UNIT 1

Square Roots and the Pythagorean Theorem

Just for Fun

What Do You Notice?

Follow the steps. An example is given.

	Example
1. Pick a 4-digit number with different digits.	3078
2. Find the greatest number that can be made with these digits.	8730
3. Find the least number that can be made with these digits.	0378
4. Subtract the least from the greatest.	8730 – 0378 = 8352
5. Repeat steps 2, 3, and 4 with the result.	8532 – 2358 = 6174
6. Continue to repeat steps 2, 3, and 4 until you notice something interesting.	7641 – 1476 = 6174

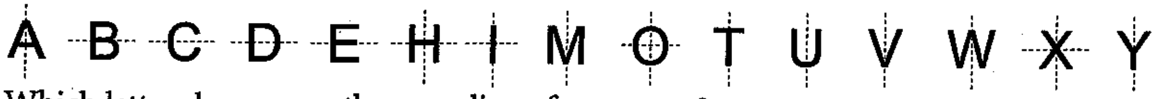
What do you notice?
The final result is 6174.

Try these steps with the number 2395. What do you notice? Pick any 4-digit number. What do you notice?
The final result is 6174, no matter what the original number.

Letter Symmetry

A letter has mirror symmetry if a straight line can be drawn through the letter so that one half of the letter is a mirror image of the other half. The straight lines can be vertical, horizontal, or slanted.
For example, the letter A has mirror symmetry, but the letter F does not.

Which letters have mirror symmetry?



Which letters have more than one line of symmetry?

HIOX

Activating Prior Knowledge

Areas of Rectangles and Triangles

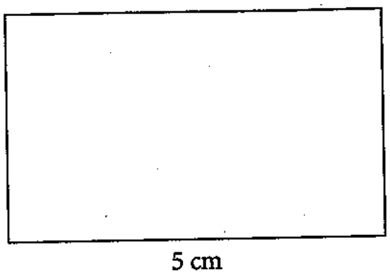
Area is the amount of surface a figure covers. It is measured in square units.

- To find the area of a rectangle, use the formula $A = bh$, where b is the base length and h the height of the rectangle.
- To find the area of a triangle use the formula $A = \frac{1}{2}bh$, where b is the base length and h is the height of the triangle.

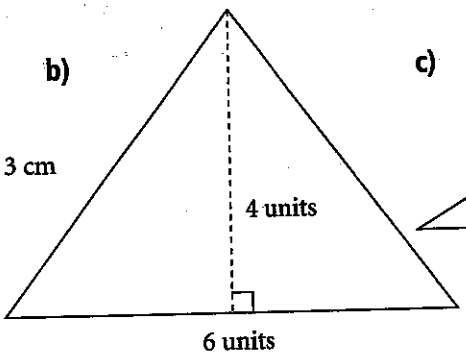
Example 1

Find the area of each figure.

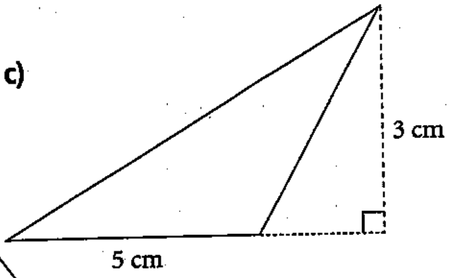
a)



b)



c)



Solution

a) The figure is a rectangle with base 5 cm and height 3 cm.
Substitute $b = 5$ cm and $h = 3$ cm into $A = bh$.

$A = 5 \text{ cm} \times 3 \text{ cm}$
$\quad = 15 \text{ cm}^2$

The area is 15 cm². The abbreviation cm² stands for "square centimetres."

b) The figure is a triangle with base 6 units and height 4 units.
Substitute $b = 6$ units and $h = 4$ units into $A = \frac{1}{2}bh$.

$A = \frac{1}{2}(6 \text{ units} \times 4 \text{ units})$
$\quad = 12$ square units

The area is 12 square units.

c) The figure is a triangle with base 5 cm and height 3 cm.
Substitute $b = 5$ cm and $h = 3$ cm into $A = \frac{1}{2}bh$.

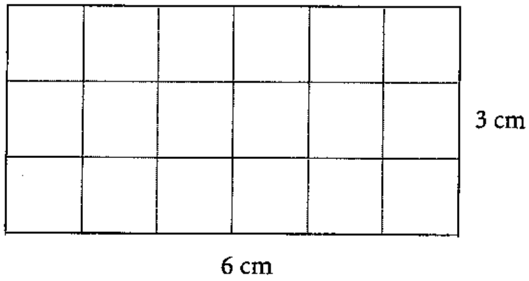
$A = \frac{1}{2}(5 \text{ cm} \times 3 \text{ cm})$
$\quad = \frac{1}{2}(15 \text{ cm}^2)$
$\quad = 7.5 \text{ cm}^2$

The area is 7.5 cm².

✓ Check

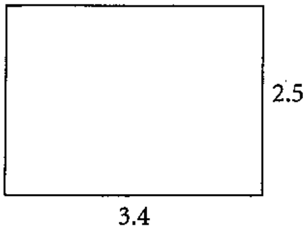
1. Find the area of each figure.

a)



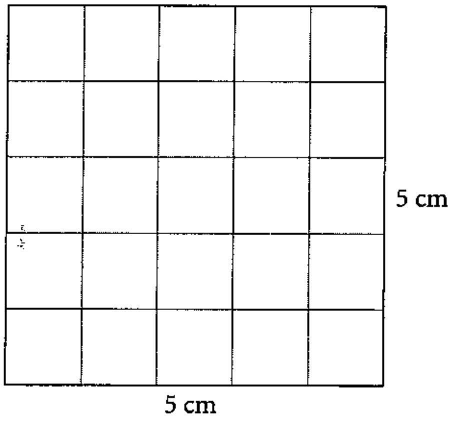
The area is 6 cm × 3 cm = **18 cm^2**

b)



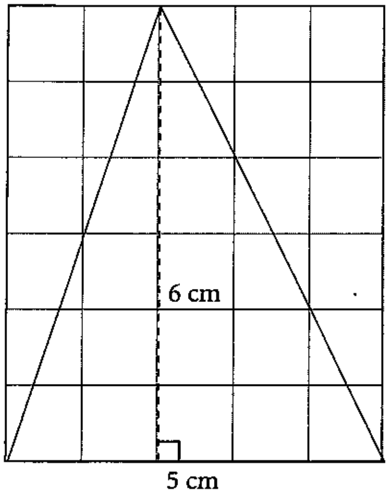
The area is **8.5 square units**

c)



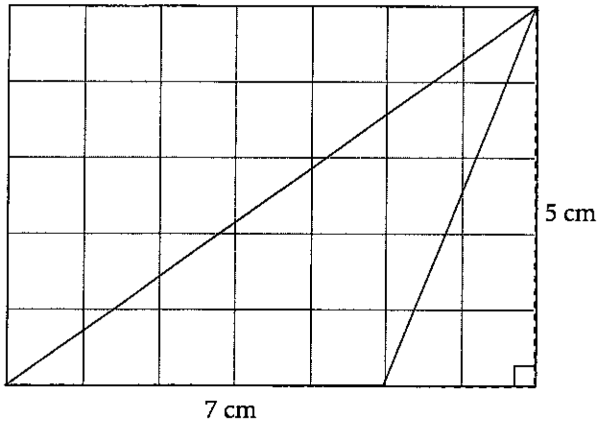
The area is **25 cm^2**

d)



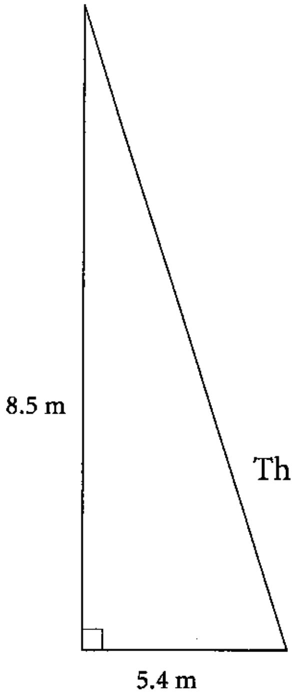
The area is $\frac{1}{2}$(5 cm × 6 cm) = **15 cm^2**

e)



The area is **17.5 cm^2**

f)

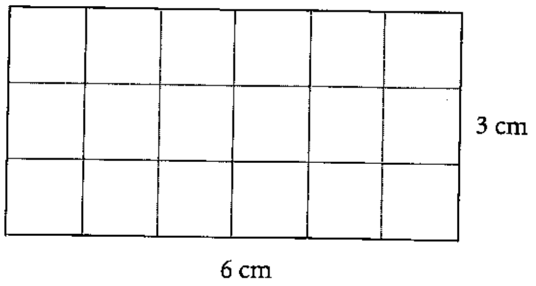


The area is **22.95 m^2**

Check

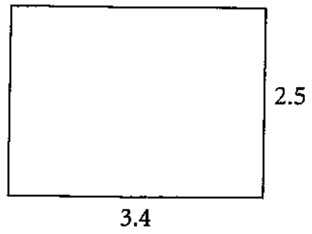
1. Find the area of each figure.

a)



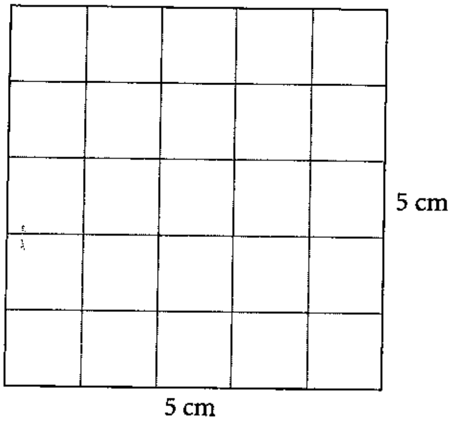
The area is <u>6</u> cm × <u>3</u> cm = **18 cm^2**

b)



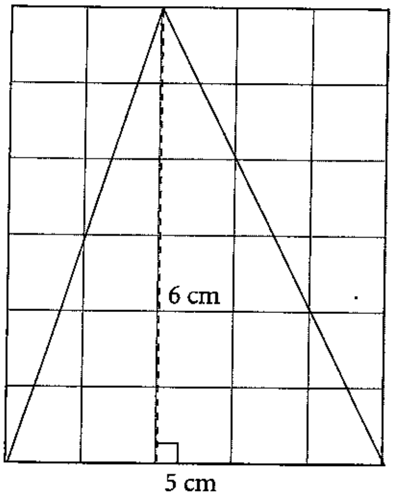
The area is **8.5 square units**

c)



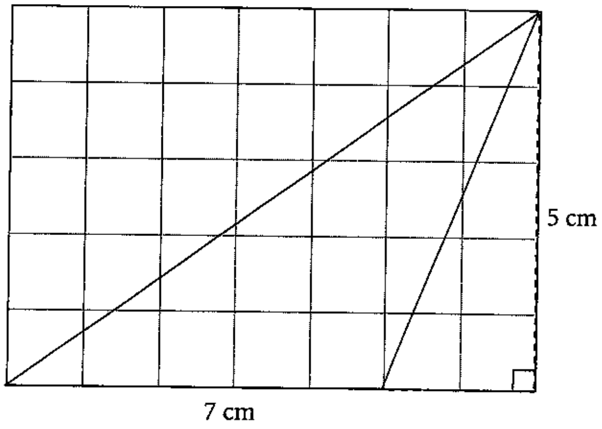
The area is **25 cm^2**

d)



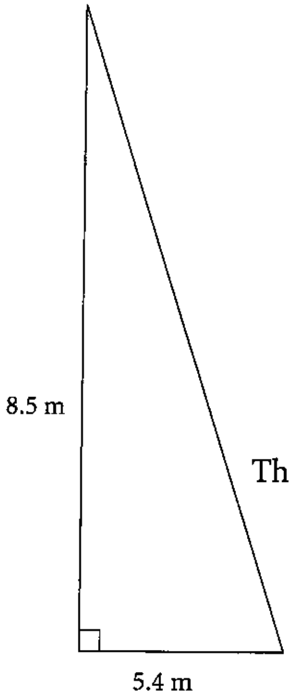
The area is $\frac{1}{2}$(<u>5</u> cm × <u>6</u> cm) = **15 cm^2**

e)



The area is **17.5 cm^2**

f)



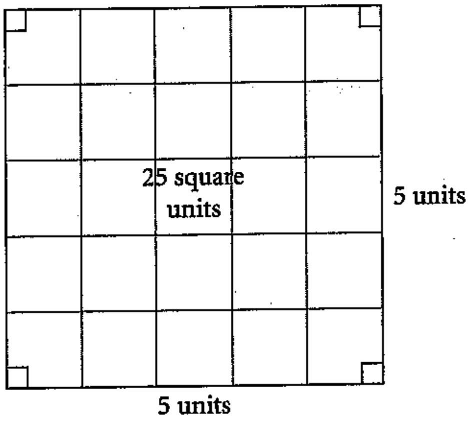
The area is **22.95 m^2**

1.1 Square Numbers and Area Models

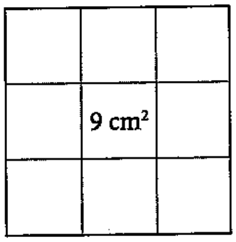
Quick Review

- When you multiply a number by itself, you *square* the number.
  The square of 5 is $5 \times 5 = 25$
  We write: $5^2 = 5 \times 5 = 25$
  We say: Five squared is twenty five.
  25 is a **square number**, or a **perfect square.**

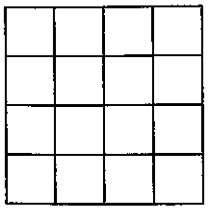
- You can model a square number by drawing a square whose area is equal to the square number.
  You can model 25 using a square with side length 5 units.



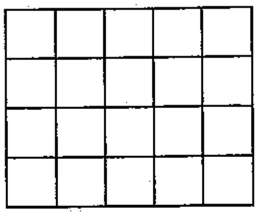
Find the perimeter of a square with area 9 cm^2.
First, find the side length of the square.
Since $3 \times 3 = 9$, the side length is 3 cm. So, the perimeter is 3 cm + 3 cm + 3 cm + 3 cm = 12 cm



16 is a perfect square because you can create a square with area 16 square units using square tiles.

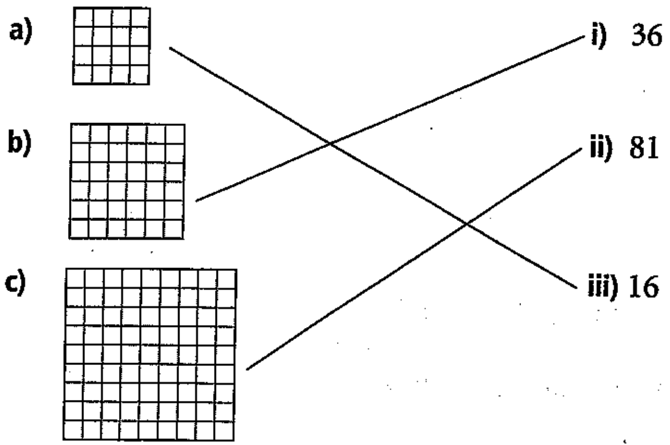


20 is not a perfect square because you cannot create a square with area 20 square units using square tiles. The 4×5 rectangle is the closest to a square that you can get.



Practice

1. Match each diagram to the correct square number.



2. Complete the statement for each square number.

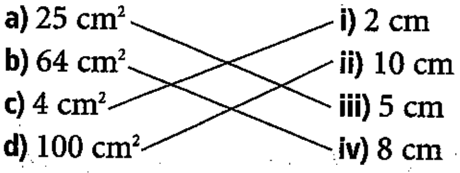
a) 64 is a square number because 64 = **8** × **8** = **8^2**

b) 49 is a square number because 49 = **7** × **7** = **7^2**

3. Complete the table. The first row has been done for you.

a)	4^2	4 × 4	16
b)	3^2	**3** × **3**	**9**
c)	**7** ^{2}	7 × 7	**49**
d)	11^2	**11** × **11**	**121**

4. Match the area of the square with the correct side length.

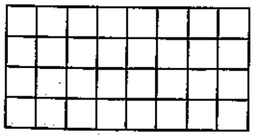


a) 25 cm²
b) 64 cm²
c) 4 cm²
d) 100 cm²

i) 2 cm
ii) 10 cm
iii) 5 cm
iv) 8 cm

5. Use square tiles to decide whether 32 is a square number.

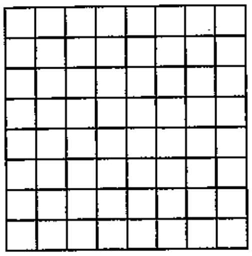
Diagrams may vary.



No, 32 is not a square number.

6. Use graph paper to decide whether 64 is a square number.

Diagrams may vary.



Yes, 64 is a square number.

7. Which of the numbers are perfect squares? How do you know?

a) 81 81 is a perfect square because 81 = 9 × 9 = 9^2

b) 18 18 is **not** a perfect square because I **cannot** draw a square with area 18 square units on grid paper.

c) 20 **20 is not a perfect square because I cannot draw a square with area 20 square units on grid paper.**

d) 25 **25 is a perfect square because 25 = 5 × 5 = 5^2**

8. Find the side length of the square with each area. Give the unit.

a) 49 cm^2 7 × 7 = 49, so the length of the side is 7 cm.

b) 900 mm^2 **30 mm**

c) 121 cm^2 **11 cm**

d) 169 m^2 **13 m**

9. Find the perimeter of each square.

a) side length 6 cm Perimeter = 6 cm + 6 cm + 6 cm + 6 cm = 24 cm

b) area 25 m^2 Side length is 5 m, because 5 × 5 = 25. So,

Perimeter = **5 m + 5 m + 5 m + 5 m** = **20 m**

c) area 144 m^2 **48 m**

10. If you multiply a perfect square by a different perfect square, is the answer also a perfect square? Give examples to explain your answer.

Sample Answer:

Yes. $5^2 \times 2^2 = 25 \times 4 = 100 = 10^2$

1.2 Squares and Square Roots

Quick Review

- When a number is multiplied by itself, the result is a square number.
  For example, 9 is a square number because $3 \times 3 = 9$.

- A number is a square number if it has an *odd* number of factors.
  For example, to check if 36 is a square number, first create a list of the factors of 36 in pairs as shown:

  1×36
  2×18
  3×12
  4×9
  6×6

  Write these factors in ascending order, starting at 1:

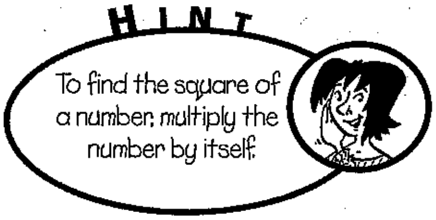
  1, 2, 3, 4, ⑥, 9, 12, 18, 36

  There are nine factors of 36. This is an odd number, so 36 is a square number.

  Tip
  A number with an even number of factors is not a square number.

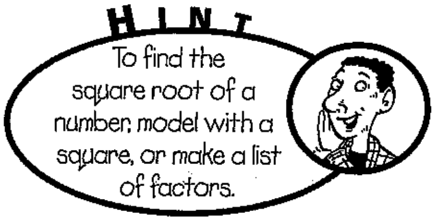
  In the ordered list of factors, notice that 6 is the middle number, and that $6 \times 6 = 36$.
  6 is called the **square root** of 36.
  We write the square root of 36 as $\sqrt{36}$

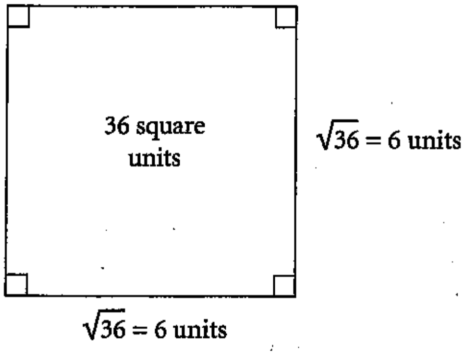
- Squaring and taking the square root are inverse operations.
  So, $\sqrt{36} = 6$ because $6^2 = 6 \times 6 = 36$.
  This means $\sqrt{6^2} = 6$



- You can find a square root using a diagram of square. The area is the square number.

- The side length of the square is the square root of the area.





Practice

1. List the factors of each number in ascending order. Which numbers are square numbers? For each of the square numbers, find the square root.

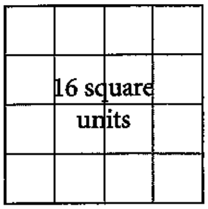
a) 196: **1, 2, 4, 7, 14, 28, 49, 98, 196** **square number with square root 14**

b) 200: **1, 2, 4, 5, 10, 20, 40, 50, 100, 200** **not a square number**

c) 441: **1, 3, 7, 9, 21, 49, 63, 147, 441** **square number with square root 21**

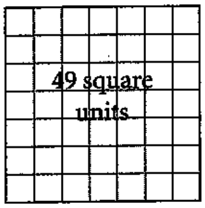
2. For each square, state the square number and the square root.

a)



square number **16**
square root **4**

b)



square number **49**
square root **7**

3. Complete the sentence for each square root. The first one has been done for you.

a) $\sqrt{25} = 5$ because $5^2 = 25$

b) $\sqrt{49} =$ **7** because **7^2** $=$ **49**

c) $\sqrt{100} =$ **10** because **10^2** $=$ **100**

d) $\sqrt{144} =$ **12** because **12^2** $=$ **144**

4. Complete each sentence. The first one has been done for you.

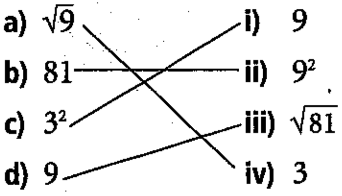
a) $\sqrt{16} = 4$ because $4^2 = 16$

b) **$\sqrt{64}$** $= 8$ because $8^2 =$ **64**

c) **$\sqrt{81}$** $= 9$ because **9^2** $=$ **81**

d) **$\sqrt{121}$** $= 11$ because **11^2** $=$ **121**

5. Match each number in column 1 to the number that is equal to it in column 2.



6. Find each square root.

a) $\sqrt{64} =$ **8** **b)** $\sqrt{400} =$ **20** **c)** $\sqrt{225} =$ **15** **d)** $\sqrt{324} =$ **18**

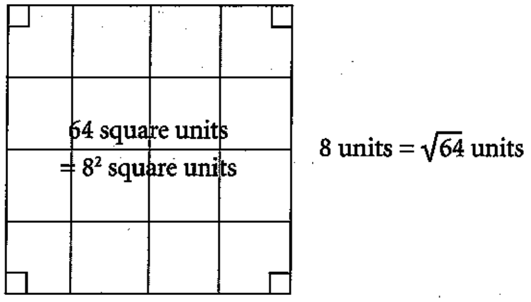
7. Find the square root of each number:

a) $5^2 =$ **5** **b)** $8^2 =$ **8** **c)** $16^2 =$ **16** **d)** $54^2 =$ **54**

8. Find the number whose square root is

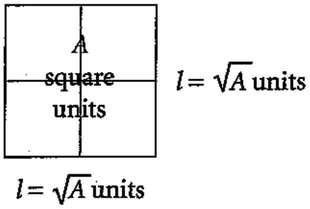
a) 17 **17^2** $=$ **289** **b)** 22 **484** **c)** 30 **900**

1.3 Measuring Line Segments



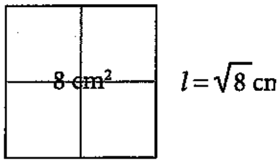
- In the square:
  - the side length is 8 units and the area is 8^2 square units
  - the area is 64 square units and the side length is $\sqrt{64}$ units

- This is true for all squares.



- In the square:
  - the side length is l units and t
    area is l^2 square units
  - the area is A square units anc
    side length is $\sqrt{A}$ units

- Squares can have areas that are not square numbers. The side length of this square is $\sqrt{8}$ cm and the area is $(\sqrt{8})^2 = 8$ cm²
  The area is 8 cm² and the side length is $\sqrt{8}$ cm.

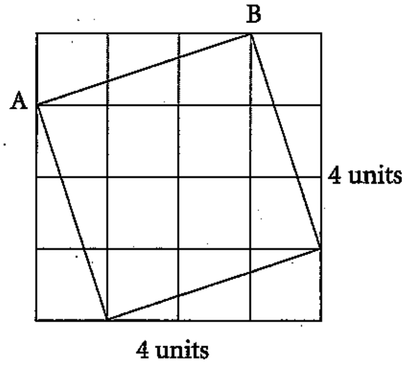


- You can find the length of a line segment AB on a grid by constructing a square on the segment. The length AB is the square root of the area of the square.

Tip

The squa
square r
number
number. For example, $(\sqrt{2})^2 = 2$. $\sqrt{8}$ is not a whole number. It is called an irrational number.

Draw an enclosing square around the square containing AB.
Then find the area of the enclosing square, and subtract the sum of the areas of the triangles.



The area of the enclosing square is 4^2 square units = 16 square units

Each triangle has area $\frac{1}{2} \times 1$ unit $\times$ 3 units = 1.5 square units

4 triangles have area 4×1.5 square units = 6 square units
The area of the square with AB as a side is
16 square units – 6 square units = 10 square units
So, the length of AB is $\sqrt{10}$ units.

