$.

2. Match each expression with its answer.

Answer

$$30 \div (5-10) \times 2$$

-14

$$30 \div (5 - 10 \times 2)$$

-12

$$(30 \div 5 - 10) \times 2$$

-8

$$30 \div 5 - 10 \times 2$$

-2

3. Simplify.

a)
$$\frac{3(5-9)}{2}$$

b) $\frac{(-4)(-2)}{-8}$

c)
$$\frac{(-6)(4)+8}{(-6)(4)+8}$$

- $=\frac{3(}{2}$
- = ____

= __

4. Evaluate each expression. Write the letter for the answer in the corresponding blank at the bottom to find out what one wall said to the other.

2(-7 + 3)		-8 + 12 ÷ 4		$3(10 \div 2) - (-4)$	
= 2		= -8 +		= 3	
=	A	=	С	=	E
$(-6)(-6) \div (-4)$		$4\times(-3)+24\div 2$		$-5 + 12 \div 4 \times (-2)$	
	Н		М		N
$19 - 3 \times 4 \div (-6)$		$\frac{6(-8)}{-12} - 1$	· · · · · · · · · · · · · · · · · · ·	$\frac{10-2(-3)}{2\times 4}$	
	0		, D	Ì	T
	U		R		1

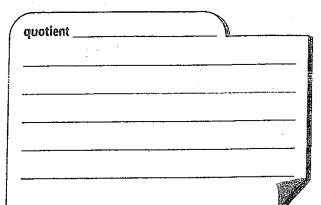
In Your Words

Here are some of the important mathematical words of this unit.

Build your own glossary by recording definitions and examples here. The first one is done for you.

integer the numbers -3, -2,
-1, 0, 1, 2, 3, ...

For example, 1, 2, 3, ... are positive
integers and -1, -2, -3, ... are negative
integers. 0 is neither positive nor
negative.



zero pair

commutative property	

zero property

order of operations _____

List other mathematical words you need to know.

Unit Review

LESSON

Write each multiplication as a repeated addition. Then illustrate using coloured tiles to find each sum.

a) $(+5) \times (-2) =$

=

b) (+3) × (+5) = _____

when the same of t

c) (+3) × (-3) =

=____

d) $(-4) \times (+2) = (+2) \times \underline{\hspace{1cm}}$

=

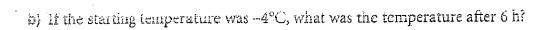
2. Use a number line. Find each product.

a) $(+5) \times (-1) =$

b) $(+3) \times (+4) =$ _____

c) $(-2) \times (+6) =$

d) $(+4) \times (-5) =$



- 4. Show how to model $(-2) \times (-5)$. Explain why you chose that model.
- Complete each statement using positive, negative, or zero.
 - a) The product of a positive integer and a negative integer is _____
 - b) The product of a negative integer and zero is ______
 - c) The product of an two negative integers is ______
- **6.** Find each product.

a)
$$(+2)(+3) =$$

d)
$$(+24)(-30) =$$

e)
$$(-36)(-5) =$$

h)
$$(-237)(0) =$$

7. Fill in the blank to make each equation true.

a)
$$(-6) \times \underline{\hspace{1cm}} = -24$$

a)
$$(-6) \times \underline{\hspace{1cm}} = -24$$
 b) $(-9) \times \underline{\hspace{1cm}} = +27$

c)
$$\times (-3) = (-21)$$
 d) $(-4) \times \underline{\hspace{1cm}} = +24$

d)
$$(-4) \times \underline{\hspace{1cm}} = +24$$

2:3 8, Write a related multiplication equation for each division equation.

a)
$$(+100) \div (-25) = -4$$

b)
$$(-28) \div (-7) = +4$$

c)
$$\frac{(-15)}{(-5)} = +3$$

d)
$$\frac{(+48)}{(+12)} = +4$$

9. Show how to model $(-12) \div 4$.

10. Decide whether each quotient will be positive, negative, or zero. Then evaluate each quotient.

a)
$$(-25) \div (-5)$$
 _____ c) $\frac{(+42)}{(-7)}$ _____

d)
$$0 \div (-5)$$

11. Evaluate each quotient and order the results from least to greatest.

a)
$$(-20) \div (+4) =$$

a)
$$(-20) \div (+4) =$$
 b) $(-18) \div (-6) =$ c) $(+48) \div (-8) =$

c)
$$(+48) \div (-8) =$$

The quotients from least to greatest are:

12. Find all of the divisors of -16. Write a division equation each time. The first one has been done for you.

Divisor	Division Equation
-1	$(-16) \div (-1) = +16$
w	

13. Write the next 3 terms in each pattern. Then write the pattern rule.

a) +1, -4, +16, -64, _____, ____, ...

Pattern rule: Start at _____ each time.

b) -128, +64, -32, 16, _____, ____, ...

Pattern rule: Start at _____ each time.

c) -3125, +625, -125, ____, ___, ...

Pattern rule: Start at _____ each time.

14. State which operation you would do first. Do not evaluate.

- a) $(+8) + (-2) \times (-3)$
- b) $(-20) \div (-4) (-2)$

c) (-2)(4-5)

d) $5-3+(-4)\times(-2)$

15. Evaluate each expression in question 14. Show all your steps.

- a)
- b)
- c)
- d)

AESSON

16. Evaluate using the order of operations.

a)
$$17 - 4 \times 4 =$$

h)
$$-48 \div 4 - 2(3 - 4) =$$

c)
$$-2 - 4 \times 9 =$$

d)
$$\frac{(-6)(8-2)}{-4}$$
 =

e)
$$(-3) \times (-3) + (-4) \times (-4) =$$

f)
$$\frac{21 + 2(3)}{(-3) \times (-3)} =$$