HINT

Use the formulas $A = s^2$ for the area of a square and $A = \frac{1}{2}bh$ for the area of a triangle.

Practice

1. Circle the correct answer for each question.

a) $16^2 = ?$ 4 [✓ 256] **b)** $\sqrt{100} = ?$ 50 [✓ 10] **c)** $25^2 = ?$ 5 [✓ 625]

2. The area of a square is given. Find its side length. Which of the side lengths are whole numbers?

a) $A = 81$ cm², $l =$ **9 cm; whole number** **b)** $A = 30$ cm², $l =$ **$\sqrt{30}$ cm**

c) $A = 144$ mm², $l =$ **12 mm; whole number** **d)** $A = 58$ m², $l =$ **$\sqrt{58}$ m**

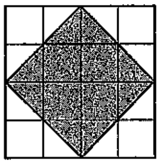
3. The side length of a square is given. Find its area.

a) $l = 7$ cm, $A =$ **49 cm²** **b)** $l = 15$ m, $A =$ **225 m²** **c)** $l = \sqrt{36}$ cm, $A =$ **36 cm²**

d) $l = \sqrt{50}$ mm, $A =$ **50 mm²** **e)** $l = \sqrt{24}$ cm, $A =$ **24 cm²** **f)** $l = \sqrt{121}$ mm, $A =$ **121 mm²**

4. Find the area of each shaded square. Then write the side length of the square.

a)



Area of large square = **16** square units

Area of each triangle = **2** square units

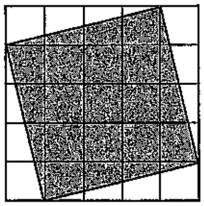
Area of shaded square = area of large square – **4** × area of each triangle

= **16 square units – 4 × 2 square units**

= **8 square units**

Side length = **$\sqrt{8}$ units**

b)



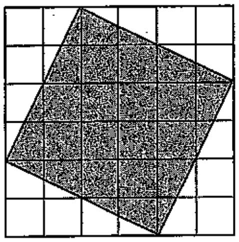
Area of square

17 square units

Side length

$\sqrt{17}$ units

c)



Area of square

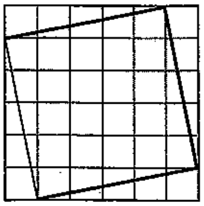
20 square units

Side length

$\sqrt{20}$ units

5. Copy each line segment and square onto grid paper. Draw a square on each line segment. Find the area of the square and the length of the line segment.

a)



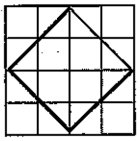
Area of square

26 square units

Length of line segment

$\sqrt{26}$ units

b)



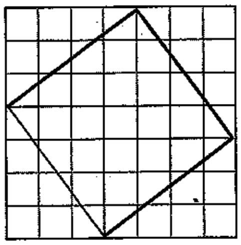
Area of square

8 square units

Length of line segment

$\sqrt{8}$ units

c)



Area of square

25 square units

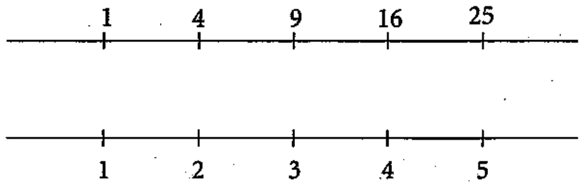
Length of line segment

5 units

1.4 Estimating Square Roots

Quick Review

➤ To estimate the square root of a number that is not a perfect square, you can use a number line.

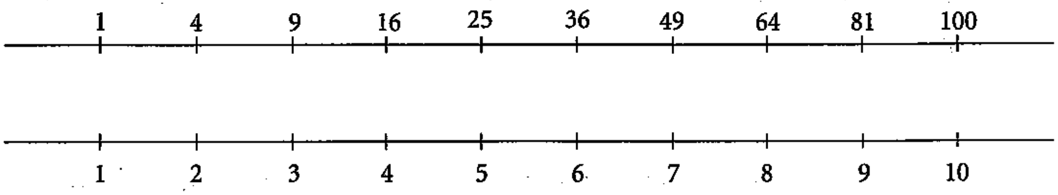


To estimate $\sqrt{10}$: Note that $\sqrt{10}$ lies between $\sqrt{9}$ and $\sqrt{16}$. So, $\sqrt{10}$ must have a value between 3 and 4, but closer to 3. Use trial and error and a calculator to get a closer approximation. Round to 2 decimal places.

Try 3.3: $3.3 \times 3.3 = 10.89$ too big
Try 3.2: $3.2 \times 3.2 = 10.24$ too big
Try 3.1: $3.1 \times 3.1 = 9.61$ too small
Try 3.16: $3.16 \times 3.16 = 9.99$ very close
$\sqrt{10}$ is approximately 3.16.

Practice

1. Use the number lines to complete each statement with whole numbers. The first one is done for you.

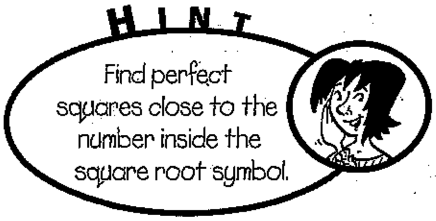


a) $\sqrt{5}$ lies between 2 and 3

b) $\sqrt{20}$ lies between 4 and 5

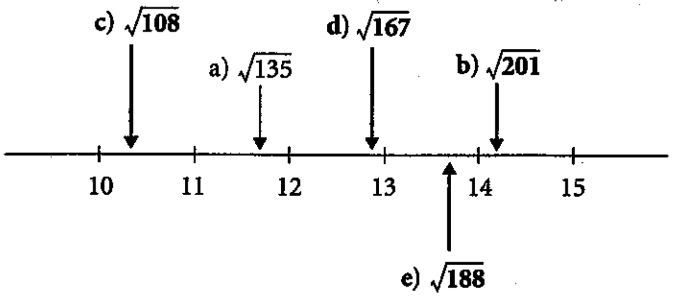
c) $\sqrt{55}$ lies between 7 and 8

d) $\sqrt{2}$ lies between 1 and 2


HINT

Find perfect squares close to the number inside the square root symbol.

2. Place the letter of the question on the number line below. The first one is done for you.



a) $\sqrt{135}$ **b)** $\sqrt{201}$ **c)** $\sqrt{108}$ **d)** $\sqrt{167}$ **e)** $\sqrt{188}$

3. Which statements are true, and which are false?

a) $\sqrt{20}$ is between 19 and 21. **False**

b) $\sqrt{20}$ is between 4 and 5. **True**

c) $\sqrt{20}$ is closer to 4 than 5. **True**

d) $\sqrt{20}$ is between $\sqrt{19}$ and $\sqrt{21}$. **True**

4. Which are good estimates of the square roots?

a) $\sqrt{19} = 4.75$ **not a good estimate**

b) $\sqrt{220} = 14.83$ **good estimate**

5. Use a calculator and the trial and error method to approximate each square root to 1 decimal place. Record each trial.

Trials may vary.

a) $\sqrt{20}$ = **4.5** **b)** $\sqrt{57}$ = **7.5** **c)** $\sqrt{115}$ = **10.7** **d)** $\sqrt{175}$ = **13.2**

6. Find the approximate side length of the square with each area.
Answer to 1 decimal place. Show your guess and checks.

a) $A = 50\text{ cm}^2$
s = **7.1 cm**

b) $A = 125\text{ cm}^2$
s = **11.2 cm**

c) $A = 18\text{ cm}^2$
s = **4.2 cm**

7. Which is the closest estimate of $\sqrt{99}$: 9.94 or [✓ 9.95] or 9.96? How did you find out?
Methods may vary.

8. What length of fencing is required to surround a square field with area 250 m²? Answer to 2 decimal places.

Side length = **$\sqrt{250\text{ m}^2}$** = **15.81 m**

Perimeter = **15.81 m** + **15.81 m** + **15.81 m** + **15.81 m** = **63.24 m**

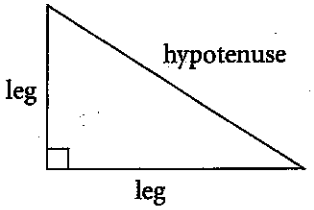
63.24 m of fencing is required.

1.5 The Pythagorean Theorem

Quick Review

At Home At School

- A right triangle has two **legs** that form the right angle. The side opposite the right angle is called the **hypotenuse.**

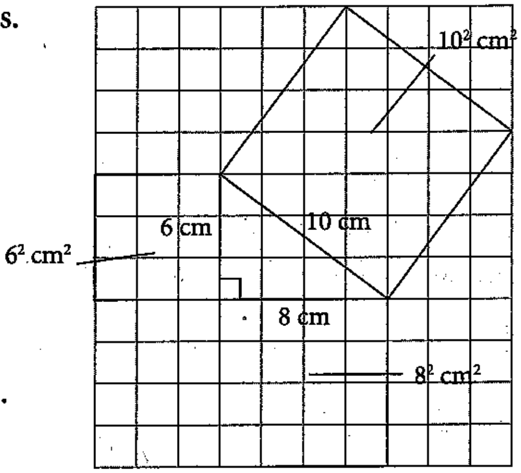


- The three sides of a right triangle form a relationship known as the **Pythagorean Theorem.**

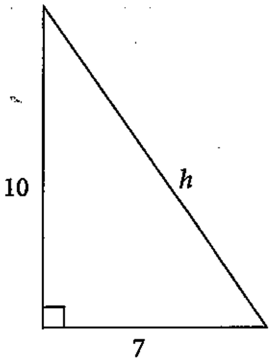
  Pythagorean Theorem: The area of the square on the hypotenuse is equal to the sum of the areas of the squares on the legs.

- In the diagram:
  Area of square on hypotenuse:
  $10^2\ \text{cm}^2 = 100\ \text{cm}^2$
  Areas of squares on legs:
  $6^2\ \text{cm}^2 + 8^2\ \text{cm}^2 = 36\ \text{cm}^2 + 64\ \text{cm}^2$
  $= 100\ \text{cm}^2$
  Notice that $10^2 = 6^2 + 8^2$.

  This theorem is true for all right triangles.

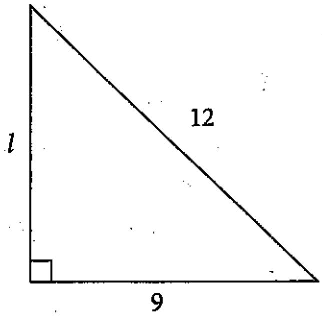


- You can use the Pythagorean Theorem to find the length of any side of a right triangle when you know the lengths of the other two sides.



To calculate the hypotenuse h, solve for h in this equation:

$h^2 = 7^2 + 10^2$
$h^2 = 49 + 100$
$h^2 = 149$
$h = \sqrt{149}$
Use a calculator: $h \doteq 12.2$



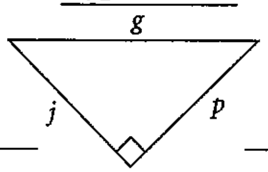
To calculate the leg with length l, solve for l in this equation:

$12^2 = l^2 + 9^2$
$144 = l^2 + 81$
$144 - 81 = l^2 + 81 - 81$
$63 = l^2$
$\sqrt{63} = l$
Use a calculator: $l \doteq 7.9$ cm

Practice

1. Identify the legs and the hypotenuse of each right triangle.

a)

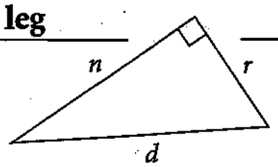


hypotenuse

leg

leg

b)



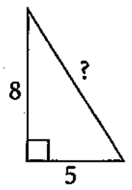
leg

leg

hypotenuse

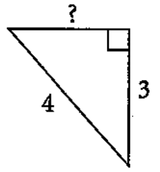
2. Circle the length of the unknown side in each right triangle.

a)



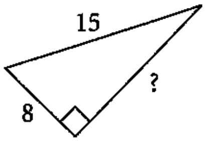
$\sqrt{13}$ [✓ $\sqrt{89}$]

b)



$\sqrt{7}$ [✓ 5]

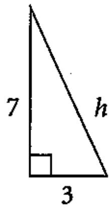
c)



[✓ 17] $\sqrt{161}$

3. Find the length of the unknown side in each right triangle. Use a calculator to approximate each length to 2 decimal places, if necessary.

a)



$h^2 = 3^2 + 7^2$

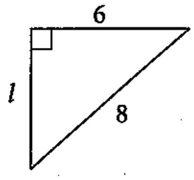
$h^2 = 9 + 49$

$h^2 = 58$

$h = \sqrt{58}$

$h \doteq 7.62$ units

b)



$8^2 = l^2 + 6^2$

$64 = l^2 + 36$

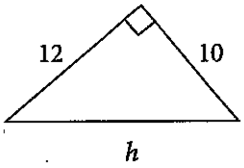
$64 - 36 = l^2 + 36 - 36$

$28 = l^2$

$\sqrt{28} = l$

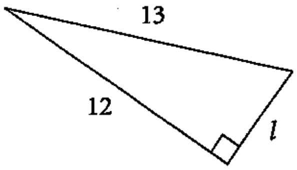
$l \doteq 5.29$ units

c)



$h \doteq 15.62$ units

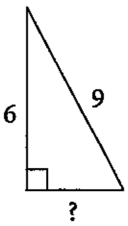
d)



$l = 5$ units

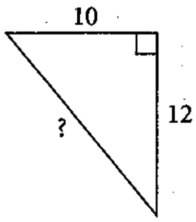
4. Find the length of the unknown side in each triangle. Use a calculator to approximate each answer to 1 decimal place.

a)



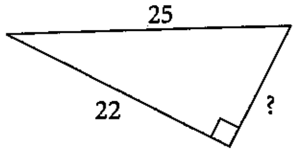
$\sqrt{45} \doteq 6.7$ units

b)



$\sqrt{244} \doteq 15.6$ units

c)

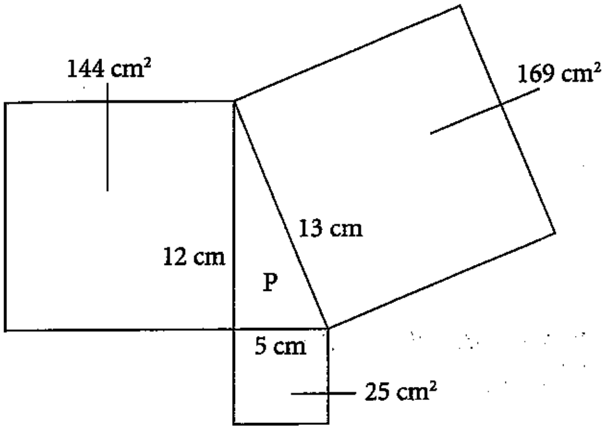


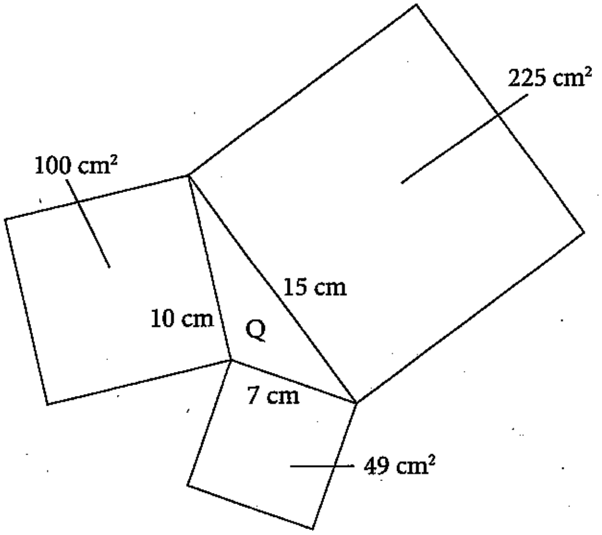
$\sqrt{141} \doteq 11.9$ units

1.6 Exploring the Pythagorean Theorem

Quick Review

- The Pythagorean Theorem is true for right triangles only.
- To see which triangle is a right triangle, check to see if the area of the square on the longest side is equal to the sum of the areas of the squares on the other two sides.

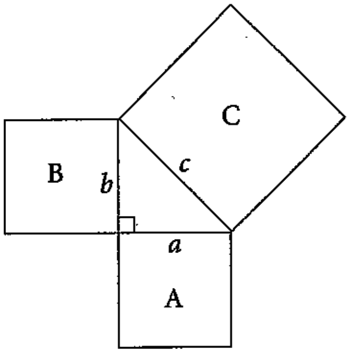




$25 \text{ cm}^2 + 144 \text{ cm}^2 = 169 \text{ cm}^2$ $\qquad$ $49 \text{ cm}^2 + 100 \text{ cm}^2 \neq 225 \text{ cm}^2$

The Pythagorean Theorem applies to triangle P, but not to triangle Q.

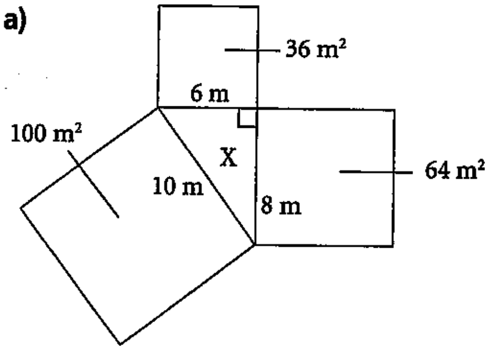
- A set of three whole numbers that satisfy the Pythagorean Theorem is called a Pythagorean triple. For example, 5-12-13 is a Pythagorean triple because $5^2 + 12^2 = 13^2$
- For a right triangle:
  area of square on the longest side (square C) = area of square A + area of square B



- For a Pythagorean triple a-b-c:
  $c^2 = a^2 + b^2$

1. Fill in the blanks from the list of choices to make the sentence true.

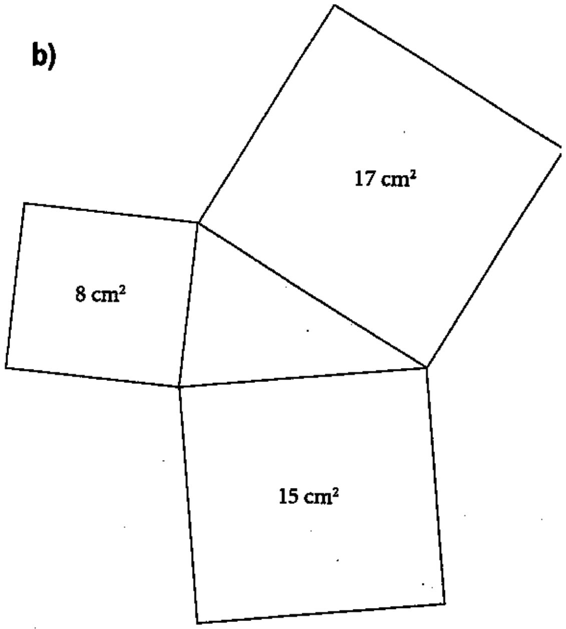
a)



Triangle X **is** a right triangle because **$100 = 64 + 36$**

is   is not $6 + 8 \neq 10$ $100 = 64 + 36$

b)

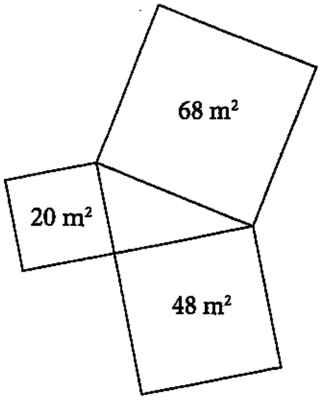


Triangle Y **is not** a right triangle because **$8 + 15 \neq 17$**

is   is not $8^2 + 15^2 = 17^2$ $8 + 15 \neq 17$

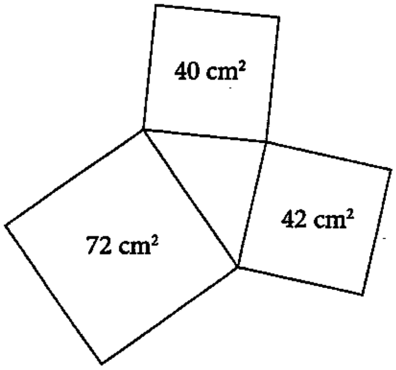
2. Which of the following triangles are right triangles? Explain.

a)



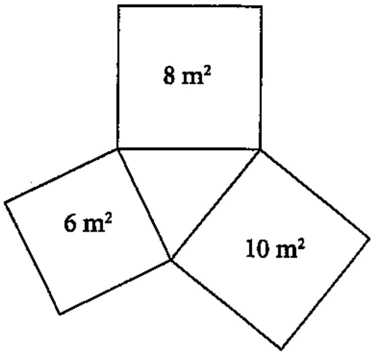
right triangle because
$20 + 48 = 68$

b)



not a right triangle
because $42 + 40 \neq 72$

c)



not a right triangle
because $6 + 8 \neq 10$

3. Determine whether the triangle with each set of side lengths is a right triangle. Justify your answer.

a) 20 cm, 30 cm, 40 cm

20^2 cm^2 + **30^2** cm^2 = **1300** cm^2

40^2 cm^2 = **1600** cm^2

The triangle is **not** a right triangle because **$20^2\ \text{cm}^2 + 30^2\ \text{cm}^2 \neq 40^2\ \text{cm}^2$**

b) 30 mm, 40 mm, 50 mm

The triangle is a right triangle because $30^2\ \text{mm}^2 + 40^2\ \text{mm}^2 = 50^2\ \text{mm}^2$

c) 20 m, 21 m, 29 m

The triangle is a right triangle because $20^2\ \text{m}^2 + 21^2\ \text{m}^2 = 29^2\ \text{m}^2$

d) 60 cm, 11 cm, 62 cm

The triangle is not a right triangle because $60^2\ \text{cm}^2 + 11^2\ \text{cm}^2 \neq 62^2\ \text{cm}^2$

4. Fill in the blanks to make the sentence true.

The set of numbers 7, 24, 25 is a Pythagorean triple because **7^2** + **24^2** = **25^2**

5. Which of these sets of numbers are Pythagorean triples? Explain.

a) 10, 50, 60

This **is not** a Pythagorean triple because $10^2 + 50^2 \neq 60^2$

b) 12, 35, 37

This is a Pythagorean triple because $12^2 + 35^2 = 37^2$

6. Two numbers of a Pythagorean triple are given. Find the missing number. The numbers are listed in ascending order.

a) 7, 24, **25**

The missing number is the **square root** of the sum of the **squares** of the first two numbers.

$$\sqrt{7^2 + 24^2} = \sqrt{49 + 576}$$
$$= \sqrt{625}$$
$$= 25$$

b) 16, 30, **34**

c) 10, **24**, 26

7. Doug wants to check that a lawn he is planting is a rectangle. He measures the width of the lawn to be 10 m, the length to be 24 m, and the diagonal to be 26 m. Is the lawn a rectangle? Explain.

If the lawn is a rectangle, then the width, length, and diagonal form a **right** triangle.

Sample Answer:

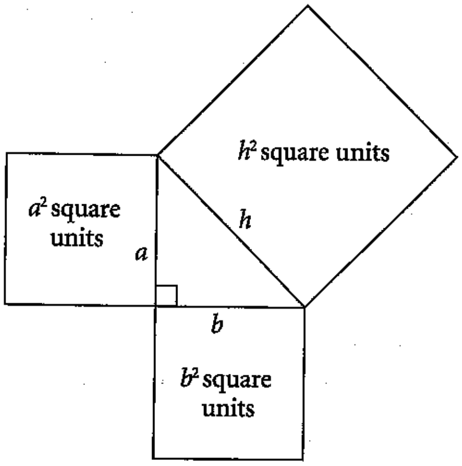
$10^2\ \text{m}^2 + 24^2\ \text{m}^2 = 676\ \text{m}^2 = 26^2\ \text{m}^2$

The triangle formed by the width, length, and diagonal is a right triangle, so the lawn is a rectangle.

1.7 Applying the Pythagorean Theorem

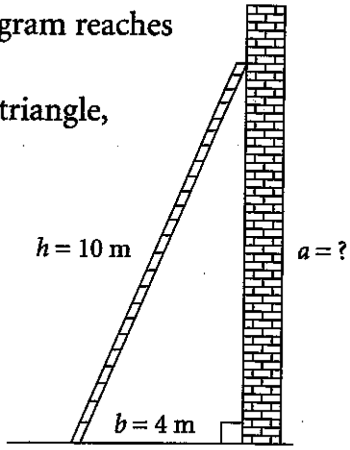
Quick Review

- The Pythagorean Theorem applies to right triangles.
- An algebraic equation for the Pythagorean Theorem is $h^2 = a^2 + b^2$, where h is the length of the hypotenuse and a and b are the lengths of the legs.



- You can apply the Pythagorean Theorem to problems involving right triangles.

You can calculate how high up the wall the ladder in the diagram reaches using the formula $h^2 = a^2 + b^2$
Since the length of the ladder is the hypotenuse of the right triangle, we label it h. The lengths of the two legs of this triangle are labelled a and b.



Substitute $b = 4$ and $h = 10$ into $h^2 = a^2 + b^2$

$$10^2 = a^2 + 4^2$$
$$100 = a^2 + 16$$
$$100 - 16 = a^2 + 16 - 16$$
$$84 = a^2$$
$$\sqrt{84} = a$$
$$9.2 \doteq a$$

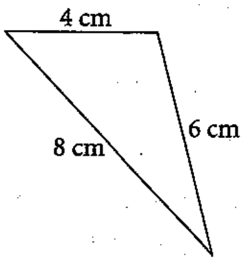
a is approximately 9.2 m. The ladder reaches approximately 9.2 m up the wall.

Tip

*It does not matter which leg is labelled a and which leg is labelled b, so long as a and b label the **legs** and h labels the **hypotenuse**.*

Practice

1. Use the Pythagorean Theorem to check if this is a right triangle.



Substitute a = **4**, b = **6**, and h = **8** into the formula $h^2 = a^2 + b^2$

h^2 = **64** $a^2 + b^2$ = **52**

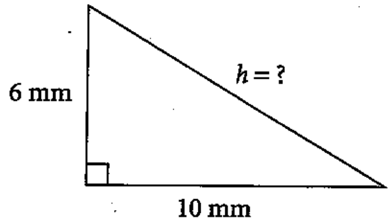
Circle the choices that make the sentence true.

Since h^2 *equals* / [✓ *does not equal*] $a^2 + b^2$, the triangle *is* / [✓ *is not*] a right triangle.

For questions 2 to 5, give each length to 1 decimal place.

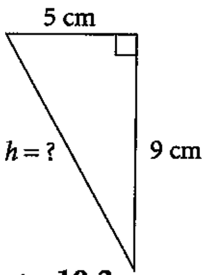
2. Use the equation $h^2 = a^2 + b^2$ to find the length of the unknown side.

a)



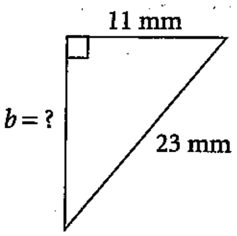
$h \doteq$ **11.7 mm**

b)



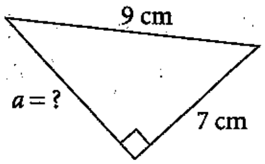
$h \doteq$ **10.3 cm**

c)



$b \doteq$ **20.2 mm**

d)



$a \doteq$ **5.7 cm**

3. An 8-m ladder leans against a wall. How far up the wall does the ladder reach if the foot of the ladder is 3 m from the base of the wall? Show your work.

$h^2 = a^2 + b^2$

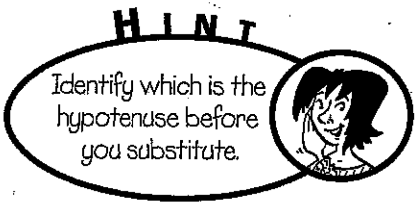
$8^2 = 3^2 + b^2$

$64 = 9 + b^2$

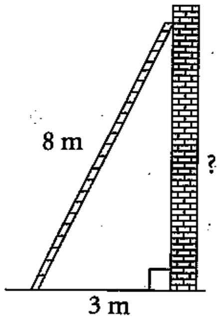
$64 - 9 = 9 + b^2 - 9$

$55 = b^2$

$b \doteq 7.4$

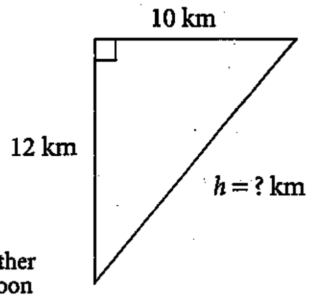

HINT

Identify which is the hypotenuse before you substitute.



The ladder can reach a height of **7.4 m**, to 1 decimal place.

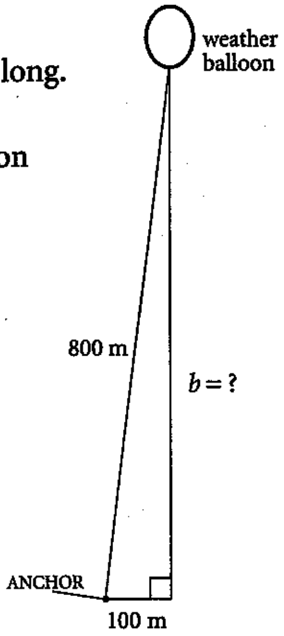
4. A ship leaves port and travels 12 km due north. It then changes direction and travels due east for 10 km. How far must it travel to go directly back to port? Sketch a diagram to explain.



Diagrams may vary.

The ship must travel **15.6 km**, to 1 decimal place, to go directly back to port.

5. A weather balloon is anchored by a cable 800 m long. The balloon is flying directly above a point that is 100 m from the anchor. How high is the balloon flying? Give your answer to the nearest metre.

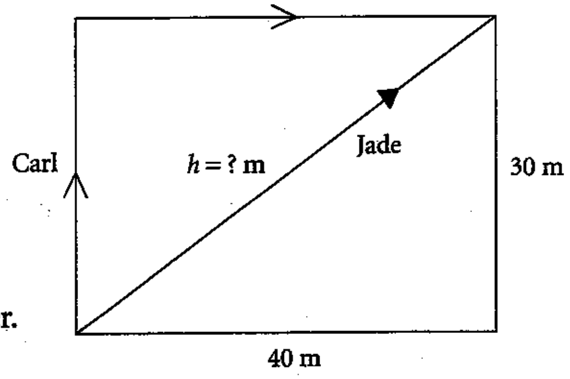


The balloon is flying at a height of **793 m**, to the nearest metre.

6. A rectangular field is 40 m long and 30 m wide. Carl walks from one corner of the field to the opposite corner, along the edge of the field. Jade walks across the field diagonally to arrive at the same corner. How much shorter is Jade's shortcut?

Tip

Sketch a diagram first.



The diagonal of the field measures **50 m**.
Jade walks **50 m**.
Carl walks **30 m** + **40 m** = **70 m**
Jade's shortcut is **70 m** − **50 m** = **20 m** shorter.

7. What is the length of a diagonal of a square with area 100 cm^2? Give your answer to 1 decimal place.

The side length of the square is the square root of **100**, or **10** cm.

The diagonal of the square is the **hypotenuse** of the right triangle with sides **10 cm** and **10 cm**.

The length of the diagonal of the square is **14.1 cm**, to 1 decimal place.

In Your Words

Here are some of the important mathematical words of this unit.
Build your own glossary by recording definitions and examples here. The first one is done for you.

perfect square (square number)
the product of a whole number multiplied by itself
For example, 25 is 5 × 5, so 25 is a perfect square.

square root a number that, when multiplied by itself, results in a given number
For example, 5 is the square root of 25.

legs of a right triangle the sides of a right triangle that form the right angle

hypotenuse the side of a right triangle that is opposite to the right angle; the longest side of a right triangle

Pythagorean Theorem the rule that states that, for any right triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the legs

Pythagorean triple three whole-number side lengths of a right triangle
For example, 3-4-5 is a Pythagorean triple.

List other mathematical words you need to know.

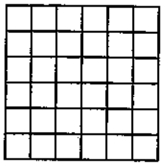
Sample answers: square number, perfect square, irrational number

Unit Review

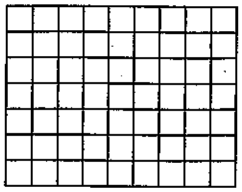
LESSON

1.1 **1.** Circle the perfect squares. Use a diagram to support your answer.
Diagrams may vary.

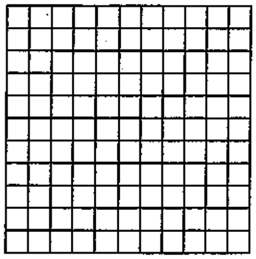
a) [✓ 36]



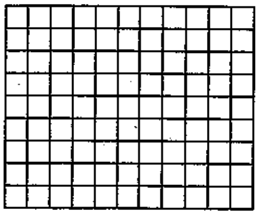
b) 63



c) [✓ 121]



d) 99



1.2 **2.** Simplify without using a calculator.

a) $8^2 =$ **64**

b) $\sqrt{49} =$ **7**

c) $12^2 =$ **144**

d) $\sqrt{121} =$ **11**

3. List the factors of each number in ascending order. Circle the numbers that are perfect squares.

a) 50
1, 2, 5, 10, 25, 50

b) [✓ 196]
1, 2, 4, 7, 14, 28, 49, 98, 196

c) 84
1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84

d) [✓ 225]
1, 3, 5, 7, 15, 25, 45, 75, 225

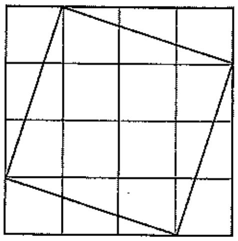
1.3 **4.** The area of a square is given. Find its side length. Circle the side lengths that are whole numbers.

a) 18 cm^2
$\sqrt{18}$ cm

b) 169 cm^2
[✓ **13 cm**]

c) 200 cm^2
$\sqrt{200}$ cm

5. Find the area of the square. Then write the side length of the square.

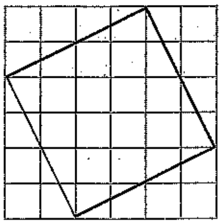


Area = **10 square units**

Side length = **$\sqrt{10}$ units**

LESSON

6. Construct a square on the line segment.
Find the length of the line segment.



Length = **$\sqrt{20}$ units**

1.4 **7.** Evaluate.

a) $\sqrt{8 \times 8}$ = **8** b) $\sqrt{54 \times 54}$ = **54** c) $\sqrt{153 \times 153}$ = **153**

8. Between which two whole numbers is each square root?

a) $\sqrt{45}$ **6 and 7** b) $\sqrt{18}$ **4 and 5** c) $\sqrt{55}$ **7 and 8** d) $\sqrt{135}$ **11 and 12**

9. Estimate each root in question 8 to 1 decimal place.

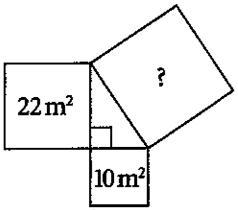
a) **6.7** b) **4.2** c) **7.4** d) **11.6**

10. Circle the better estimate.

a) $\sqrt{75} \doteq$ 8.65 or [✓ 8.66]? b) $\sqrt{90} \doteq$ [✓ 9.49] or 9.50?

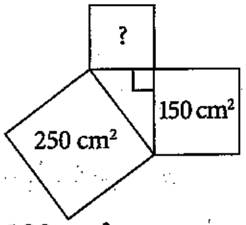
1.5 **11.** Find the area of each indicated square.

a)



32 m^2

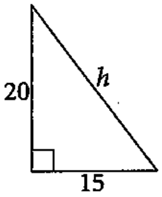
b)



100 cm^2

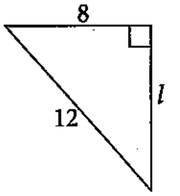
12. Find the length of each side labelled with a variable. Give answers to 1 decimal place, if necessary.

a)



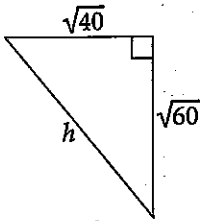
25 units

b)



8.9 units

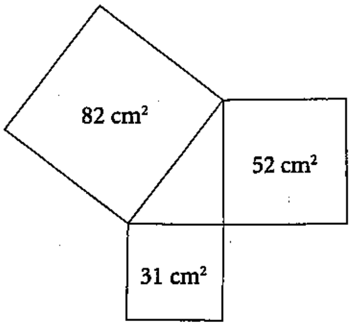
c)



10 units

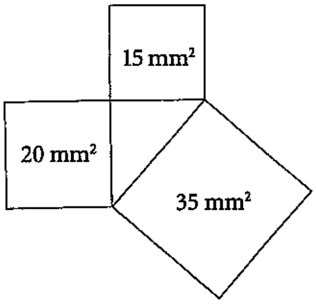
1.6 **13.** Which of the following are right triangles? Justify your answer.

a)



not a right triangle because
52 cm^2 + 31 cm^2 $\neq$ 82 cm^2

b)



right triangle because
15 mm^2 + 20 mm^2 = 35 mm^2

LESSON

14. Circle the sets of numbers that are Pythagorean triples.

a) [✓ 10, 24, 26]

b) 12, 15, 20

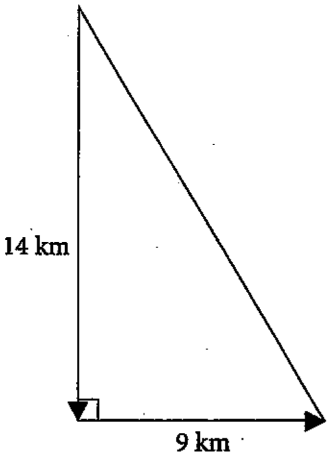
c) 7, 24, 26

d) [✓ 11, 60, 61]

1.7 **15.** A ship travels for 14 km toward the south. It then changes direction and travels for 9 km toward the east. How far does the ship have to travel to return directly to its starting point? Answer correct to 2 decimal places.

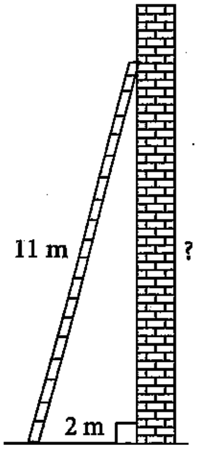
Tip

Draw a diagram.



The ship must travel **16.64 km**

16. How high up the wall does the ladder reach? Answer correct to 2 decimal places.



10.82 m

UNIT 2

Integers

Just for Fun

Modified Sudoku

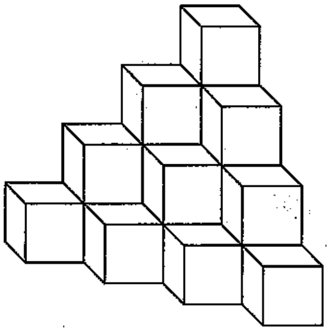
This is a modified version of a Sudoku puzzle, which originated in Japan.

Complete this grid so that every row, column, and 2 × 3 box contains every digit from 1 to 6.

4	1	6	3	5	2
5	3	2	6	4	1
1	2	5	4	6	3
6	4	3	2	1	5
3	6	1	5	2	4
2	5	4	1	3	6

Cube Count

How many cubes are in this figure? Look for a pattern to find the answer.



Sample Answer:

Layer	**Number of Cubes**
1	**4 + 3 + 2 + 1 = 10**
2	**3 + 2 + 1 = 6**
3	**2 + 1 = 3**
4	**1**

Total = 10 + 6 + 3 + 1 = 20

There are **20** cubes.

Add 'Em Up

Find the value of this expression without using a calculator. Explain your work.

1 − 2 + 3 − 4 + 5 − 6 + . . . + 99 − 100 = **−50**

Sample Answer: Each pair of numbers adds to give −1.

There are 50 pairs. So, the sum is −50.

Activating Prior Knowledge

Using Models to Add Integers

➤ You can use coloured tiles to model integers.
A black tile models –1. A white tile models +1.

■ -1 □ +1

A black tile and a white tile combine to model 0.
They form a **zero pair**: (+1) + (–1) = 0

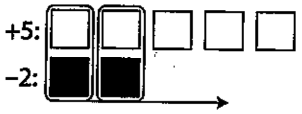
□ +1
■ -1

To add: (–4) + (–2)
Model –4 with 4 black tiles.
Model –2 with 2 black tiles.

-4: ■■■■
-2: ■■

There are 6 black tiles altogether.
They model –6.
So, (–4) + (–2) = –6

To add: (+5) + (–2)
Model +5 with 5 white tiles.
Model –2 with 2 black tiles.



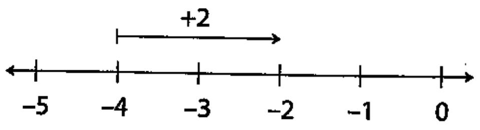
Circle zero pairs.
3 white tiles remain.
They model +3.
So, (+5) + (–2) = +3

➤ You can also use a number line to add integers.

Find the first integer on the number line.

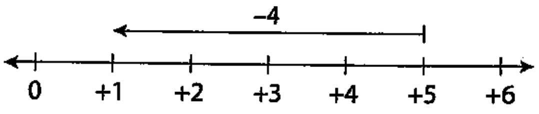
- To add a positive integer, move right on the number line.
- To add a negative integer, move left on the number line.

To add: (–4) + (+2)
Start at –4.
Move 2 units right to add +2.



The arrow ends at –2.
So, (–4) + (+2) = –2

To add: (+5) + (–4)
Start at +5.
Move 4 units left to add –4.

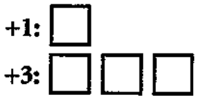


The arrow ends at +1.
So, (+5) + (–4) = +1

Check

1. Use tiles to add.

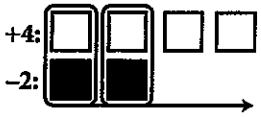
a) (+1) + (+3) = +4



b) (–2) + (–3) = –5

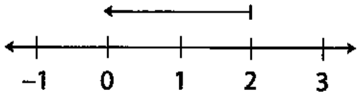
c) (–4) + (+3) = –1

d) (+4) + (–2) = +2

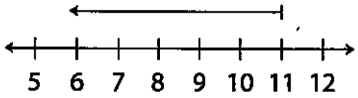


2. Use a number line to add.

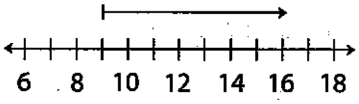
a) (+2) + (–2) = 0



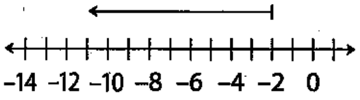
b) (+11) + (–5) = +6



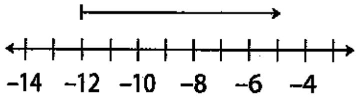
c) (+9) + (+7) = +16



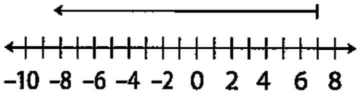
d) (–2) + (–9) = –11



e) (–12) + (+7) = –5



f) (+7) + (–15) = –8



Using Models to Subtract Integers

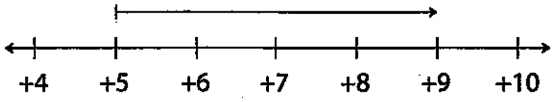
➤ To subtract, you take away tiles.
If there are not enough tiles to remove, add zero pairs.
To subtract: (–3) – (+2)
Model –3 with 3 black tiles.
To take away +2, 2 white tiles are needed.
Add 2 zero pairs of tiles to provide 2 white tiles.

5 black tiles remain. They model –5.
So, (–3) – (+2) = –5

HINT

Adding a zero pair is equivalent to adding 0. It does not change the value represented by the tiles.

➤ To subtract an integer on a number line, move in the opposite direction of adding the same integer.
To subtract a positive integer, move left.
To subtract a negative integer, move right.
To subtract: (+5) – (–4)
Start at +5.
Move 4 units right to subtract –4.



The arrow ends at +9.
So, (+5) – (–4) = +9

Check

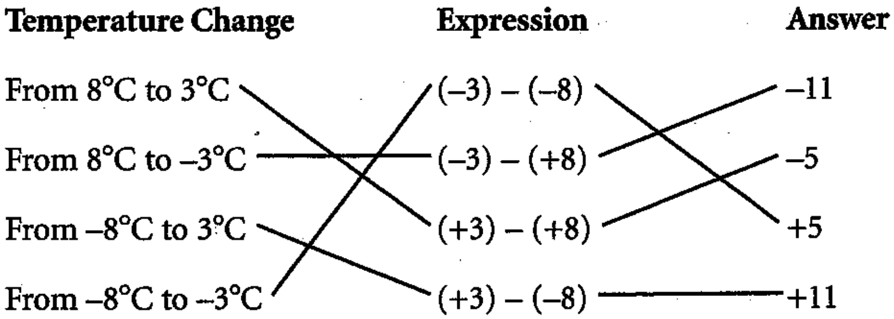
3. Subtract using a model of your choice

Models may vary.

a) (+2) – (–7) = **+9** **b)** (–3) – (–4) = **+1** **c)** (–5) – (–5) = **0**

d) (+10) – (–4) = **+14** **e)** (–5) – (+6) = **–11** **f)** (–3) – (–5) = **+2**

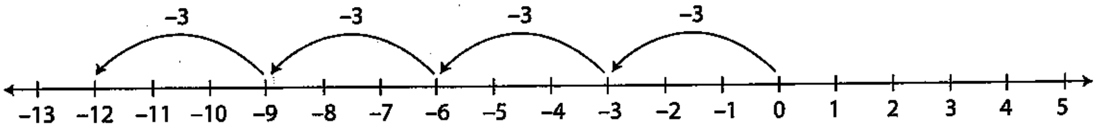
4. Match each description with the correct subtraction expression and answer.



2.1 Using Models to Multiply Integers

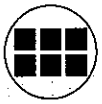
Quick Review

- You can think of multiplication as repeated addition.
  4 × (–3) is the same as adding –3 four times.
  As a sum: (–3) + (–3) + (–3) + (–3) = –12
  As a product: 4 × (–3) = –12
  On a number line:



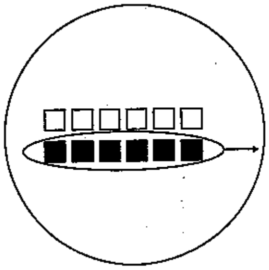
- You can use tiles to multiply integers.
  Let a circle represent the bank. The bank has zero value at the start.

  Multiply: (+2) × (–3)
  +2 is a positive integer.
  –3 is modelled with 3 black tiles.
  So, put 2 sets of 3 black tiles into the circle.



  The 6 black tiles in the circle represent –6.
  So, (+2) × (–3) = –6

- Multiply: (–2) × (–3)
  –2 is a negative integer.
  –3 is modelled with 3 black tiles.
  So, we need to take 2 sets of 3 black tiles from the circle.



  Add zero pairs until there are enough black tiles to remove.
  Take out 2 sets of 3 black tiles.

  There now are 6 white tiles left in the circle.
  So, (–2) × (–3) = 6

Practice

1. Write a multiplication expression for each repeated addition.

a) (–2) + (–2) + (–2) + (–2) + (–2) = 5 × (–2)

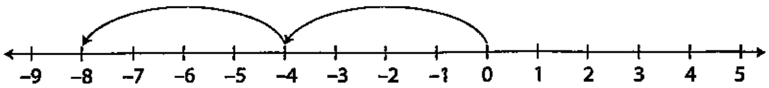
b) (+11) + (+11) + (+11) = 3 × (+11)

c) (–5) + (–5) + (–5) = 3 × (–5)

2. Write each multiplication expression as a repeated addition. Then use a number line to find each sum.

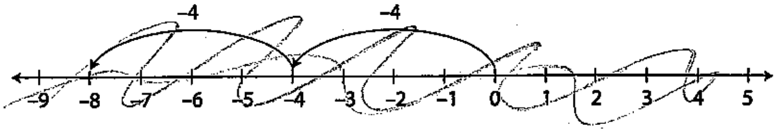
a) (+2) × (–4) = (–4) + (–4)

= ~~–4~~ –8



b) (+5) × (+4) = **(+4) + (+4) + (+4) + (+4) + (+4)**

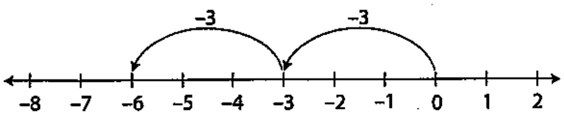
= **+20**



c) (–3) × (+2) = ~~(+2) × (–3)~~

= **(–3) + (–3)** ✓

= **–6**



3. Write a multiplication equation for each model. Find the product.

a) Deposit 3 sets of 2 black tiles.

3 × (–2) = **–6**

b) Deposit 5 sets of 2 white tiles.

5 × (+2) = **+10**

c) Withdraw 2 sets of 3 black tiles.

(–2) × **(–3)** = **+6**

d) Withdraw 9 sets of 2 black tiles.

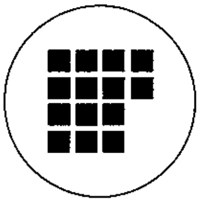
(–9) × (–2) = +18

e) Deposit 4 sets of 3 black tiles.

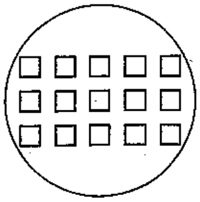
4 × (–3) = –12

4. Use a tile model to find each product.

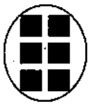
a) (+7) × (–2) = **–14**



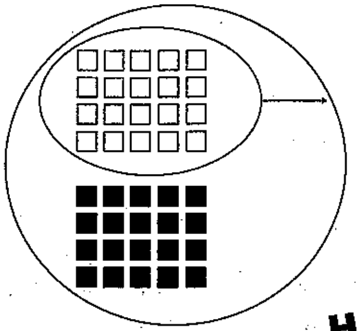
b) (+3) × (+5) = **+15**



c) (+2) × (–3) = **–6**



d) (–4) × (+5) = **–20**



HINT

Add enough zero pairs to take away the appropriate number of white tiles.

5. Use a model to represent each product. Draw the model you used each time.
Models will vary.

a) (–3) × (–4) = **+12**

b) (+2) × (–5) = **–10**

c) (+7) × (+2) = **+14**

d) (–3) × (+6) = **–18**

6. The temperature dropped 2°C each hour for 4 h. Use integers to find the total change in temperature.

–2 represents a fall of 2°C.

4 represents 4 h.

4 × (–2) = –8

The temperature dropped 8°C in 4 h.

2.2 Developing Rules to Multiply Integers

At Home At School

Quick Review

➤ Integers have these properties of whole numbers.

- **Multiplying by 0:** 4 × 0 = 0 and 0 × 4 = 0
  So, (–4) × 0 = 0 and 0 × (–4) = 0
- **Multiplying by 1:** 4 × 1 = 4 and 1 × 4 = 4
  So, (–4) × (+1) = –4 and (+1) × (–4) = –4
- **Commutative Property:** 4 × 2 = 8 and 2 × 4 = 8
  So, (–4) × (+2) = –8 and (+2) × (–4) = –8
- **Distributive Property:** 4 × (2 + 3) = 4 × 2 + 4 × 3 = 20
  So, (–4) × [(+2) + (+3)] = (–4) × (+2) + (–4) × (+3) = –20

➤ You can write the product of integers without the use of the × sign.
(–4) × (+2) can simply be written as: (–4)(+2)

➤ When 2 integers with the same sign are multiplied, their product is positive.
(+2)(+3) = +6 (–2)(–3) = +6

When 2 integers with different signs are multiplied, their product is negative.
(+2)(–3) = –6 (–2)(+3) = –6

Practice

1. Find a pattern rule for each multiplication pattern.
Extend the pattern for 3 more rows.

HINT

To find a pattern rule, look for a pattern in the integer factors and in the products.

a) (+3)(+3) = +9
(+2)(+3) = +6
(+1)(+3) = +3
(0)(+3) = **0**
(**–1**)(+3) = **–3**
(–2)(+3) = –6
(–3)(+3) = –9

b) (–3)(+3) = –9
(–3)(+2) = –6
(–3)(+1) = –3
(–3)(0) = **0**
(–3)(–1) = +3
(–3)(–2) = +6
(–3)(–3) = +9

2. In this chart, write the sign of each product of multiplying 2 integers.

×	positive integer	negative integer
positive integer	**positive**	**negative**
negative integer	**negative**	**positive**

- When 2 integer factors have the same sign, their product is **positive**.
- When 2 integer factors have different signs, their product is **negative**.

3. Find each product.

a) (+7)(–2) = **–14** **b)** (–4)(–3) = **+12** **c)** (–8)(+9) = **–72**

d) (+10)(–5) = **–50** **e)** (+5)(–7) = **–35** **f)** (–9)(–4) = **+36**

i) (–7)(–1) = **+7** **j)** (+5)(0) = **0** **k)** (+20)(–20) = **–400**

4. Fill in the blank to make each equation true.

a) (+7) × **–5** = –35 **b)** **–11** × (–9) = +99 **c)** (–10) × **+32** = –320

d) **–4** × (–5) = +20 **e)** (+7) × **–7** = –49 **f)** **–5** × (+13) = –65

g) **+12** × (–15) = –180 **h)** (+14) × **–10** = –140 **k)** **–8** × (–7) = 56

5. Match each pattern rule with the corresponding pattern.
Complete each pattern and pattern rule.

Number Pattern	**Pattern Rule**
–3, +9, –27, + 81, . . .	Start at 2. Multiply by **–5** each time.
+2, –10, +50, –250, . . .	Start at 1. Multiply by –10 each time.
+3, –3, **+3**, **–3**, . . .	Start at **–3**. Multiply by –3 each time.
+1, –10, **100**, **–1000**, . . .	Start at 3. Multiply by –1 each time.
–1, –2, –4, –8, –16, . . .	Start at –1. Multiply by **2** each time.

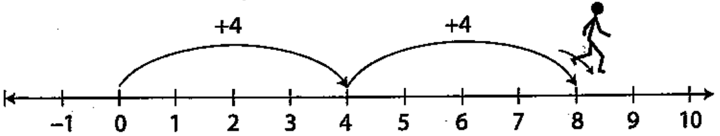
2.3 Using Models to Divide Integers

Quick Review

Division is the inverse of multiplication.
So, 10 ÷ 5 = ? is the same as ? × 5 = 10.
The product means, "how many sets of 5 produce 10?"

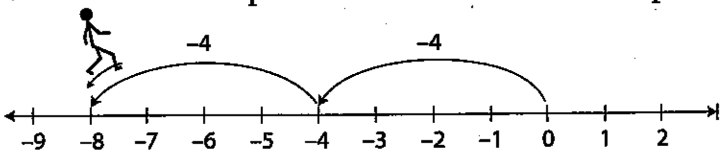
You can "walk" a number line to model the division of two integers.
If the step size is positive, walk forward. If the step size is negative, walk backward.
The number of steps is the quotient and the direction you are facing at the end determines its sign.

- **Positive ÷ Positive**
  Divide: (+8) ÷ (+4)
  Start at 0. Take steps of size 4 forward to end up at +8.



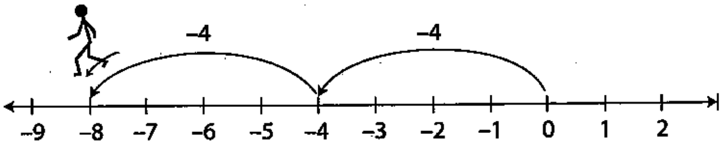
You took 2 steps and are facing the positive end of the line. So, (+8) ÷ (+4) = +2

- **Negative ÷ Negative:**
  Divide: (–8) ÷ (–4)
  Start at 0. Take steps of size 4 backward to end up at –8.



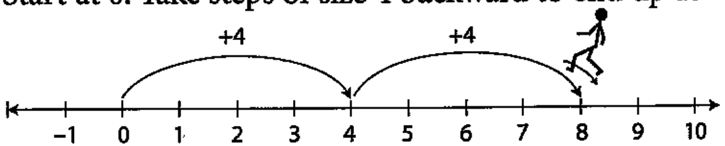
You took 2 steps and are facing the positive end of the line. So, (–8) ÷ (–4) = +2.

- **Negative ÷ Positive:**
  Divide: (–8) ÷ (+4)
  Start at 0. Take steps of size 4 forward to end up at –8.



You took 2 steps and are facing the negative end of the line. So, (–8) ÷ (+4) = –2.

- **Positive ÷ Negative:**
  Divide: (+8) ÷ (–4)
  Start at 0. Take steps of size 4 backward to end up at +8.



You took 2 steps. You are facing the negative end of the line. So, (–8) ÷ (+4) = –2.

Practise

1. Write 2 related multiplication equations for each division equation.

a) (+60) ÷ (+10) = +6 **(+6) × (+10) = +60**, **(+10) × (+6) = +60**

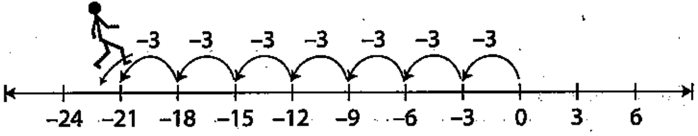
b) (+36) ÷ (–9) = –4 **(–4) × (–9) = +36**, **(–9) × (–4) = +36**

c) (–45) ÷ (–9) = +5 **(+5) × (–9) = –45**, **(–9) × (+5) = –45**

d) (–16) ÷ (+2) = –8 **(–8) × (+2) = –16**, **(+2) × (–8) = –16**

2. Suzanne wanted to model division using a number line. She started at zero and took steps backward of size 3. She ended up at –21.

a) Illustrate this problem using a number line.

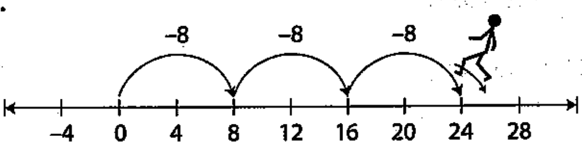


b) Model this problem using a division equation. **(–21) ÷ (–3) = +7**

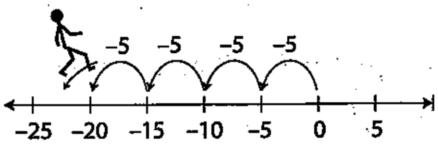
c) How many steps did Suzanne take? **Suzanne took 7 steps.**

3. Use a number line. Find each quotient.

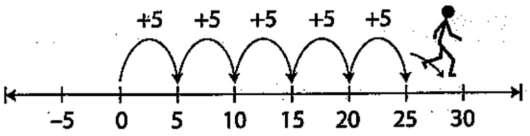
a) (+24) ÷ (–8) = **–3**



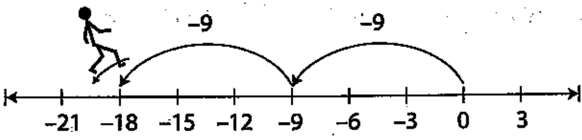
b) (–20) ÷ (–5) = **+4**



c) (+25) ÷ (+5) = **+5**



d) (–18) ÷ (–9)= **+2**



4. Find each quotient.

a) (–12) ÷ (+4) = **–3** **b)** (–12) ÷ (–6) = **+2** **c)** (–8) ÷ (+4) = **–2**

5. The water level in a well dropped 4 cm each hour. The total drop in the water level was 28 cm. Use an integer model to find out how long it took for the water level to change.

–4 represents the drop each hour.

–28 represents the total drop.

(–28) ÷ (–4) = +7

It took 7 h for the water level to drop 28 cm.

6. Use coloured tiles, a number line, or another model to clearly show your thinking. Find each quotient.

Models may vary.

a) (+10) ÷ (+2) **+5** **b)** (–10) ÷ (–2) **+5**

c) (+10) ÷ (–2) **–5** **d)** (–10) ÷ (+2) **–5**

Compare the quotients. What do you notice? **They are all either +5 or –5.**

7. The temperature dropped a total of 12°C over a 4-h period. The temperature dropped the same amount each hour. Using a model, show the hourly drop in temperature.

Models may vary.

–12 represents a drop of 12°C.

+4 represents 4 h.

(–12) ÷ (+4) = –3

The temperature dropped 3°C every hour.

2.4 Developing Rules to Divide Integers

Quick Review

- ➤ For any multiplication of 2 different factors, there are 2 related division facts:
For 4 × 3 = 12, the related division facts are: 12 ÷ 3 = 4 and 12 ÷ 4 = 3

The same rules apply to the product of 2 integers.
For (–2)(+5) = –10, the related division facts are:

$$\underset{\text{dividend}}{\underset{\downarrow}{(-10)}} \div \underset{\text{divisor}}{\underset{\downarrow}{(-2)}} = \underset{\text{quotient}}{\underset{\downarrow}{+5}} \quad \text{and} \quad (-10) \div (+5) = -2$$

- ➤ The quotient of 2 integers with the same sign is positive.
(+10) ÷ (+2) = +5 (–10) ÷ (–2) = +5

- ➤ The quotient of 2 integers with different signs is negative.
(+10) ÷ (–2) = –5 (–10) ÷ (+2) = –5

- ➤ A division expression can be written using a division sign, (–24) ÷ (–6), or it can be written as a fraction, $\frac{(-24)}{(-6)}$.

[illegible]

1. For each product, complete the 2 related division facts and name the sign of the quotient.

Multiplication Fact	Related Division Facts	Sign of Quotient
(+2)(+3) = +6	(+6) ÷ (+2) = +3	positive
	(+6) ÷ (+3) = +2	positive
(–2)(–3) = +6	(+6) ÷ (–2) = –3	negative
	(+6) ÷ (–3) = –2	negative
(+2)(–3) = –6	(–6) ÷ (+2) = –3	negative
	(–6) ÷ (–3) = +2	positive
(–2)(+3) = –6	(–6) ÷ (–2) = +3	positive
	(–6) ÷ (+3) = –2	negative

/16

2. Use your results in question 1. Complete these 2 statements.

When 2 integers have the same sign, their quotient is **positive**.

When 2 integers have different signs, their quotient is **negative**.

3. Find a pattern rule for each division pattern.
Extend the pattern 3 more rows.

HINT

To find a pattern rule, look for a pattern in the dividends and in the quotients.

a) (+6) ÷ (–2) = –3	**b)** (–12) ÷ (–4) = +3
(+4) ÷ (–2) = –2	(–8) ÷ (–4) = +2
(+2) ÷ (–2) = –1	(–4) ÷ (–4) = +1
(0) ÷ (–2) = **0**	(0) ÷ (–4) = **0**
(–2) ÷ (–2) = +1	**(+4) ÷ (–4) = –1**
(–4) ÷ (–2) = +2	**(+8) ÷ (–4) = –2**
(–6) ÷ (–2) = +3	**(+12) ÷ (–4) = –3**

Use the last 3 rows of each pattern. Complete these statements.

When both the dividend and divisor are negative, the quotient is **positive**.

When the dividend is positive and the divisor is negative, the quotient is **negative**.

4. Find each quotient.

a) (+15) ÷ (–3) = **–5** **b)** (–32) ÷ (+4) = **–8** **c)** (+72) ÷ (–8) = **–9**

d) (–54) ÷ (–9) = **+6** **e)** (–72) ÷ (+6) = **–12** **f)** (+88) ÷ (+11) = **+8**

g) (–42) ÷ (–6) = **+7** **h)** (+108) ÷ (+9) = **+12** **i)** (–56) ÷ (+7) = **–8**

5. Use 2 of these 5 integers. Write a division fact with each quotient.
–2 +3 +12 –1 +4

a) a quotient of –2	**(+4) ÷ (–2) = –2**
b) the greatest quotient	**(+12) ÷ (+3) = +4**
c) the least quotient	**(+12) ÷ (–1) = –12**
d) a quotient between –5 and –10	**(+12) ÷ (–2) = –6**

6. Use a calculator to divide.

Tip

Look for the ⊟ *or* +/– *key on your calculator to key in negative numbers.*

a) (+247) ÷ (–13) = **–19** **b)** (–851) ÷ (–37) = **+23**

c) $\frac{(-748)}{(-68)}$ = **+11** **d)** $\frac{(-1485)}{(+33)}$ = **–45**

2.5 Order of Operations with Integers

Quick Review

- The order of operations with whole numbers also applies to integers.

① Perform operations in brackets first.
② Divide and multiply, in order, from left to right.
③ Add and subtract, in order, from left to right.

$$\overset{\textcircled{1}}{(1 + 2)} \overset{\textcircled{3}}{-} 3 \overset{\textcircled{2}}{\times} 4$$

① **B** $= 3 - 3 \times 4$
② **DM** $= 3 - 12$
③ **AS** $= -9$

Tip
The letters BDMAS can help you remember the order of operations.
B–Brackets
DM–Divide, Multiply
AS–Add, Subtract

- A fraction bar indicates division.
It also acts like brackets.
Evaluate the numerator and denominator separately before dividing.

For example, $\frac{12 + 8}{2 - 6} = \frac{20}{-4} = -5$

- If an integer does not have a sign, it is assumed to be positive: $2 = +2$

Practice

1. Simplify.

a) $5 - 2 - 6$
$= \underline{\textbf{3}} - 6$
$= \underline{\textbf{-3}}$

b) $3(8 - 12)$
$= 3 \times \underline{\textbf{-4}}$
$= \underline{\textbf{-12}}$

Tip
Brackets symbolize multiplication as well as grouping. 3(8 – 12) means 3 × (8 – 12).

c) $-4 + 2 \times 3$
$= -4 + \underline{\textbf{6}}$
$= \underline{\textbf{2}}$

d) $21 \div (-7) \times 5$
$= \underline{\textbf{-3} \times \textbf{5}}$
$= \underline{\textbf{-15}}$

e) $10 - [(5 - 3) + 9]$

$\underline{\textbf{-1}}$

f) $-8 + 15 \div (-3) + 7$

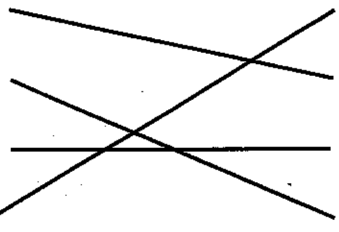
$\underline{\textbf{-6}}$

g) $(-3)(-8) + 24 \div (-2)$

$\underline{\textbf{12}}$

2. Match each expression with its answer.

Expression	**Answer**
$30 \div (5 - 10) \times 2$	-14
$30 \div (5 - 10 \times 2)$	-12
$(30 \div 5 - 10) \times 2$	-8
$30 \div 5 - 10 \times 2$	-2



3. Simplify.

a) $\frac{3(5-9)}{2}$

$= \frac{3(-4)}{2}$

$= \frac{-12}{2}$

$= -6$

b) $\frac{(-4)(-2)}{-8}$

-1

-1

c) $\frac{(-6)(4)+8}{(-2)\times 4}$

2

4. Evaluate each expression. Write the letter for the answer in the corresponding blank at the bottom to find out what one wall said to the other.

$2(-7+3)$ $= 2(-4)$ $= -8$ A	$-8 + 12 \div 4$ $= -8 + 3$ $= -5$ C	$3(10 \div 2) - (-4)$ $= 3(5) + 4$ $= 15 + 4$ $= 19$ E
$(-6)(-6) \div (-4)$ $= (6)^2 \div (-4)$ $= 36 \div (-4)$ $= -9$ H	$4 \times (-3) + 24 \div 2$ $= -12 + 24 \div 2$ $= -12 + 12$ $= 0$ M	$-5 + 12 \div 4 \times (-2)$ $= -5 + 3 \times (-2)$ $= -5 + (-6)$ $= -11$ N
$19 - 3 \times 4 \div (-6)$ $= 19 - 12 \div (-6)$ $= 19 - (-2)$ $= 19 + 2$ $= 21$ O	$\frac{6(-8)}{-12} - 1$ $= \frac{-48}{-12} - 1$ $= 4 - 1$ $= 3$ R	$\frac{10-2(-3)}{2\times 4}$ $= \frac{10+6}{9-1}$ $= \frac{16}{8}$ $= 2$ T

M	E	E	T	M	E	A	T	T	H	E	C	O	R	N	E	R		
0	19	19	2	0	19	−8	2	2	−9	19	−5	21	3	−11	19	3		

In Your Words

Here are some of the important mathematical words of this unit.
Build your own glossary by recording definitions and examples here. The first one is done for you.

integer *the numbers ..., -3, -2, -1, 0, 1, 2, 3, ...*

For example, 1, 2, 3, ... are positive integers and -1, -2, -3, ... are negative integers. 0 is neither positive nor negative.

quotient *the answer to a division question*

For example, in the division equation $\frac{(+8)}{(-2)} = -4$*, -4 is the quotient.*

zero pair *two opposite integers, such as +4 and -4, whose sum is 0*

commutative property *order does not matter when multiplying integers*

For example, $(-2) \times (+3) = (+3) \times (-2)$

zero property *the answer when multiplying an integer by 0 is 0*

For example, $4 \times 0 = 0 \times 4 = 0$

order of operations *the order to perform operations in a mathematical expression*

For example, in $[3 + (-2)] + 5 \times (-4)$*, perform the addition in brackets first, then do the multiplication, and finally do the addition outside the brackets.*

$$
\begin{aligned}
[3 + (-2)] + 5 \times (-4) &= 1 + 5 \times (-4) \\
&= 1 + (-20) \\
&= -19
\end{aligned}
$$

List other mathematical words you need to know.

number line, opposite integers, multiplying by 1 property, distributive property, numerator, denominator

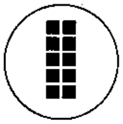
Unit Review

LESSON

2.1 **1.** Write each multiplication as a repeated addition. Then illustrate using coloured tiles to find each sum.

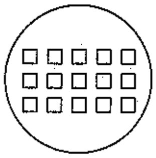
a) $(+5) \times (-2) = (-2) + (-2) + (-2) + (-2) + (-2)$

$= -10$



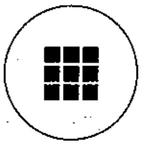
b) $(+3) \times (+5) = (+5) + (+5) + (+5)$

$= +15$



c) $(+3) \times (-3) = (-3) + (-3) + (-3)$

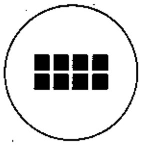
$= -9$



d) $(-4) \times (+2) = (+2) \times (-4)$

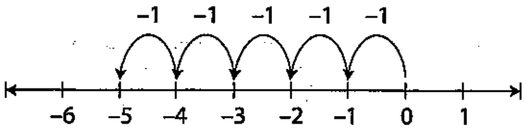
$= (-4) + (-4)$

$= (-8)$

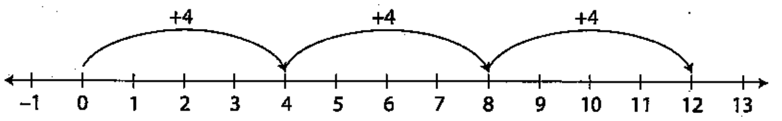


2. Use a number line. Find each product.

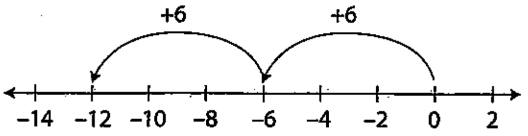
a) $(+5) \times (-1) = -5$



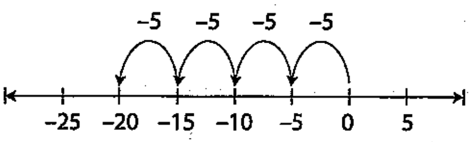
b) $(+3) \times (+4) = +12$



c) $(-2) \times (+6) = -12$



d) $(+4) \times (-5) = -20$



3. a) The temperature rose 2°C each hour for 6 h. Use integers to find the total change in temperature.

+2 represents a rise of 2°C.

+6 represents 6 h.

(+6) × (+2) = +12

The temperature rose 12°C in 6 h.

b) If the starting temperature was –4°C, what was the temperature after 6 h?

(–4) + (+12) = +8

The temperature after 6 h was 8°C.

4. Show how to model (–2) × (–5). Explain why you chose that model.

Models may vary.

2.2 **5.** Complete each statement using positive, negative, or zero.

a) The product of a positive integer and a negative integer is **negative**.

b) The product of a negative integer and zero is **zero**.

c) The product of an two negative integers is **positive**.

6. Find each product.

a) (+2)(+3) = **+6**

b) (–6)(+4) = **–24**

c) (–22)(–10) = **+220**

d) (+24)(–30) = **–720**

e) (–36)(–5) = **+180**

f) (+42)(+3) = **+126**

g) (–81)(+2) = **–162**

h) (–237)(0) = **0**

7. Fill in the blank to make each equation true.

a) (–6) × **(+4)** = –24

b) (–9) × **(–3)** = +27

c) **(+7)** × (–3) = (–21)

d) (–4) × **(–6)** = +24

e) (+20) × **(+15)** = +300

f) (–32) × **(+5)** = –160

LESSON

2.3 **8.** Write a related multiplication equation for each division equation.

a) (+100) ÷ (−25) = −4

(−4) × (−25) = +100 or (−25) × (−4) = +100

b) (−28) ÷ (−7) = +4

(+4) × (−7) = −28 or (−7) × (+4) = −28

c) $\frac{(-15)}{(-5)} = +3$

(+3) × (−5) = −15 or (−5) × (+3) = −15

d) $\frac{(+48)}{(+12)} = +4$

(+4) × (+12) = +48 or (+12) × (+4) = +48

9. Show how to model (−12) ÷ 4.

Models may vary.

2.4 **10.** Decide whether each quotient will be positive, negative, or zero. Then evaluate each quotient.

a) (−25) ÷ (−5) **positive, +5**

c) $\frac{(+42)}{(-7)}$ **negative, −6**

b) (−36) ÷ (+9) **negative, −4**

d) 0 ÷ (−5) **zero, 0**

11. Evaluate each quotient and order the results from least to greatest.

a) (−20) ÷ (+4) = **−5**

b) (−18) ÷ (−6) = **+3**

c) (+48) ÷ (−8) = **−6**

The quotients from least to greatest are: **−6, −5, +3**

12. Find all of the divisors of −16. Write a division equation each time. The first one has been done for you.

Divisor	Division Equation
−1	(−16) ÷ (−1) = +16
+1	**(−16) ÷ (+1) = −16**
−2	**(−16) ÷ (−2) = +8**
+2	**(−16) ÷ (+2) = −8**
−4	**(−16) ÷ (−4) = +4**
+4	**(−16) ÷ (+4) = −4**
−8	**(−16) ÷ (−8) = +2**
+8	**(−16) ÷ (+8) = −2**
−16	**(−16) ÷ (−16) = +1**
+16	**(−16) ÷ (+16) = −1**

13. Write the next 3 terms in each pattern. Then write the pattern rule.

a) +1, −4, +16, −64, **+256**, **−1024**, **+4096**, …

Pattern rule: Start at **+1**. **Multiply by −4** each time.

b) −128, +64, −32, 16, **−4**, **+1**, $-\frac{1}{4}$, …

Pattern rule: Start at **−128**. **Divide by −4** each time.

c) −3125, +625, −125, **+25**, **−5**, $-\frac{1}{5}$, …

Pattern rule: Start at **−3125**. **Divide by −5** each time.

2.5 **14.** State which operation you would do first. Do not evaluate.

a) (+8) + (−2) × (−3)

multiply (−2) × (−3)

b) (−20) ÷ (−4) − (−2)

divide (−20) ÷ (−4)

c) (−2)(4 − 5)

subtract (4 − 5)

d) 5 − 3 + (−4) × (−2)

multiply (−4) × (−2)

15. Evaluate each expression in question 14. Show all your steps.

a) **(+8) + (−2) × (−3) = (+8) + (+6)**

= +14

b) **(−20) ÷ (−4) − (−2) = (+5) − (−2)**

= (+5) + (+2)

= +7

c) **(−2)(4 − 5) = (−2)(−1)**

= +2

d) **5 − 3 + (−4) × (−2) = 5 − 3 + 8**

= 2 + 8

= 10

16. Evaluate using the order of operations.

a) $17 - 4 \times 4 = \mathbf{17 - 16}$

$= \mathbf{1}$

b) $-48 \div 4 - 2(3 - 4) = \mathbf{-48 \div 4 - 2(-1)}$

$= \mathbf{-12 - 2(-1)}$

$= \mathbf{-12 - (-2)}$

$= \mathbf{-12 + 2}$

$= \mathbf{-10}$

c) $-2 - 4 \times 9 = \mathbf{-2 - 36}$

$= \mathbf{-38}$

d) $\frac{(-6)(8-2)}{-4} = \frac{(-6)(6)}{-4}$

$= \frac{-36}{-4}$

$= \mathbf{9}$

e) $(-3) \times (-3) + (-4) \times (-4) = \mathbf{(+9) + (-4) \times (-4)}$

$= \mathbf{(+9) + (+16)}$

$= \mathbf{+25}$

f) $\frac{21 + 2(3)}{(-3) \times (-3)} = \frac{21 + 6}{(-3) \times (-3)}$

$= \frac{27}{(-3) \times (-3)}$

$= \frac{27}{9}$

$= \mathbf{3}$

UNIT 3

Operations with Fractions

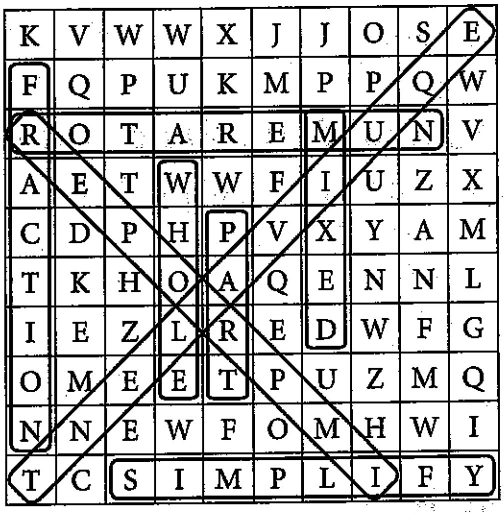
Just for Fun

Fraction Word Search

Can you find this list of words in the word search table at the right?

Words can be horizontal, vertical, or diagonal.

SIMPLIFY	FRACTION
IMPROPER	MIXED
NUMERATOR	PART
EQUIVALENT	WHOLE



K	V	W	W	X	J	J	O	S	E
F	Q	P	U	K	M	P	P	Q	W
R	O	T	A	R	E	M	U	N	V
A	E	T	W	W	F	I	U	Z	X
C	D	P	H	P	V	X	Y	A	M
T	K	H	O	A	Q	E	N	N	L
I	E	Z	L	R	E	D	W	F	G
O	M	E	E	T	P	U	Z	M	Q
N	N	E	W	F	O	M	H	W	I
T	C	S	I	M	P	L	I	F	Y

A Game for or more

Compose It

Make as many words as you can from the letters of the word "fraction." Words must contain at least four letters. The person with the most words after 3 minutes wins!

Sample answer: cart, raft, craft, ratio, ration, . . .

A Game for or more

Winfrac

Play with one or more classmates.

You will need two 8- or 10-sided dice, a pencil, and paper.

Take turns to roll the two dice. Use the 2 numbers to create a proper fraction.

The player with the larger fraction wins a point.

The first player to reach 10 points wins the game.

Activating Prior Knowledge

Equivalent Fractions

➤ $\frac{1}{3}$, $\frac{2}{6}$, $\frac{3}{9}$, and $\frac{4}{12}$ are equivalent fractions. To find equivalent fractions, multiply or divide the numerator and denominator by the same number.

✓ Check

1. Write 3 equivalent fractions for each fraction. **Sample answers:**

a) $\frac{6}{24}$ $\mathbf{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}}$

b) $\frac{21}{14}$ $\mathbf{\frac{3}{2}, \frac{6}{4}, \frac{42}{28}}$

c) $\frac{30}{72}$ $\mathbf{\frac{5}{12}, \frac{25}{60}, \frac{50}{120}}$

Relating Mixed Numbers and Improper Fractions

➤ To convert $3\frac{5}{8}$ to an improper fraction:

$$3\frac{5}{8} = 3 + \frac{5}{8}$$
$$= \frac{24}{8} + \frac{5}{8}$$
$$= \frac{29}{8}$$

➤ To convert $\frac{17}{5}$ to a mixed number:

$$\frac{17}{5} = \frac{15}{5} + \frac{2}{5}$$
$$= 3\frac{2}{5}$$

✓ Check

2. Convert each mixed number to an improper fraction.

a) $3\frac{4}{5} = \mathbf{3 + \frac{4}{5}}$

$= \mathbf{\frac{15}{5}} + \mathbf{\frac{4}{5}}$

$= \mathbf{\frac{19}{5}}$

b) $5\frac{4}{9} = \mathbf{\frac{49}{9}}$

c) $3\frac{7}{20} = \mathbf{\frac{67}{20}}$

d) $2\frac{1}{24} = \mathbf{\frac{49}{24}}$

3. Convert each improper fraction to a mixed number.

a) $\frac{27}{8} = \frac{24}{8} + \frac{3}{8}$

$= \mathbf{3\frac{3}{8}}$

b) $\frac{41}{18} = \mathbf{2\frac{5}{18}}$

c) $\frac{41}{15} = \mathbf{2\frac{11}{15}}$

d) $\frac{29}{12} = \mathbf{2\frac{5}{12}}$

Adding and Subtracting Fractions

➤ To add or subtract fractions with the same denominator, add or subtract the numerators.

$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$ $\frac{9}{13} - \frac{3}{13} = \frac{6}{13}$

➤ To add or subtract fractions with different denominators:

- Use the least common multiple of the denominators as the common denominator.
- Write equivalent fractions with this common denominator.

To add $\frac{1}{4}+\frac{5}{6}$, find the least common multiple of 4 and 6.

The least common multiple of 4 and 6 is 12.

$$\frac{1}{4}+\frac{5}{6}=\frac{3}{12}+\frac{10}{12}$$
$$=\frac{13}{12}$$

To subtract $\frac{5}{8}-\frac{1}{12}$, find the least common multiple of 8 and 12.

The least common multiple of 8 and 12 is 24.

$$\frac{5}{8}-\frac{1}{12}=\frac{15}{24}-\frac{2}{24}$$
$$=\frac{13}{24}$$

➤ To add or subtract mixed numbers, add or subtract the fractions and then add or subtract the whole numbers. Sometimes, you need to regroup a whole number to subtract the fractions. Simplify if necessary.

$$2\frac{1}{4}+3\frac{5}{6}=2\frac{3}{12}+3\frac{10}{12}$$
$$=5\frac{13}{12}$$
$$=5+1\frac{1}{12}$$
$$=6\frac{1}{12}$$

$$5\frac{1}{8}-3\frac{1}{2}=5\frac{1}{8}-3\frac{4}{8}$$
$$=4\frac{9}{8}-3\frac{4}{8}$$
$$=1\frac{5}{8}$$

✓ Check

4. Add. Write the answer in simplest form. Write improper fractions as mixed numbers.

a) $\frac{7}{10}+\frac{1}{6}=$ $\mathbf{\frac{21}{30}}$ + $\mathbf{\frac{5}{30}}$

$=$ $\mathbf{\frac{26}{30}}$

$=$ $\mathbf{\frac{13}{15}}$

b) $\frac{1}{2}+\frac{3}{7}=$ $\mathbf{\frac{13}{14}}$

c) $3\frac{1}{3}+4\frac{1}{2}=$ $\mathbf{3+4+\frac{1}{3}+\frac{1}{2}}$

$=$ $\mathbf{7\frac{5}{6}}$

d) $2\frac{5}{6}+1\frac{3}{8}=$ $\mathbf{4\frac{5}{24}}$

5. Subtract. Write the answer in simplest form. Write improper fractions as mixed numbers.

a) $\frac{3}{4}-\frac{3}{10}=$ $\mathbf{\frac{15}{20}-\frac{6}{20}}$

$=$ $\mathbf{\frac{9}{20}}$

b) $\frac{5}{8}-\frac{1}{6}=$ $\mathbf{\frac{11}{24}}$

c) $4\frac{1}{9}-2\frac{2}{3}=4\frac{1}{9}-2\frac{6}{9}$

$=3\frac{10}{9}-$ $\mathbf{2\frac{6}{9}}$

$=$ $\mathbf{1\frac{4}{9}}$

d) $7\frac{1}{4}-3\frac{5}{6}=$ $\mathbf{3\frac{5}{12}}$

3.1 Using Models to Multiply Fractions and Whole Numbers

Quick Review

➤ Repeated addition can be written as multiplication.

$$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=5\times\frac{1}{3}$$

$$=\frac{5}{3}$$

$$=\frac{3}{3}+\frac{2}{3}$$

$$=1\frac{2}{3}$$

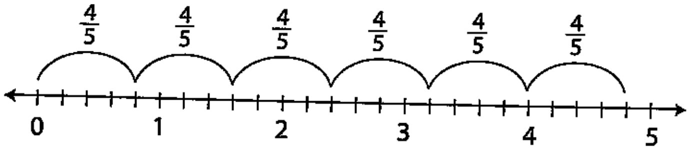
$$\frac{4}{5}+\frac{4}{5}+\frac{4}{5}+\frac{4}{5}+\frac{4}{5}=6\times\frac{4}{5}$$

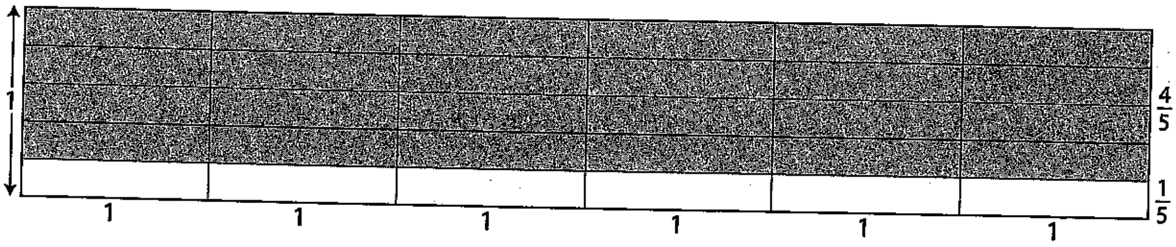
$$=\frac{24}{5}$$

$$=\frac{20}{5}+\frac{4}{5}$$

$$=4\frac{4}{5}$$

$6\times\frac{4}{5}=4\frac{4}{5}$ can also be shown on a number line or using a rectangle.





Practice

1. Write each addition statement as a multiplication statement and determine the product.

a) $\frac{5}{7}+\frac{5}{7}+\frac{5}{7}+\frac{5}{7}=\underline{\ 4\ }\times\frac{5}{7}$

$=\frac{20}{7}$

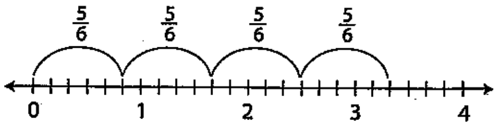
$=2\frac{6}{7}$

b) $\frac{7}{8}+\frac{7}{8}+\frac{7}{8}+\frac{7}{8}+\frac{7}{8}=\underline{4\frac{3}{8}}$

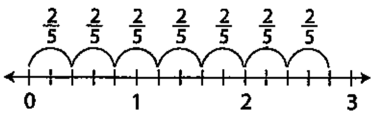
c) $\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=\underline{2\frac{2}{3}}$

d) $\frac{7}{12}+\frac{7}{12}+\frac{7}{12}+\frac{7}{12}+\frac{7}{12}+\frac{7}{12}=\underline{3\frac{1}{2}}$

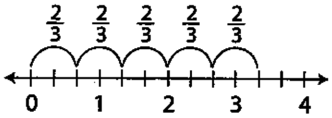
2. Write the multiplication sentence represented by each number line.

a) 

$4 \times \frac{5}{6} = \frac{20}{5}$, or $3\frac{1}{3}$

b) 

$7 \times \frac{2}{5} = \frac{14}{5}$, or $2\frac{4}{5}$

c) 

$5 \times \frac{2}{3} = \frac{10}{3}$, or $3\frac{1}{3}$

3. Multiply. Use a model to help.

a) $3 \times \frac{7}{12} =$ $1\frac{3}{4}$

b) $20 \times \frac{3}{4} =$ 15

c) $\frac{2}{3} \times 18 =$ 12

d) $\frac{4}{9} \times 10 =$ $4\frac{4}{9}$

e) $6 \times \frac{3}{4} =$ $3\frac{3}{5}$

f) $\frac{5}{8} \times 9 =$ $5\frac{5}{8}$

4. Match each multiplication to the correct product.

a) $2 \times \frac{4}{5}$	**i)** $7\frac{1}{2}$
b) $\frac{3}{8}$ of 13	**ii)** $4\frac{2}{3}$
c) $5 \times \frac{3}{4}$	**iii)** $1\frac{3}{5}$
d) $9 \times \frac{5}{6}$	**iv)** $3\frac{3}{4}$
e) $\frac{2}{3} \times 7$	**v)** $4\frac{7}{8}$

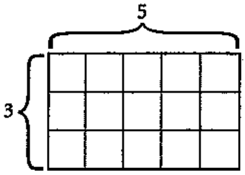
5. It takes $\frac{3}{4}$ h to frame a picture. How long will it take to frame 13 pictures?

It will take $9\frac{3}{4}$ h to frame 13 pictures.

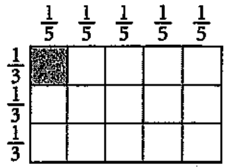
3.2 Using Models to Multiply Fractions

Quick Review

Area models are useful for visualizing multiplication.

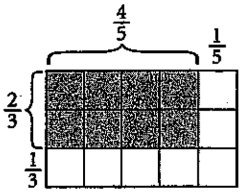


- ➤ The area of a rectangle is length multiplied by width.
  A 5 by 3 rectangle covers 15 unit squares.
  So, $5 \times 3 = 15$.



- ➤ To model $\frac{1}{5} \times \frac{1}{3}$, draw a 5 by 3 rectangle.
  The rectangle has 15 equal parts.
  A horizontal row of 5 squares represents $\frac{1}{3}$ of the rectangle.
  $\frac{1}{5}$ of this row of $\frac{1}{3}$ covers 1 of the 15 parts.

  So, $\frac{1}{5} \times \frac{1}{3} = \frac{1}{15}$.



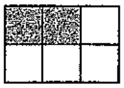
- ➤ 2 horizontal rows of 5 squares represent $\frac{2}{3}$ of the rectangle.
  $\frac{4}{5}$ of these 2 horizontal rows of 5 covers 8 of the 15 parts.
  So, $\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$.

Practice

1. Write the multiplication sentence modelled by the shaded region in each rectangle.

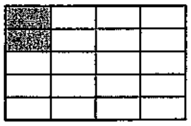
Tip
Write all fractions in simplest form.

a)



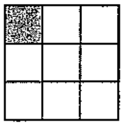
$\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$

b)



$\frac{1}{4} \times \frac{2}{5} = \frac{1}{10}$

c)



$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

d)

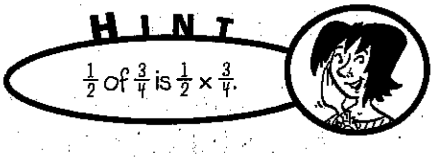
$\frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$

2. Draw an area model for each product. Then find the product. Write all fractions in simplest form.

Models may vary.

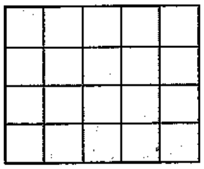
a) $\frac{1}{4} \times \frac{3}{4} = \underline{\mathbf{\frac{3}{16}}}$ **b)** $\frac{1}{2} \times \frac{2}{3} = \underline{\mathbf{\frac{1}{3}}}$

3. Tom took $\frac{3}{4}$ of a pie. He could only eat $\frac{1}{2}$ of what he took. What fraction of the pie did Tom eat?



Tom ate $\frac{3}{8}$ of the pie.

4. Use the area model below to calculate each product.



$\frac{1}{5} \times \frac{1}{4} = \underline{\mathbf{\frac{1}{20}}}$ $\frac{2}{5} \times \frac{3}{4} = \underline{\mathbf{\frac{6}{20}}}$ $\frac{3}{5} \times \frac{1}{4} = \underline{\mathbf{\frac{3}{20}}}$ $\frac{1}{5} \times \frac{1}{2} = \underline{\mathbf{\frac{1}{10}}}$ $\frac{3}{5} \times \frac{1}{2} = \underline{\mathbf{\frac{3}{10}}}$

Look for a pattern in the numbers. Describe a relationship between the numerator and the denominator of each answer fraction and those of the fractions being multiplied.

The numerator of the answer fraction is the product of the numerators of the fractions being multiplied.

The denominator of the answer fraction is the product of the denominators of the fractions being multiplied.

5. Determine each product.

a) $\frac{3}{4} \times \frac{2}{5} = \underline{\mathbf{\frac{3}{10}}}$ **b)** $\frac{5}{8} \times \frac{2}{3} = \underline{\mathbf{\frac{5}{12}}}$ **c)** $\frac{4}{7} \times \frac{2}{3} = \underline{\mathbf{\frac{8}{21}}}$ **d)** $\frac{2}{3} \times \frac{7}{10} = \underline{\mathbf{\frac{7}{15}}}$

3.3 Multiplying Fractions

Quick Review

- To multiply fractions without using a model, multiply the numerators and multiply the denominators.

  $\frac{3}{4} \times \frac{2}{5} = \frac{3 \times 2}{4 \times 5} = \frac{6}{20} = \frac{3}{10}$

- If the numerators and denominators have common factors, divide by the common factors before multiplying.

$$\frac{5}{12} \times \frac{8}{12} = \frac{5 \times 8}{12 \times 15}$$

$$= \frac{\overset{1}{\cancel{5}} \times \overset{2}{\cancel{8}}}{\overset{3}{\cancel{12}} \times \overset{3}{\cancel{15}}}$$

$$= \frac{1 \times 2}{3 \times 3}$$

$$= \frac{2}{9}$$

5 ÷ 5 = 1

8 ÷ 4 = 2

12 ÷ 4 = 3

15 ÷ 5 = 3

Practice

1. Multiply.

a) $\frac{3}{8} \times \frac{4}{15} = \frac{3 \times 4}{8 \times 15}$

3 is the greatest common factor for 3 and 15.

4 is the greatest common factor for 4 and 8.

$$\frac{\overset{1}{\cancel{3}} \times \overset{1}{\cancel{4}}}{\overset{2}{\cancel{8}} \times \overset{5}{\cancel{15}}} = \frac{1 \times 1}{2 \times 5}$$

$$= \frac{1}{10}$$

b) $\frac{3}{5} \times \frac{5}{6} = \frac{1}{2}$

c) $\frac{3}{4} \times \frac{7}{9} = \frac{7}{12}$

d) $\frac{8}{9} \times \frac{3}{10} = \frac{4}{15}$

d) $\frac{13}{9} \times \frac{3}{26} = \frac{1}{6}$

2. Simplify before multiplying. Express products as proper fractions.

a) $\frac{7}{3} \times \frac{9}{14} = \underline{1\frac{1}{2}}$ **b)** $\frac{15}{8} \times \frac{10}{9} = \underline{2\frac{1}{12}}$ **c)** $\frac{15}{4} \times \frac{2}{9} = \underline{\frac{5}{6}}$ **d)** $\frac{12}{5} \times \frac{10}{9} = \underline{2\frac{2}{3}}$

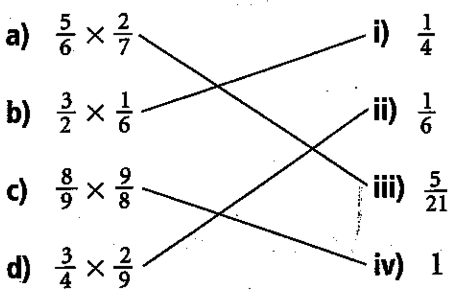
3. Multiply.

a) $\frac{15}{8} \times \frac{3}{5} = \underline{1\frac{1}{8}}$ **b)** $\frac{6}{7} \times \frac{2}{3} = \underline{\frac{4}{7}}$ **c)** $\frac{5}{6} \times \frac{3}{10} = \underline{\frac{1}{4}}$ **d)** $\frac{7}{15} \times \frac{10}{21} = \underline{\frac{2}{9}}$

4. Multiply. Estimate to check that each product is reasonable.

a) $\frac{44}{35} \times \frac{7}{33} = \underline{\frac{4}{15}}$ **b)** $\frac{34}{33} \times \frac{22}{17} = \underline{1\frac{1}{3}}$ **c)** $\frac{57}{91} \times \frac{14}{19} = \underline{\frac{6}{13}}$ **d)** $\frac{39}{64} \times \frac{24}{13} = \underline{1\frac{1}{8}}$

5. Match each multiplication to the correct product.



6. In the school band, $\frac{3}{5}$ of the students play the trumpet. Of these, $\frac{1}{6}$ also play the trombone. What fraction of the students in the band play both trumpet and trombone?

$\underline{\frac{1}{10}}$ of the students in the band play both trumpet and trombone.

7. Jeremy ate $\frac{1}{3}$ of an apple pie. Sara ate $\frac{1}{4}$ of the remainder. What fraction of the pie did Sara eat?

$\underline{\frac{2}{3}}$ of the pie was left after Jeremy had his share. So, Sara ate $\underline{\frac{1}{4}} \times \underline{\frac{2}{3}} = \underline{\frac{1}{6}}$ of the pie.

8. Leona spent $\frac{5}{8}$ of $\frac{2}{3}$ of her allowance on magazines. What fraction of her total allowance did she spend on magazines? What fraction did she have left?

Leona spent $\underline{\frac{5}{12}}$ of her allowance on magazines. She had $1 - \underline{\frac{5}{12}} = \underline{\frac{7}{12}}$ of her allowance left.

3.4 Multiplying Mixed Numbers

Quick Review

➤ An area model is often useful for visualizing a multiplication.

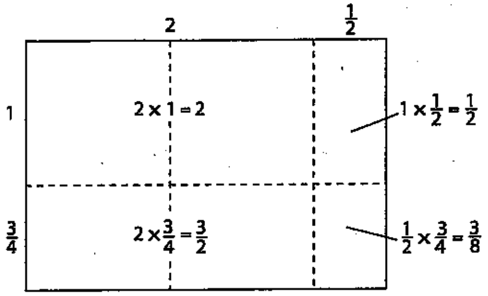
$$2\tfrac{1}{2} \times 1\tfrac{3}{4} = (2\times1)+\left(\tfrac{1}{2}\times1\right)+\left(2\times\tfrac{3}{4}\right)+\left(\tfrac{1}{2}\times\tfrac{3}{4}\right)$$

$$= 2 + \tfrac{1}{2} + \tfrac{3}{2} + \tfrac{3}{8}$$

$$= \tfrac{16}{8} + \tfrac{4}{8} + \tfrac{12}{8} + \tfrac{3}{8}$$

$$= \tfrac{35}{8}$$

$$= 4\tfrac{3}{8}$$



➤ Another way to multiply mixed numbers is to first convert to improper fractions.

Multiply: $1\tfrac{1}{5} \times 3\tfrac{1}{8}$

$$1\tfrac{1}{5} \times 3\tfrac{1}{8} = \tfrac{6}{5} \times \tfrac{25}{8}$$

$$= \frac{\overset{3}{\cancel{6}} \times \overset{5}{\cancel{25}}}{\overset{1}{\cancel{5}} \times \overset{4}{\cancel{8}}}$$

$$= \tfrac{15}{4}$$

$$= 3\tfrac{3}{4}$$

$6 \div 2 = 3$ $25 \div 5 = 5$
$5 \div 5 = 1$ $8 \div 2 = 4$

Practice

1. Write each mixed number as an improper fraction.

a) $2\frac{3}{5} = \underline{\mathbf{\frac{13}{5}}}$ b) $4\frac{3}{4} = \underline{\mathbf{\frac{19}{4}}}$ c) $3\frac{1}{6} = \underline{\mathbf{\frac{19}{6}}}$ d) $1\frac{7}{12} = \underline{\mathbf{\frac{19}{12}}}$

2. Write each improper fraction as a mixed number.

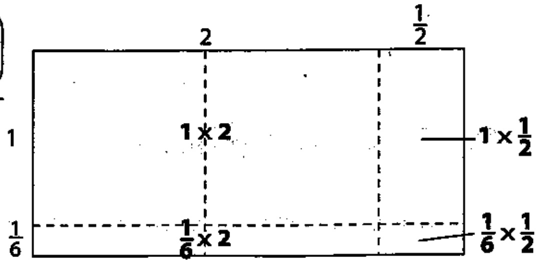
a) $\frac{43}{8} = \underline{\mathbf{5\frac{3}{8}}}$ b) $\frac{19}{6} = \underline{\mathbf{3\frac{1}{6}}}$ c) $\frac{17}{3} = \underline{\mathbf{5\frac{2}{3}}}$ d) $\frac{27}{4} = \underline{\mathbf{6\frac{3}{4}}}$

3. **a)** Show the product $1\frac{1}{6} \times 1\frac{1}{2}$ on the rectangle. State the product.

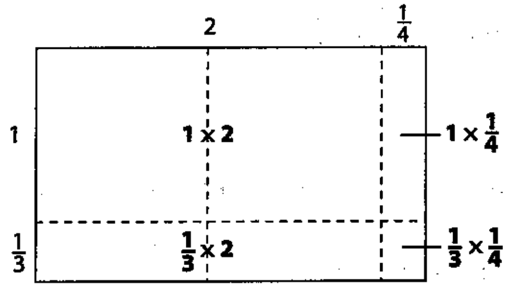
$1\frac{1}{6} \times 2\frac{1}{2} = (\underline{1\times2}) + \underline{\left(1\times\frac{1}{2}\right)} + \underline{\left(\frac{1}{6}\times2\right)} + \underline{\left(\frac{1}{6}\times\frac{1}{2}\right)}$

$= 2 + \underline{\frac{1}{2}+\frac{1}{3}+\frac{1}{12}}$

$= \underline{2\frac{11}{12}}$



b) Draw an area model to show the product $2\frac{1}{4} \times 1\frac{1}{3}$. Determine the product.



$2\frac{1}{4} \times 1\frac{1}{3} = \underline{3}$

4. Multiply. Express answers as proper fractions.

a) $2\frac{5}{8} \times 1\frac{5}{7} = \underline{\frac{21}{8} \times \frac{12}{7}}$

$= \underline{\frac{9}{2}}$

$= \underline{4\frac{1}{2}}$

b) $2\frac{1}{10} \times 2\frac{2}{3} = \underline{5\frac{3}{5}}$

c) $1\frac{1}{8} \times 3\frac{1}{3} = \underline{3\frac{3}{4}}$

d) $2\frac{1}{4} \times 2\frac{2}{3} = \underline{6}$

e) $4\frac{2}{5} \times 2\frac{1}{7} = \underline{9\frac{3}{7}}$

f) $4\frac{4}{5} \times 2\frac{1}{4} = \underline{10\frac{4}{5}}$

5. George practises his guitar for $1\frac{1}{5}$ h per day on school days. On Saturdays, he increases his practice time to $2\frac{1}{2}$ times his normal time. How many hours does he practise on Saturdays?

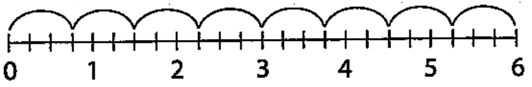
George practises for 3 h on Saturdays.

3.5 Dividing Whole Numbers and Fractions

Quick Review

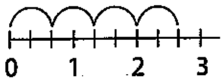
A number line can be used to help divide a whole number by a fraction.

- ➤ To determine how many three-quarters there are in 6, divide 6 into quarters.
Arrange 24 quarters into groups of three.
There are 8 groups of three-quarters.



$6 \div \frac{3}{4} = 8$

- ➤ To determine how many two-thirds there are in 3, divide 6 into thirds.
Arrange 18 thirds into groups of two-thirds.
There are 4 groups of two-thirds and one-third left over.



$\frac{1}{3}$ is $\frac{1}{2}$ of $\frac{2}{3}$.

So, $3 \div \frac{2}{3} = 4\frac{1}{2}$

To find $\frac{4}{5} \div 3$, think of sharing $\frac{4}{5}$ into 3 equal parts.

Each part is $\frac{1}{3}$ of $\frac{4}{5}$, or $\frac{1}{3} \times \frac{4}{5} = \frac{4}{15}$

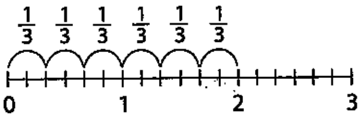
So, $\frac{4}{5} \div 3 = \frac{4}{15}$

/3b

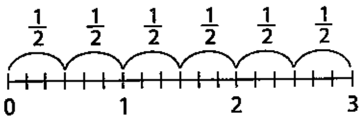
Practice

1. Use the number line to determine each quotient.

a) $2 \div \frac{1}{3} =$ 6

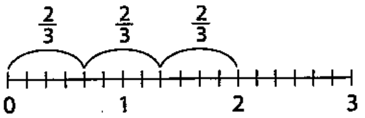


b) $3 \div \frac{1}{2} =$ 6

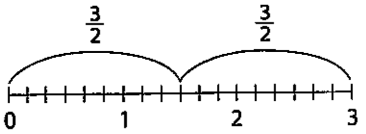


/4

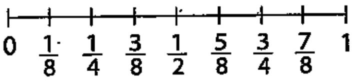
c) $2 \div \frac{2}{3} =$ 3



d) $3 \div \frac{3}{2} =$ 2



2. Use the number line to determine each quotient.



/4

a) $\frac{1}{2} \div 4 =$ $\frac{1}{8}$

b) $\frac{1}{4} \div 2 =$ $\frac{1}{8}$

c) $\frac{3}{4} \div 2 =$ $\frac{3}{8}$

d) $\frac{7}{8} \div 2 =$ $\frac{7}{16}$

16
12
3 8
36

3. Use fraction circles, a number line, or a picture to determine each quotient.

12

a) $2 \div \frac{1}{7} =$ **14**

b) $3 \div \frac{1}{3} =$ **9**

c) $5 \div \frac{5}{6} =$ **6**

d) $6 \div \frac{3}{5} =$ **10**

4. Determine each quotient.

/8

a) $2 \div \frac{3}{4} =$ **$2\frac{2}{3}$**

b) $3 \div \frac{2}{3} =$ **$4\frac{1}{2}$**

c) $2 \div \frac{3}{8} =$ **$5\frac{1}{3}$**

d) $2 \div \frac{3}{5} =$ **$3\frac{1}{3}$**

e) $\frac{3}{5} \div 2 =$ **$\frac{3}{10}$**

f) $\frac{3}{4} \div 5 =$ **$\frac{3}{20}$**

g) $\frac{5}{6} \div 2 =$ **$\frac{5}{12}$**

h) $\frac{1}{2} \div 2 =$ **$\frac{1}{4}$**

6. **a)** Two-thirds of a bag of candies is shared equally among 6 people. What fraction of the candies does each person receive?

Each person receives **$\frac{1}{9}$** of the bag of candies.

/4

b) How many two-thirds cup servings are in 12 cups of fruit?

There are **18** two-thirds cup servings in 12 cups of fruit.

7. **a)** Write the digits 3, 4, and 12 in the boxes to obtain the greatest quotient. Is there more than one answer?

$\square \div \frac{\square}{\square} =$ ______

2

/4

Sample answers:

$4 \div \frac{3}{12} = 16$ or $12 \div \frac{3}{4} = 16$

b) Write the digits 3, 4, and 12 in the boxes to obtain the least quotient. Is there more than one answer?

2 $\square \div \frac{\square}{\square} =$ ______

Sample answers:

$3 \div \frac{12}{4} = 1$ or $4 \div \frac{12}{3} = 1$

3.6 Dividing Fractions

Quick Review

There are at least two ways to divide fractions.

- ➤ Use common denominators

  To divide: $\frac{3}{4} \div \frac{1}{6}$

  Write the fractions with common denominator 12:

  $\frac{3}{4} \div \frac{1}{6} = \frac{9}{12} \div \frac{2}{12}$

  How many two-twelfths are in nine-twelfths?

  $9 \div 2 = 4\frac{1}{2}$

  So, $\frac{3}{4} \div \frac{1}{6} = 4\frac{1}{2}$

- ➤ You can also divide by a fraction by multiplying by the reciprocal.

  To divide: $\frac{4}{5} \div \frac{2}{3}$

  The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

  $$\frac{4}{5} \div \frac{2}{3} = \frac{4}{5} \times \frac{3}{2}$$
  $$= \frac{6}{5}$$
  $$= 1\frac{1}{5}$$

/42

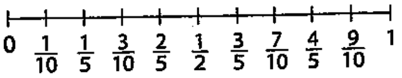
Practice

1. Write the reciprocal of each fraction.

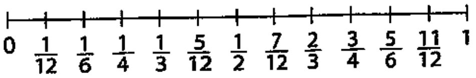
/4 **a)** $\frac{4}{7}$ $\underline{\frac{7}{4}}$ **b)** $\frac{3}{8}$ $\underline{\frac{8}{3}}$ **c)** $\frac{11}{15}$ $\underline{\frac{15}{11}}$ **d)** $\frac{7}{8}$ $\underline{\frac{8}{7}}$

2. Use the number line to determine each quotient.

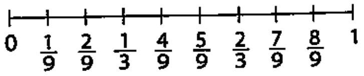
a) $\frac{9}{10} \div \frac{2}{5} = \underline{2\frac{1}{4}}$



/6 **b)** $\frac{2}{3} \div \frac{1}{4} = \underline{2\frac{2}{3}}$



c) $\frac{2}{3} \div \frac{4}{9} = \underline{1\frac{1}{2}}$



3. Divide. Estimate to check that each quotient is reasonable.

a) $\frac{6}{7} \div \frac{3}{7}$ There are 2 three-sevenths in six-sevenths. So, $\frac{6}{7} \div \frac{3}{7} =$ 2

/4

b) $\frac{8}{9} \div \frac{5}{9} = 1\frac{3}{5}$ **c)** $\frac{4}{5} \div \frac{3}{5} = 1\frac{1}{3}$ **d)** $\frac{7}{8} \div \frac{3}{8} = 2\frac{1}{3}$

4. Use common denominators to determine each quotient.

a) $\frac{7}{8} \div \frac{1}{4} = \frac{7}{8} \div \frac{2}{8}$ There are $3\frac{1}{2}$ **two** -eighths in seven-eighths. So, $\frac{7}{8} \div \frac{1}{4} = 3\frac{1}{2}$

/8

b) $\frac{4}{5} \div \frac{1}{10} = 8$ **c)** $\frac{3}{4} \div \frac{2}{5} = 1\frac{7}{8}$ **d)** $\frac{6}{7} \div \frac{1}{3} = 2\frac{4}{7}$

5. Divide by multiplying by the reciprocal.

a) $\frac{9}{4} \div \frac{2}{3} = \frac{9}{4} \times \frac{3}{2}$ **b)** $\frac{7}{3} \div \frac{4}{5} = 2\frac{11}{12}$ **c)** $\frac{5}{2} \div \frac{3}{8} = 6\frac{2}{3}$ **d)** $\frac{3}{4} \div \frac{9}{2} = \frac{1}{6}$

/8

$= \frac{27}{8}$

$= 3\frac{3}{8}$

6. Suppose you have $\frac{3}{4}$ of a cake. How many servings of each size can you make?

a) $\frac{1}{4}$ of the cake 3 servings **b)** $\frac{1}{6}$ of the cake $4\frac{1}{2}$ servings

/8

c) $\frac{1}{3}$ of the cake $2\frac{1}{4}$ servings **d)** $\frac{1}{2}$ of the cake $1\frac{1}{2}$ servings

7. How many pieces of ribbon, each $\frac{1}{6}$ m long, can be cut from a ribbon $\frac{7}{8}$ m long?

4

$5\frac{1}{4}$ pieces of ribbon can be cut. That is 5 whole pieces of ribbon with $\frac{1}{4}$ of a piece,

or $\frac{1}{24}$ m, left over.

3.7 Dividing Mixed Numbers

Quick Review

To divide mixed numbers without using a model, use either of the following methods.

Divide: $2\frac{5}{6} \div 1\frac{1}{4}$

- Write each mixed number as an improper fraction and then use common denominators.

$$2\frac{5}{6} \div 1\frac{1}{4} = \frac{17}{6} \div \frac{5}{4}$$

$$= \frac{34}{12} \div \frac{15}{12}$$

$$= 34 \div 15$$

$$= \frac{34}{15}\text{, or } 2\frac{4}{15}$$

- Use multiplication.

$$2\frac{5}{6} \div 1\frac{1}{4} = \frac{17}{6} \div \frac{5}{4}$$

Dividing by $\frac{5}{4}$ is the same as multiplying by $\frac{4}{5}$.

So, $\frac{17}{6} \div \frac{5}{4} = \frac{17}{6} \times \frac{4}{5}$

$$= \frac{17 \times \overset{2}{\cancel{4}}}{\overset{3}{\cancel{6}} \times 5}$$

$$= \frac{34}{15}\text{, or } 2\frac{4}{15}$$

Practice

1. Write each mixed number as an improper fraction.

a) $4\frac{5}{8} = \underline{\frac{37}{8}}$ **b)** $3\frac{5}{7} = \underline{\frac{26}{7}}$ **c)** $2\frac{5}{12} = \underline{\frac{29}{12}}$ **d)** $3\frac{4}{9} = \underline{\frac{31}{9}}$

2. Write each pair of mixed numbers as improper fractions with the same denominator.

a) $2\frac{1}{3}, 4\frac{1}{6}$

Write the fraction part of each mixed number with the same denominator, $\underline{6}$:

$2\frac{1}{3} = 2\,\underline{\frac{2}{6}}$, $4\frac{1}{6} = 4\,\underline{\frac{1}{6}}$

Write each mixed number as an improper fraction: $\underline{\frac{14}{6}, \frac{25}{6}}$

b) $3\frac{3}{5}, 2\frac{1}{10}$ $\underline{\frac{36}{10}, \frac{21}{10}}$ **c)** $4\frac{1}{5}, 2\frac{1}{2}$ $\underline{\frac{42}{10}, \frac{25}{10}}$ **d)** $2\frac{1}{4}, 1\frac{2}{3}$ $\underline{\frac{27}{12}, \frac{20}{12}}$

3. Use common denominators to determine each quotient. Estimate to check that the quotients are reasonable.

a) $4\frac{3}{4} \div 2\frac{1}{2} = \frac{19}{4} \div \frac{5}{2}$

$= \frac{19}{4} \div \frac{10}{4}$

$= \underline{\mathbf{19}} \div \underline{\mathbf{10}}$

$= \underline{\mathbf{\frac{19}{10}}}$, or $\underline{\mathbf{1\frac{9}{10}}}$

b) $5\frac{1}{3} \div 2\frac{2}{9} = \underline{\mathbf{2\frac{2}{5}}}$ c) $3\frac{1}{2} \div 2\frac{1}{3} = \underline{\mathbf{1\frac{1}{2}}}$ d) $4\frac{4}{5} \div 1\frac{1}{2} = \underline{\mathbf{3\frac{1}{5}}}$

4. Divide by multiplying by the reciprocal. Estimate to check that the quotients are reasonable.

a) $3\frac{1}{2} \div 2\frac{1}{4} = \frac{7}{2} \div \frac{9}{4}$

$= \frac{7}{2} \times \underline{\mathbf{\frac{4}{9}}}$

$= \underline{\mathbf{\frac{14}{9}}}$, or $\underline{\mathbf{1\frac{5}{9}}}$

b) $5\frac{5}{6} \div 2\frac{7}{9} = \underline{\mathbf{2\frac{1}{10}}}$ c) $4\frac{1}{4} \div 1\frac{5}{12} = \underline{\mathbf{3}}$ d) $4\frac{4}{5} \div 2\frac{2}{3} = \underline{\mathbf{1\frac{4}{5}}}$

5. Divide. Estimate to check that the quotients are reasonable.

a) $2\frac{3}{4} \div 1\frac{1}{8} = \underline{\mathbf{2\frac{4}{9}}}$ b) $3\frac{3}{4} \div 2\frac{2}{5} = \underline{\mathbf{1\frac{9}{16}}}$ c) $5\frac{1}{2} \div 2\frac{1}{3} = \underline{\mathbf{2\frac{5}{14}}}$ d) $5\frac{1}{5} \div 2\frac{1}{10} = \underline{\mathbf{2\frac{10}{21}}}$

6. Paula ran $3\frac{1}{3}$ laps in $13\frac{1}{3}$ min. If she ran at a steady pace, how long did it take her to run one lap?

It took Paula __**4**__ min to run one lap.

7. Jonathon took $7\frac{1}{2}$ h to complete his project. He worked on the project for $1\frac{1}{2}$ h each evening.

How many evenings did Jonathon take to complete the project?

Jonathon took __**5**__ evenings to complete the project.

8. Which of the following quotients is the greatest? What is its value?

A $6\frac{2}{7} \div 1\frac{5}{7} = \underline{\mathbf{3\frac{2}{3}}}$ B $6\frac{3}{10} \div 1\frac{3}{4} = \underline{\mathbf{3\frac{3}{5}}}$ C $7\frac{1}{5} \div 2\frac{2}{15} = \underline{\mathbf{3\frac{3}{8}}}$

The product in part __**A**__ is the greatest. Its value is $\underline{\mathbf{3\frac{2}{3}}}$.

3.8 Solving Problems with Fractions

Quick Review

➤ When solving word problems, it is important to understand the problem. This is best done by explaining in your own words, drawing diagrams, or listing the steps required to obtain a solution. It also helps if you identify key words and related mathematical operations.

Parent-teacher interviews were held on Tuesday. Mr. Smith had 3 interviews that each lasted $\frac{3}{4}$ h, 5 interviews that lasted $\frac{1}{4}$ h each, and 4 that lasted $\frac{1}{3}$ h each. How long did Mr. Smith spend on interviews?

The question is asking for the total time.

3 interviews at $\frac{3}{4}$ h each suggests multiplying: $3 \times \frac{3}{4}\text{ h} = \frac{9}{4}\text{ h}$

5 interviews at $\frac{1}{4}$ h each suggests multiplying: $5 \times \frac{1}{4}\text{ h} = \frac{5}{4}\text{ h}$

4 interviews at $\frac{1}{3}$ h each suggests multiplying: $4 \times \frac{1}{3}\text{ h} = \frac{4}{3}\text{ h}$

Total time suggests adding:

$$\begin{aligned}\left(\frac{9}{4} + \frac{5}{4} + \frac{4}{3}\right)\text{h} &= \left(\frac{27}{12} + \frac{15}{12} + \frac{16}{12}\right)\text{h} \\ &= \frac{58}{12}\text{ h} \\ &= \frac{29}{6}\text{ h} \\ &= 4\frac{5}{6}\text{ h}\end{aligned}$$

Mr. Smith spent $4\frac{5}{6}$ h in interviews.

Practice

1. Which operation (addition, subtraction, multiplication, division) would you use to solve each problem?

a) Jon ate one-third of a bag of candies and Monika ate one-quarter of the bag. What fraction of the bag of candies did they eat? **addition**

b) How many three-quarter cups of milk can be poured from a 6-cup jug of milk? **division**

c) There are 186 students in grade 8. Two thirds of them have brown eyes. How many students have brown eyes? **multiplication**

2. Solve each problem in question 1.

a) Jon and Monika ate $\frac{1}{3}$ + $\frac{1}{4}$ of the bag of candies.

$\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12}$

$= \frac{7}{12}$

Jon and Monika ate $\frac{7}{12}$ of the bag of candies.

b) 8 three-quarter cups of milk can be poured from a 6-cup jug of milk

c) 124 students have brown eyes.

3. Maribeth works in a dog rescue centre. At feeding time, 5 of the dogs each get $\frac{3}{4}$ kg of food and 3 dogs each get $\frac{3}{5}$ kg of food. How much food does Maribeth feed to the dogs?

Maribeth feeds $5\frac{11}{20}$ kg of food to the dogs.

4. A recipe calls for $3\frac{3}{4}$ cups of flour and 2 cups of sugar. Teri wants to make $\frac{1}{3}$ of the recipe. How much flour and sugar does she need?

Teri needs $1\frac{1}{4}$ cups of flour and $\frac{2}{3}$ cups of sugar.

5. Fiona has $2\frac{4}{7}$ kg of rice that she wants to share equally among 6 people. How much rice does each person get?

Each person gets $\frac{3}{7}$ kg of rice.

6. Vonnie works in an engine repair shop where she replaces the oil after the engines have been repaired. She has $11\frac{2}{3}$ L of oil and each engine requires $1\frac{1}{6}$ L of oil. How many engines can she fill with oil?

Vonnie can fill 10 engines with oil.

7. A jug contains $3\frac{3}{4}$ cups of juice. Halla pours $\frac{5}{8}$ cups of juice into each of three glasses.

a) How much juice does she pour into the glasses? $\underline{1\frac{7}{8}}$ cups

b) How much juice is left in the jug? $\underline{1\frac{7}{8}}$ cups

8. How many $1\frac{5}{8}$-m pieces of ribbon can be cut from a ribbon $9\frac{3}{4}$ m long?

$\underline{6}$ pieces of ribbon

9. Sara spends $\frac{2}{5}$ of her salary on rent and $\frac{1}{3}$ of the remainder on food.

a) What fraction of her salary is left after Sara pays the rent?

$\underline{\frac{3}{5}}$ of her salary is left after she pays the rent.

b) What fraction of her salary is left after she pays for rent and food?

$\underline{\frac{2}{5}}$ of her salary is left after she pays for rent and food.

10. Justin spent $\frac{1}{3}$ of his money on clothes and $\frac{1}{4}$ on music videos. He then spent $\frac{3}{5}$ of the remainder on food.

a) What fraction of his money did Justin spend on clothes and videos? $\underline{\frac{7}{12}}$ of his money

b) What fraction of his money did Justin spend on food? $\underline{\frac{1}{4}}$ of his money

c) What fraction of his money did Justin spend altogether? $\underline{\frac{5}{6}}$ of his money

3.9 Order of Operations with Fractions

Quick Review

➤ The order of operations for fractions is the same as for whole numbers.
Do the operations in brackets first.
Then divide and multiply, in order, from left to right.
Then add and subtract, in order, from left to right.

$\frac{3}{14} \div \left(\frac{5}{8} - \frac{1}{4}\right) + \frac{2}{7} = \frac{3}{14} \div \left(\frac{5}{8} - \frac{2}{8}\right) + \frac{2}{7}$ Write the fractions in the brackets with common denominators.

$= \frac{3}{14} \div \frac{3}{8} + \frac{2}{7}$ Do the operation in the brackets first.

$= \frac{3}{14} \times \frac{8}{3} + \frac{2}{7}$ Divide by multiplying by the reciprocal.

$= \frac{\cancel{3}^{1} \times \cancel{8}^{4}}{\cancel{14}_{7} \times \cancel{3}_{1}} + \frac{2}{7}$

$= \frac{4}{7} + \frac{2}{7}$

$= \frac{6}{7}$ Add.

Practice

1. Which operation would you do first?

a) $\frac{7}{8} \div \left(\frac{3}{4} + \frac{3}{8}\right)$ **add the fractions in the brackets**

b) $\frac{7}{9} - \frac{5}{9} \times \frac{1}{4}$ **multiply**

c) $\left(\frac{9}{16} - \frac{3}{4}\right) \times \frac{5}{8}$ **subtract the fractions in the brackets**

d) $\frac{3}{4} \times \left(\frac{3}{4} - \frac{1}{4} \div \frac{1}{2}\right)$ **divide the fractions in the brackets**

2. Elise was asked to evaluate $1\frac{1}{3} \div \frac{3}{4} \times \frac{2}{3}$. Her work is shown below. Is her answer correct? Explain.

$1\frac{1}{3} \div \frac{3}{4} \times \frac{2}{3} = \frac{4}{3} \div \frac{1}{2}$

$= \frac{4}{3} \times \frac{2}{1}$

$= \frac{8}{3}$

$= 2\frac{2}{3}$

Her answer *is*/[✓ *is not*] correct.

She multiplied first instead of dividing first.

3. Evaluate. Show all steps.

a) $$\begin{aligned}\left(\frac{1}{2}+\frac{2}{3}\right)\times\frac{1}{7}&=\left(\frac{3}{6}+\frac{4}{6}\right)\times\frac{1}{7}\\&=\frac{7}{6}\times\frac{1}{7}\\&=\frac{1}{6}\end{aligned}$$

b) $$\begin{aligned}\left(1-\frac{1}{4}\right)\div\left(1+\frac{3}{4}\right)&=\frac{3}{4}\div\frac{7}{4}\\&=\frac{3}{4}\times\frac{4}{7}\\&=\frac{3}{7}\end{aligned}$$

c) $$\begin{aligned}\frac{1}{3}\div\left(\frac{5}{6}\times\frac{1}{4}\right)&=\frac{1}{3}\div\frac{5}{24}\\&=\frac{1}{3}\times\frac{24}{5}\\&=\frac{8}{5}\\&=1\frac{3}{5}\end{aligned}$$

d) $$\begin{aligned}\frac{4}{7}\times\frac{3}{5}-\frac{1}{5}&=\frac{12}{35}-\frac{1}{5}\\&=\frac{12}{35}-\frac{7}{35}\\&=\frac{5}{35}\\&=\frac{1}{7}\end{aligned}$$

4. Evaluate.

a) $\frac{7}{9}\times\frac{3}{5}-\frac{1}{6}\div\frac{5}{2}=\underline{\ \frac{2}{5}\ }$ b) $\frac{1}{8}+\frac{3}{4}\div\frac{5}{8}-\frac{4}{5}=\underline{\ \frac{21}{40}\ }$

c) $\frac{6}{7}\div\frac{3}{22}\times\frac{7}{11}\div\frac{8}{9}=\underline{\ 4\frac{1}{2}\ }$ d) $\frac{11}{12}+\frac{5}{6}\times\frac{3}{4}-\frac{5}{6}=\underline{\ \frac{17}{24}\ }$

5. Evaluate.

a) $3\frac{1}{3}\div4\frac{1}{6}\times2\frac{1}{4}=\underline{\ 1\frac{4}{5}\ }$ b) $\frac{4}{5}\times\frac{5}{8}\div\frac{5}{8}\times\frac{3}{4}=\underline{\ \frac{3}{5}\ }$

c) $\frac{5}{12}\div\frac{3}{8}\div\frac{3}{4}\times\frac{9}{10}=\underline{\ 1\frac{1}{3}\ }$ d) $3\frac{1}{2}\div5\frac{1}{3}\times1\frac{1}{3}\div1\frac{1}{6}=\underline{\ \frac{3}{4}\ }$

HINT

Convert mixed numbers to improper fractions first.

6. Evaluate.

a) $\left(\frac{5}{9}+\frac{2}{3}\right)\div\left(\frac{3}{4}+\frac{5}{8}\right)=\underline{\ \frac{8}{9}\ }$ b) $\frac{9}{16}-\left(\frac{3}{4}-\frac{2}{3}\right)\times\frac{3}{4}=\underline{\ \frac{1}{2}\ }$

c) $1\frac{3}{5}\times\left(\frac{5}{8}+\frac{3}{4}-\frac{5}{6}\right)=\underline{\ \frac{13}{15}\ }$ d) $\left(\frac{9}{16}\div\frac{5}{12}\right)-\left(\frac{2}{5}\times\frac{7}{8}\right)=\underline{\ 1\ }$

e) $2\frac{2}{3}\times1\frac{1}{8}+\left(2\frac{3}{4}+1\frac{3}{8}\right)=\underline{\ 7\frac{1}{8}\ }$ f) $\left(4\frac{5}{8}-2\frac{3}{4}\right)\div\left(2\frac{1}{3}+1\frac{1}{6}\right)=\underline{\ \frac{15}{28}\ }$

In Your Words

Here are some of the important mathematical words of this unit.
Build your own glossary by recording definitions and examples here. The first one is done for you.

proper and improper fractions

proper fractions have numerator less than denominator; improper fractions have numerator greater than denominator

simplest form of a fraction

a fraction in which the numerator and denominator have been divided by their greatest common factor

reciprocal of a fraction

a fraction, either proper or improper that is inverted.

For example, the reciprocal of $\frac{5}{7}$ *is* $\frac{7}{5}$.

mixed number

a number consisting of a whole number and a fraction

For example, $5\frac{3}{8}$ *is a mixed number.*

quotient

the result when one number is divided by another

order of operations

the rules that are followed when simplifying or evaluating an expression; brackets, multiplication and division, addition and subtraction

List other mathematical words you need to know.

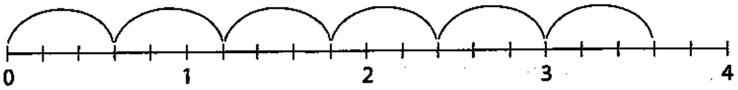
product, factor, equivalent fraction, divisor, dividend

Unit Review

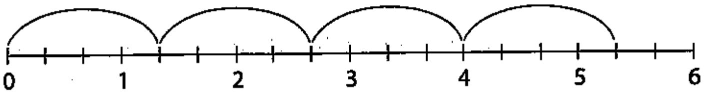
LESSON

3.1 **1.** Write the multiplication sentence represented by each number line.

a) $6 \times \frac{3}{5} = \frac{18}{5}$, or $3\frac{3}{5}$

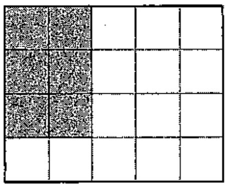


b) $4 \times \frac{4}{3} = \frac{16}{3}$, or $5\frac{1}{3}$



3.2 **2.** Shade each rectangle to show the product.

a) $\frac{3}{4} \times \frac{2}{5}$



b) $\frac{1}{3} \times \frac{5}{6}$

3.3 **3.** Multiply. Estimate to check that the solutions are reasonable.

a) $\frac{3}{4} \times \frac{8}{9} = \frac{2}{3}$ **b)** $\frac{5}{16} \times \frac{4}{15} = \frac{1}{12}$ **c)** $\frac{7}{6} \times \frac{8}{21} = \frac{4}{9}$

4. Claude mowed $\frac{1}{4}$ of the lawn before lunch. After lunch he mowed $\frac{2}{3}$ of the uncut lawn. What fraction of the lawn did Claude mow altogether?

Before he started mowing after lunch, Claude had $\frac{3}{4}$ of the lawn left to mow.

Claude mowed $\frac{1}{2}$ of the lawn altogether.

3.4 **5.** Write each mixed number as an improper fraction.

a) $3\frac{3}{5} = \frac{18}{5}$ **b)** $4\frac{7}{8} = \frac{39}{8}$ **c)** $1\frac{11}{16} = \frac{27}{16}$

LESSON

6. Multiply.

a) $3\frac{3}{8} \times 3\frac{1}{3} = \underline{\mathbf{11\frac{1}{4}}}$ b) $2\frac{2}{5} \times 6\frac{2}{3} = \underline{\mathbf{16}}$ c) $1\frac{5}{12} \times 2\frac{5}{8} = \underline{\mathbf{3\frac{23}{32}}}$

3.5 **7.** Use a model to determine each quotient.
Models may vary.

a) $4 \div \frac{2}{3} = \underline{\mathbf{6}}$ b) $5 \div \frac{3}{4} = \underline{\mathbf{6\frac{2}{3}}}$ c) $\frac{3}{5} \div \frac{3}{4} = \underline{\mathbf{\frac{4}{5}}}$

3.6 **8.** Divide.

a) $\frac{5}{12} \div \frac{10}{11} = \underline{\mathbf{\frac{11}{24}}}$ b) $\frac{3}{7} \div \frac{9}{14} = \underline{\mathbf{\frac{2}{3}}}$ c) $\frac{3}{5} \div \frac{5}{6} = \underline{\mathbf{\frac{18}{25}}}$

3.7 **9.** Divide. Estimate to check that the quotients are reasonable.

a) $2\frac{1}{4} \div 1\frac{7}{8} = \underline{\mathbf{1\frac{1}{5}}}$ b) $1\frac{3}{4} \div 2\frac{4}{5} = \underline{\mathbf{\frac{5}{8}}}$ c) $3\frac{3}{4} \div 2\frac{1}{12} = \underline{\mathbf{1\frac{4}{5}}}$

10. A recipe for chocolate cake calls for $1\frac{1}{4}$ cups of chocolate chips. Hasim has $7\frac{1}{2}$ cups of chocolate chips. How many cakes can he make?

Hasim can make **6** cakes.

LESSON

3.8 **11.** On Tuesday, $\frac{5}{12}$ of the grade 8 students attended the computer club meeting and $\frac{3}{8}$ of the grade 8 students attended the science club meeting. The meetings were at the same time. What fraction of the grade 8 students attended one of the meetings? What fraction did not attend either of the meetings?

$\mathbf{\frac{19}{24}}$ of the grade 8 students attended one of the meetings.

$\mathbf{\frac{5}{24}}$ of the grade 8 students did not attend either of the meetings.

12. Grace has $6\frac{3}{4}$ L of maple syrup that she wants to pour into $\frac{3}{4}$-L containers. How many containers can she fill?

Grace can fill **9** containers.

3.9 **13.** Evaluate.

a) $\frac{3}{5} + \frac{7}{15} \times \frac{9}{14} = \mathbf{\frac{9}{10}}$ **b)** $\left(\frac{3}{5} + \frac{7}{15}\right) \times \frac{9}{14} = \mathbf{\frac{24}{35}}$

14. Evaluate: $\frac{4}{7} \times \left(\frac{9}{5} - \frac{3}{4}\right) \div \frac{3}{8} = \mathbf{1\frac{3}{5}}$

UNIT 4

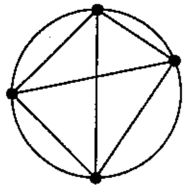
Measuring Prisms and Cylinders

Just for Fun

Handshakes

People are standing in a circle.
Each person shakes hands with every other person in the circle.

Draw a circle.
Then draw dots to represent the people.
Join any 2 dots to represent a handshake.



Record your results in the table.

Write a pattern for the number of handshakes.

Number of People	Number of Handshakes
1	0
2	1
3	3
4	**6**
5	**10**
6	**15**
7	**21**

Sample Answer: Starting from 1, as the number of people increases by 1, the number of handshakes increases in this pattern 1, 2, 3, 4, . . .

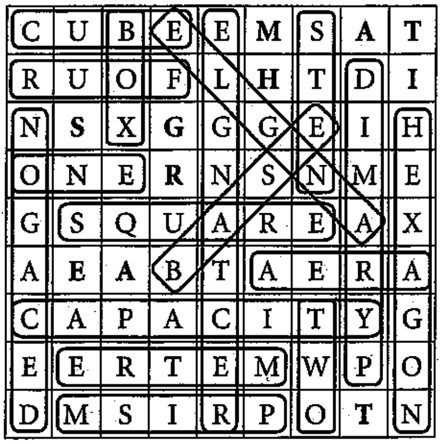
Word Search

1. Find the list of words in the word search table on the right. Words can be horizontal, vertical, or diagonal.

   ANGLE, AREA, BASE, BOX, CAPACITY, CUBE, DECAGON, FOUR, HEXAGON, METRE, NETS, ONE, PRISM, PYRAMID, RECTANGLE, SQUARE, TWO

2. Write all unused letters in order, row by row, from left to right. Separate the letters to form a phrase.

   MATH IS GREAT

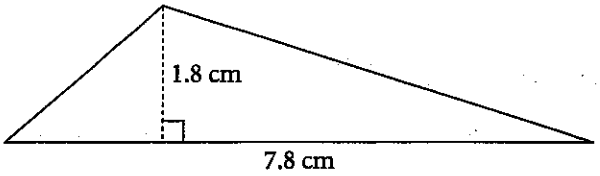


C	U	B	E	E	M	S	A	T
R	U	O	F	L	H	T	D	I
N	S	X	G	G	G	E	I	H
O	N	E	R	N	S	N	M	E
G	S	Q	U	A	R	E	A	X
A	E	A	B	T	A	E	R	A
C	A	P	A	C	I	T	Y	G
E	E	R	T	E	M	W	P	O
D	M	S	I	R	P	O	T	N

Activating Prior Knowledge

Area of Two-Dimensional Shapes

To calculate the area of this triangle, use the formula Area = $\frac{1}{2} \times$ base $\times$ height or $A = \frac{1}{2}bh$.



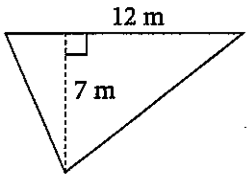
Substitute $b = 7.8$ and $h = 1.8$.

$A = \frac{1}{2}bh = \frac{1}{2}(7.8 \times 1.8) = 7.02$

The area is about 7 cm², to the nearest square centimetre.

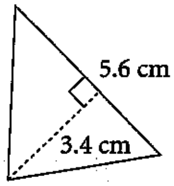
✓ Check

1. Calculate the area of each triangle.



a) $A = \frac{bh}{2} = \frac{12 \times 7}{2} =$ **42**

The area is **42** m².

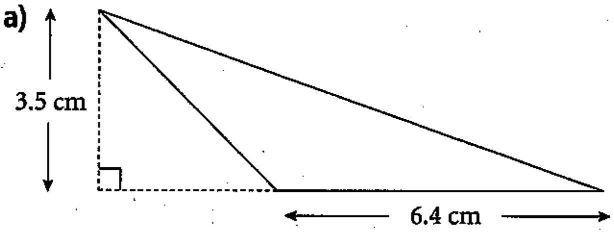


b) $A = \frac{bh}{2}$

The area is **9.52 cm²**.

2. Calculate the area of each triangle.

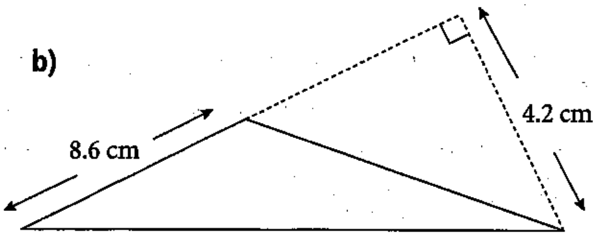
a)



$b =$ **6.4 cm** $h =$ **3.5 cm**

$A =$ **11.2 cm²**

b)



$b =$ **8.6 cm** $h =$ **4.2 cm**

$A =$ **18.06 cm²**

To calculate the area of a circle with diameter 14 cm, use the formula Area = $\pi \times$ radius² or $A = \pi r^2$. The diameter of the circle is 14 cm, so the radius is 7 cm.

Substitute $r = 7$ cm.

$A \doteq \pi r^2 = \pi \times 7^2 \doteq 153.938$

The area is about 154 cm², to the nearest square centimetre.

Tip

For π, use the π key on a calculator.

✓ Check

3. Calculate the area of each circle, to the nearest square unit.

a) diameter = 24 cm

$r = \frac{d}{2} = \frac{24}{2} = \underline{\ 12\ }$

$A = \pi r^2 \doteq \underline{\ 452.389\ }$ The area of the circle is **452 cm²**, to the nearest square **centimetre**.

b) radius = 9 m

$A = \pi r^2 \doteq \underline{\ 254.469\ }$ The area of the circle is **254 m²**, to the nearest square **metre**.

c) diameter = 11 mm The area of the circle is **95 mm²**, to the nearest square **millimetre**.

d) radius = 8 km The area of the circle is **201 km²**, to the nearest square **kilometre**.

Circumference of a Circle

To calculate the circumference of a circle with diameter 4.8 cm, use the formula Circumference = pi × diameter, or $C = \pi d$.

Substitute $d = 4.8$.

$C = \pi \times d = \pi \times 4.8 \doteq 15.080$

The circumference of the circle is about 15.1 cm, to one decimal place.

To calculate the circumference of a circle with radius 5.2 cm, use the formula Circumference = 2 × pi × radius or $C = 2\pi r$.

Substitute $r = 5.2$.

$C = 2 \times \pi \times r = 2 \times \pi \times 5.2 \doteq 32.673$

The circumference of the circle is about 32.7 cm, to one decimal place.

✓ Check

4. Calculate the circumference of each circle, to one decimal place.

a) $d = 12$ cm $C = \pi \times d = \pi \times \underline{\ 12\ } \doteq 37.699$

The circumference of the circle is **37.7 cm**, to one decimal place.

b) $r = 8$ m $C = 2 \times \pi \times r = 2 \times \pi \times \underline{\ 8\ } \doteq 50.265$

The circumference of the circle is **50.3 m**, to one decimal place.

c) $d = 5.6$ mm The circumference of the circle is **17.6 mm**, to one decimal place.

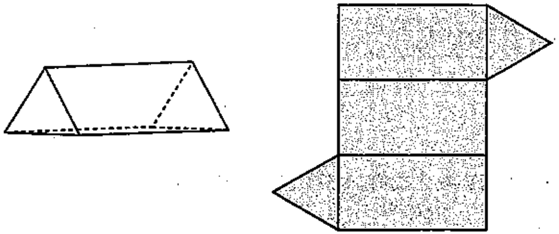
d) $r = 3.8$ m The circumference of the circle is **23.9 m**, to one decimal place.

4.1 Exploring Nets

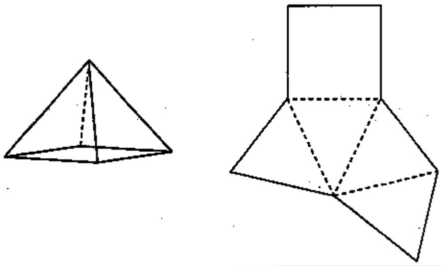
Quick Review

- ➤ A prism has two congruent bases and is named for its bases.
  A pyramid has one base and the other faces are congruent triangles.
- ➤ A net is a diagram that can be folded to make an object.

The diagram shows a triangular prism and its net.

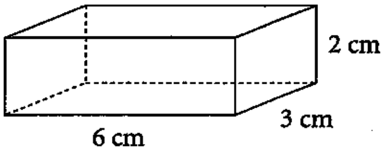


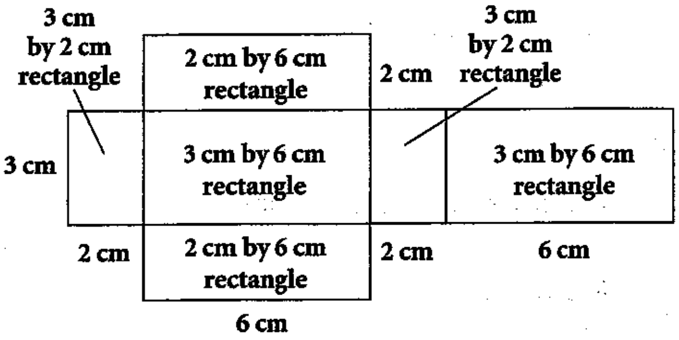
The diagram shows a square pyramid and its net.



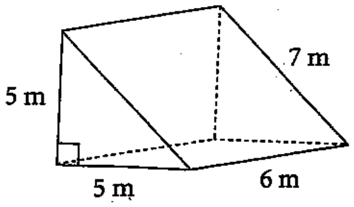
Practice

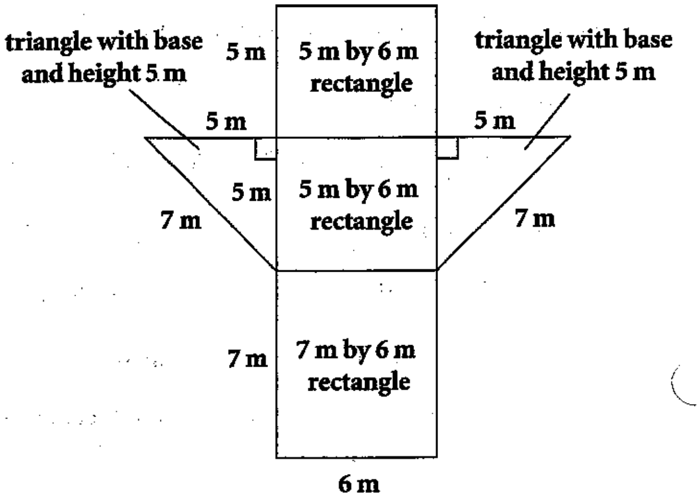
1. Sketch a net for the right rectangular prism. Identify and name each face.





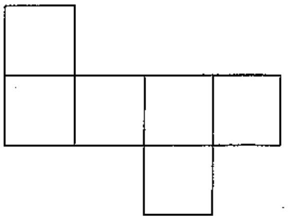
2. Sketch a net for the right triangular prism. Identify and name each face.



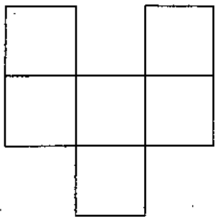


3. Which of the following diagrams is **not** the net of a cube?

A



B



C

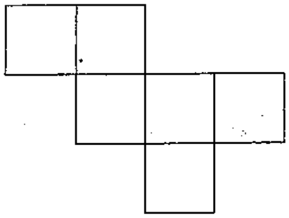
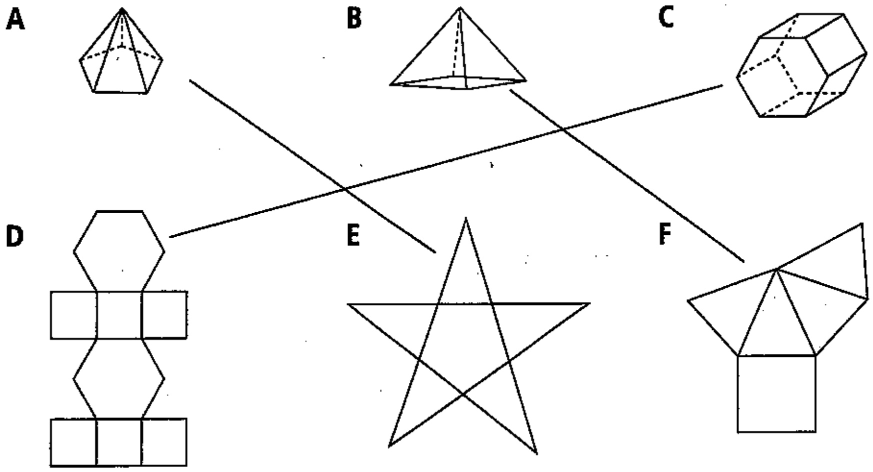


Diagram **B** is not the net of a cube.

4. **a)** Match each object to its net.



b) Identify and name each face of each object.

A and E represent a regular pentagonal pyramid with one pentagonal base and five isosceles triangles.

B and F represent a regular square pyramid with one square base and four isosceles triangles.

C and D represent a hexagonal prism with two hexagonal bases and six squares.

5. Use the descriptions to identify the object that has each set of faces.

a) six congruent triangles and one hexagon **hexagonal pyramid**

b) four congruent equilateral triangles **triangular pyramid**

c) two congruent squares and four congruent rectangles **square prism**

d) two congruent triangles and three rectangles **right triangular prism**

4.2 Creating Objects from Nets

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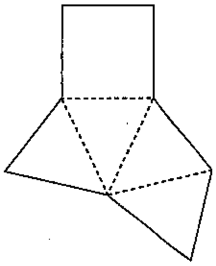
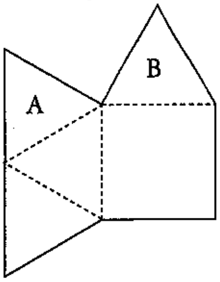
Quick Review

- To determine if a diagram is a net for an object, look at each shape and at how the shapes are arranged.

This is the net of a square pyramid.



This is not the net of a square pyramid. If the design is cut out and folded, triangles A and B will coincide.

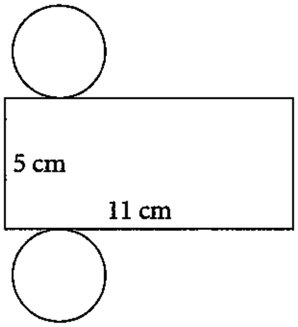


Practice

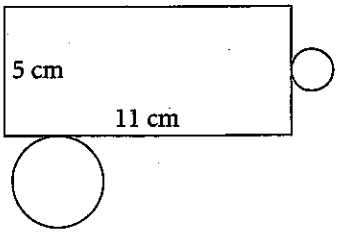
1. Which of the following diagrams is not the net of a right cylinder?

1

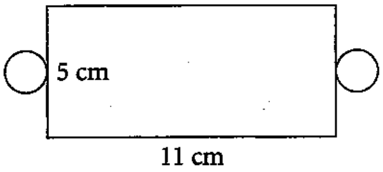
A



B



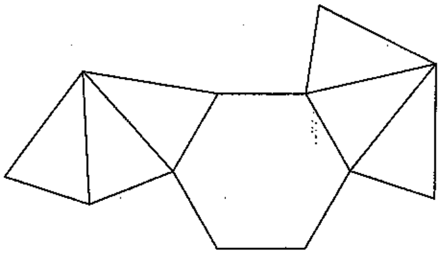
C



The figure in part B is not the net of a right cylinder.

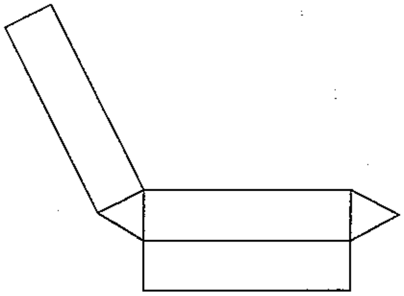
2. Is each diagram the net of an object?
If your answer is yes, name and describe the object.
If your answer is no, what changes could you make so it could be a net?

a)



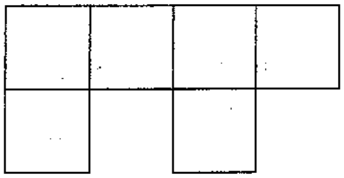
The diagram __is__ the net of an object. **It is the net of a hexagonal pyramid.** /2

b)



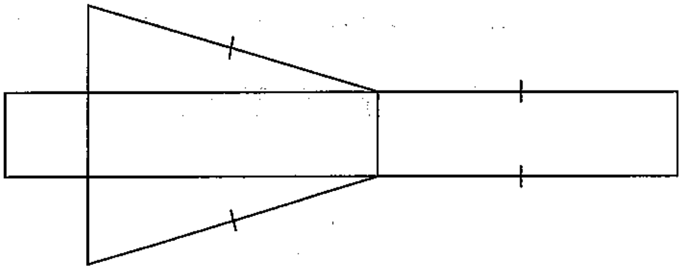
The diagram __is__ the net of an object. **It is the net of a triangular prism.** /2

c)



The diagram __is not__ the net of an object. **Move either square on the bottom edge to anywhere along the top edge to make the net of a cube.** /2

3. Name and describe the object that can be made from the net.

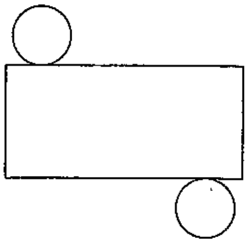


The object is a **triangular prism with two congruent right triangle faces and three rectangular faces.** /2

/8

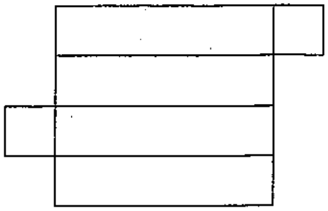
4. Identify the object that each net folds to form.

a)



right cylinder

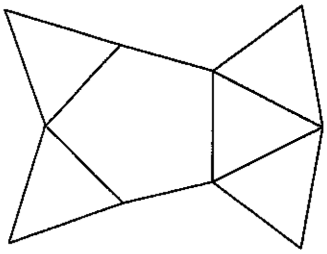
b)



square prism

/3

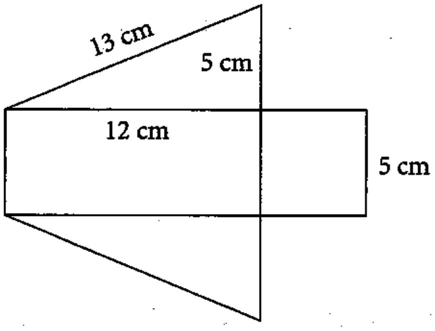
c)



pentagonal pyramid

5. Describe the changes that have to be made to each diagram to make it a net. Name the object that can be made from the new net.

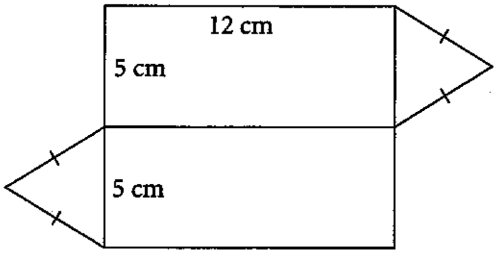
a)



Add a 13 cm by 5 cm rectangle to form the net of a right triangular prism.

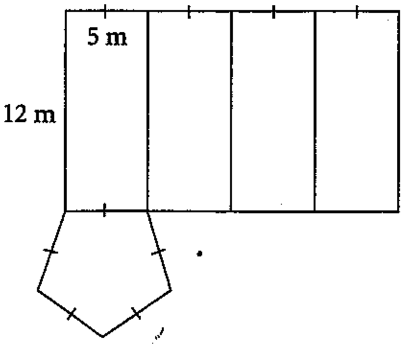
/6

b)



Add a 12 cm by 5 cm rectangle to form the net of a triangular prism.

c)



Add a regular pentagon of edge length 5 m to form the net of a pentagonal prism.

and a rectangle 5x12)

9

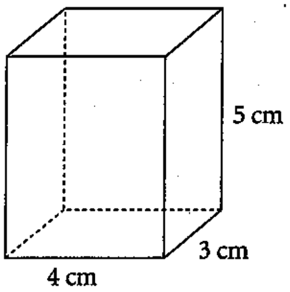
4.3 Surface Area of a Right Rectangular Prism

Quick Review

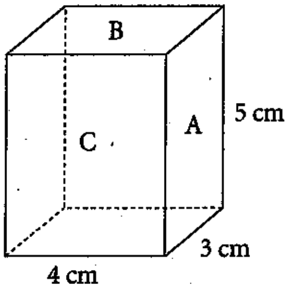
/12

➤ The surface area of a rectangular prism is the sum of the areas of its rectangular faces. The surface area is the same as the area of the prism's net.

To determine the surface area of this rectangular prism:



Identify each rectangle with a letter.



Rectangle A has area 3 cm × 5 cm = 15 cm^2

Rectangle B has area 4 cm × 3 cm = 12 cm^2

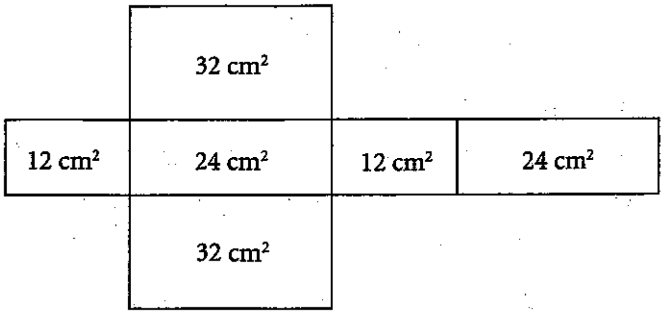
Rectangle C has area 4 cm × 5 cm = 20 cm^2

Surface area = 2 × 15 cm^2 + 2 × 12 cm^2 + 2 × 20 cm^2
= 30 cm^2 + 24 cm^2 + 40 cm^2
= 94 cm^2

The surface area of the rectangular prism is 94 cm^2.

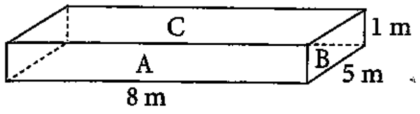
Practice

1. The diagram shows the net of a right rectangular prism. The area of each face is given. Calculate the surface area of the prism.



Area = 12 cm^2 + 32 cm^2 + 24 cm^2 + 32 cm^2 + 12 cm^2 + 24 cm^2 = 136 cm^2

2. Determine the surface area of the rectangular prism.



Rectangle A has area **8 m** × **1 m** = **8 m²**

Rectangle B has area **5 m** × **1 m** = **5 m²**

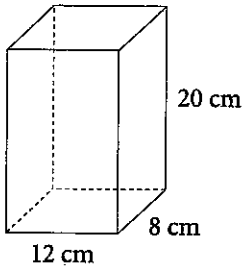
Rectangle C has area **8 m** × **5 m** = **40 m²**

Surface area = 2 × **8 m²** + 2 × **5 m²** + 2 × **40 m²**

= **106 m²**

3. Glenda and Louis each design a rectangular package.
Whose package has the greater surface area? Show your work.

Glenda's package:



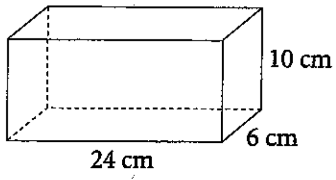
$SA =$ **2(12 × 20)** + **2(12 × 8)** + **2(20 × 8)**

= **480 + 192 + 320**

= **992**

The surface area is 992 cm².

Louis's package:



$SA =$ **2(24 × 10)** + **2(24 × 6)** + **2(10 × 6)**

= **480 + 288 + 120**

= **888**

The surface area is 888 cm².

992 > **888** So, **Glenda's** package has the greater surface area.

4. The surface area of a cube is 294 cm².

a) What is the area of each face of the cube?

Area of each face = **294 cm²** ÷ **6** = **49 cm²**

b) What is the length of one edge of the cube?

Edge length = **7 cm**

5. An office building is in the shape of a right rectangular prism with height 200 m, length 60 m, and width 40 m. The top quarter of each vertical face of the building is to be covered with a large banner advertising a major sporting event. What is the total surface area to be covered with banners?

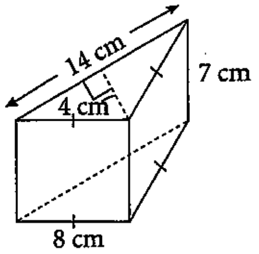
$\frac{1}{4}$ × **200 m** = **50 m**

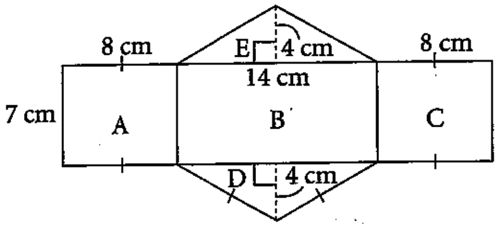
Total area to be covered = 2 × **60 m** × **50 m** + 2 × **40 m** × **50 m** = **10 000 m²**

4.4 Surface Area of a Right Triangular Prism

Quick Review

➤ To calculate the surface area of this right triangular prism, calculate the area of each face, and then sum the results.





Rectangle A has area 8 cm × 7 cm = 56 cm^2

Rectangle B has area 14 cm × 7 cm = 98 cm^2

Rectangle C has area 8 cm × 7 cm = 56 cm^2

Triangle D has area = $\frac{1}{2}$ × 14 cm × 4 cm = 28 cm^2

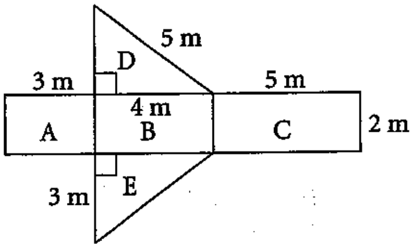
Triangle E has area = $\frac{1}{2}$ × 14 cm × 4 cm = 28 cm^2

Surface area = 56 cm^2 + 98 cm^2 + 56 cm^2 + 28 cm^2 + 28 cm^2
= 266 cm^2

The surface area of the triangular prism is 266 cm^2.

Practice

1. The diagram shows the net of a right triangular prism.



Calculate the area of the net.

Rectangle A has area **2 m** × **3 m** = **6 m^2**

Rectangle B has area **2 m** × **4 m** = **8 m^2**

Rectangle C has area **2 m** × **5 m** = **10 m^2**

Triangle D has area $\frac{1}{2}$ × **4 m** × **3 m** = **6 m^2**

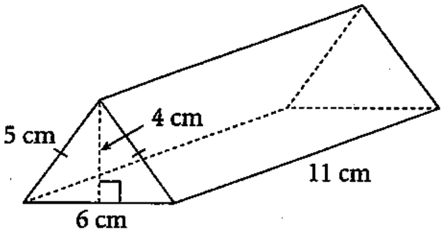
Triangle E has area $\frac{1}{2}$ × **4 m** × **3 m** = **6 m^2**

Area = **6 m^2** + **8 m^2** + **10 m^2** + **6 m^2** + **6 m^2** = **36 m^2**

The area of the net is **36** m^2.

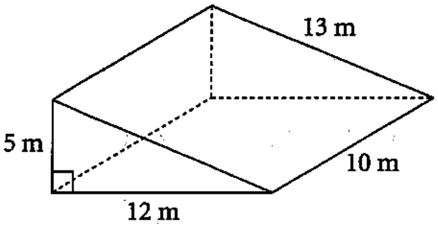
2. Calculate the surface area of each prism.

a)



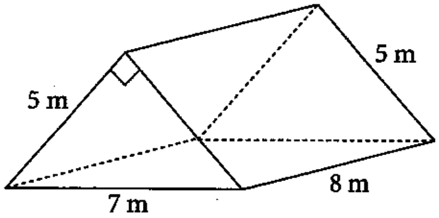
The surface area is **200** cm².

b)



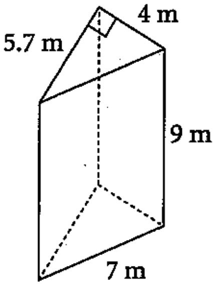
The surface area is **360** m².

c)



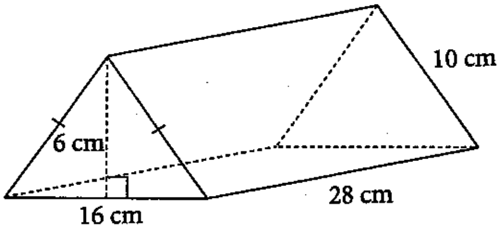
The surface area is **161** cm².

d)



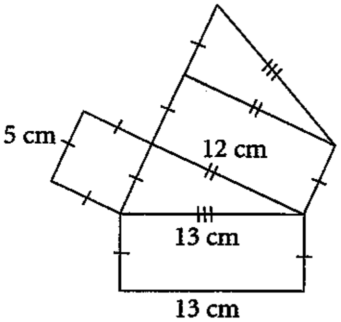
The surface area is **173.1** m².

3. Calculate the total surface area of the right triangular prism.



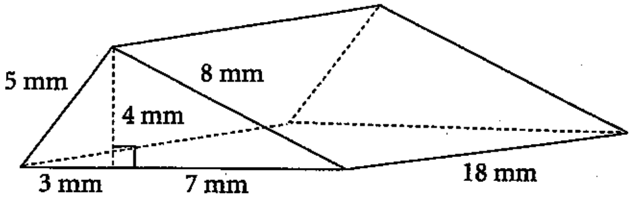
The surface area is **1104** cm².

4. Calculate the area of the net of a prism.



The area of the net is **210 cm²**.

5. Calculate the surface area of the prism.



The surface area is **454 mm²**.

4.5 Volume of a Right Rectangular Prism

Quick Review

➤ To find the volume of this rectangular prism:

Let the base be one of the rectangles with length 10 cm and width 4 cm.

$A = 10 \times 4$

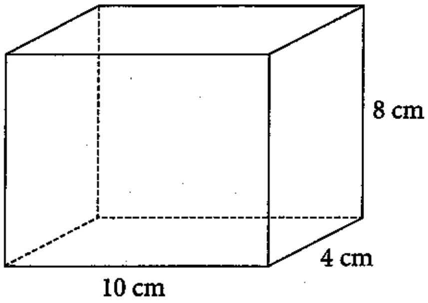
$= 40$

The area of the base is 40 cm².
The height of the prism is 8 cm.
Use the formula $V = Ah$.

$V = 40 \times 8$

$= 320$

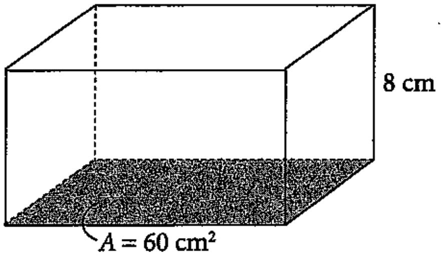
The volume of the prism is 320 cm³.



Practice

1. The area of the base and the height are shown on each rectangular prism. Determine the volume of each prism.

a)



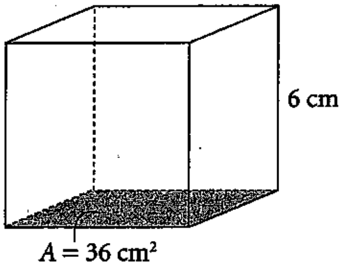
$V = Ah$

= **60 × 8**

= **480**

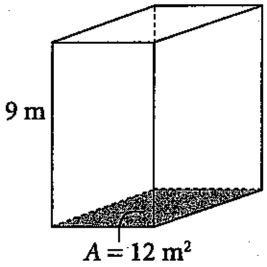
The volume is **480** cm³.

b)



The volume is **216** cm³.

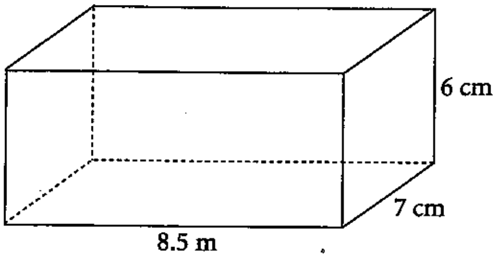
c)



The volume is **108** m³.

2. Determine the volume of each prism.

a)



$A =$ **8.5** × **7**

$=$ **59.5**

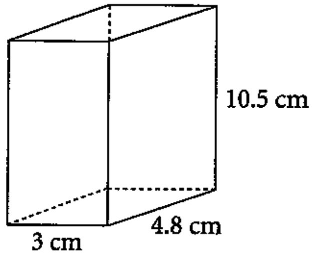
$V = Ah$

$=$ **59.5** × **6**

$=$ **480**

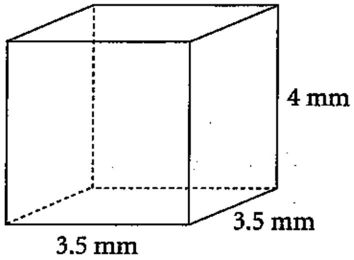
The volume is **357** m^3.

b)



The volume is **151.2** cm^3.

c)



The volume is **49** mm^3.

3. A right rectangular prism has length 16 cm, width 12 cm, and height 5 cm.

a) What is the volume of the prism?

The volume is **960 cm^3**.

b) If the length is halved and the height is doubled, what is the new volume?

The new length is **8 cm** and the new height is **10 cm**.

The new volume is **960 cm^3**.

4. Which right rectangular prism has the greater volume?

A length 6 m, width 4.5 m, height 3.6 m

The volume is **97.2 m^3**.

B a cube with edge 4.6 m

The volume is **97.336 m^3**.

The volume of prism **B** is greater.

5. A fish pond in the shape of a rectangular prism is 4 m long, 3 m wide, and 2 m deep.

a) What is the volume of the empty pond?

The volume is **24 m^3**.

b) If the pond is filled to a depth of 1.5 m, what is the volume of water in the pond, in litres? Remember that 1000 cm^3 = 1 L.

The height for this calculation is **1.5 m**.

Convert the dimensions to centimetres. The length is **400 cm**, the width is **300 cm**, and the height is **150 cm**.

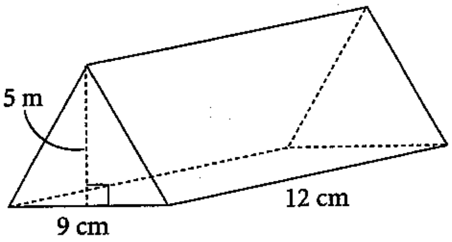
The volume is **18 000 000 cm^3**. This is the same as **18 000** L.

4.6 Volume of a Right Triangular Prism

At Home At School

Quick Review

➤ To determine the volume of this triangular prism:



The base of the triangle is $b = 9$.
The height of the triangle is $h = 5$.
The length of the prism is $\ell = 12$.
Use the formula $V = A\ell$.
First find A.

$A = \frac{1}{2}bh$

Substitute $b = 9$ and $h = 5$.

$A = \frac{1}{2} \times 9 \times 5$
$= 22.5$

Now find V.
Substitute $A = 22.5$ and $\ell = 12$ into $V = A\ell$.

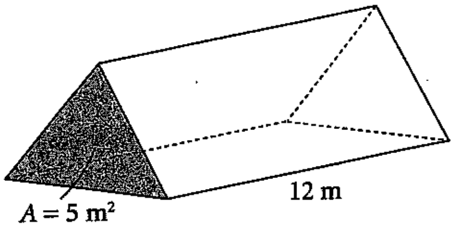
$V = 22.5 \times 12$
$= 270$

The volume of the prism is 270 cm³.

Practice

1. The area of the base and the length of each prism are shown. Calculate the volume of each prism.

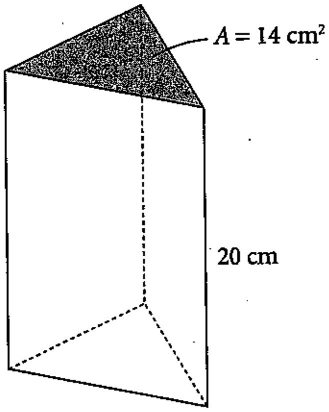
a)



$V = A\ell$
$= \underline{\ 5\ } \times \underline{\ 12\ }$
$= \underline{\ 60\ }$

The volume is 60 cm³.

b)



The volume is 280 cm³.

2. Determine the volume of each prism.

a) $A = \frac{1}{2}bh$

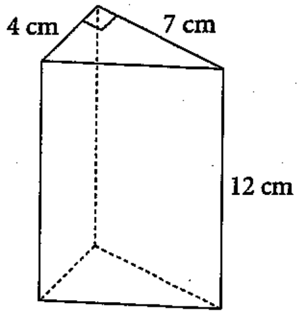
$= \frac{1}{2} \times \underline{\ \ 4\ \ } \times \underline{\ \ 7\ \ }$

$= \underline{\ \ 14\ \ }$

$V = A\ell$

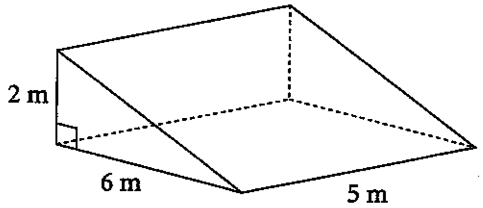
$= \underline{\ \ 14\ \ } \times \underline{\ \ 12\ \ }$

$= \underline{\ \ 168\ \ }$



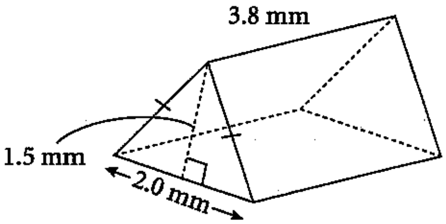
The volume is **168 cm³**.

b)



The volume is **30 m³**.

c)



The volume is **5.7 mm³**.

3. The volume of a right triangular prism is 27.8 cm³. The length of the prism is 5 cm. What is the area of each triangular face?

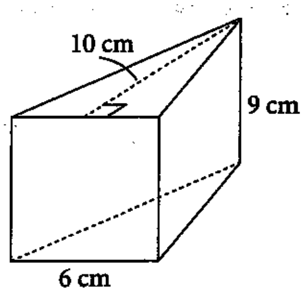
$V = A\ell$, so $A = \frac{V}{\ell}$.

The area of each triangular face is **5.56 cm²**.

4. The volume of a right triangular prism is 6 cm³. Determine the possible whole-number values for A and ℓ. How many different solutions can you find? Use a table to organize your solutions.

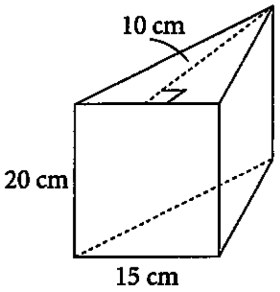
A	ℓ
1 cm²	**6 cm**
2 cm²	**3 cm**
3 cm²	**2 cm**
6 cm²	**1 cm**

5. Determine the volume of the prism.



The volume is **270 cm³**.

6. **a)** Determine the volume of the prism.



The volume is **1500 cm³**.

b) Suppose the prism contains 1200 mL of water. What is the depth of the water?

Let ℓ represent the depth. Remember that $1\ \text{cm}^3 = 1\ \text{mL}$.

$V = 1200\ \text{mL} = \underline{\mathbf{1200}}\ \text{cm}^3$

$A = \frac{1}{2} \times \underline{\mathbf{10}} \times \underline{\mathbf{15}}$

$= \underline{\mathbf{75}}$

$V = A\ell$

$\underline{\mathbf{1200}} = \underline{\mathbf{75}} \times \ell$

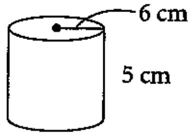
$\ell = \underline{\mathbf{16}}$

The depth of the water is **16 cm**.

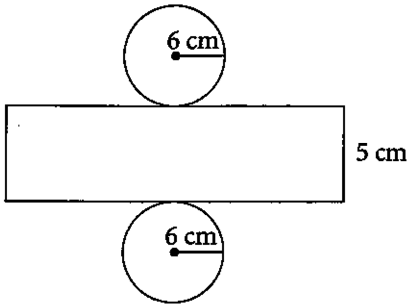
4.7 Surface Area of a Right Cylinder

Quick Review

➤ To find the surface area of this cylinder:



Sketch the net.

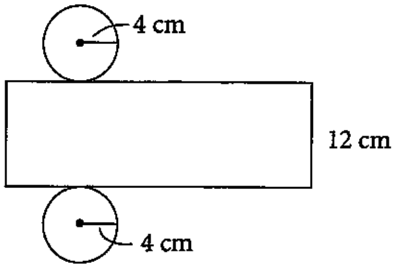


Surface area = 2 × area of one circle + area of the rectangle
The area of the circle is $A = \pi r^2$
Substitute $r = 6$.
$A = \pi \times 6^2$
$\doteq 113.10$
The area of the rectangle = circumference × height
$= 2\pi r \times h$
Substitute $r = 6$ and $h = 5$.
The area of the rectangle $= 2\pi \times 6 \times 5$
$\doteq 188.50$
Surface area $\doteq 2 \times 113.10 + 188.50$
$= 414.70$
The surface area of the cylinder is about 415 cm^2.

Practice

1. Determine the area of each net, to the nearest square centimetre.

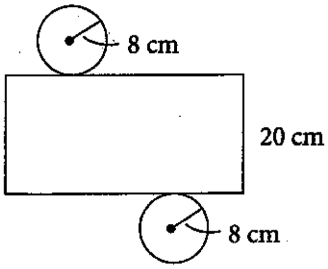
a)



Area of net = 2 × area of one circle
+ area of the rectangle
$= 2 \times \pi r^2 + 2\pi r \times h$
$= 2 \times \pi \times \underline{4}^2 + 2 \times \pi \times \underline{4} \times \underline{12}$
$\doteq \underline{402.1}$

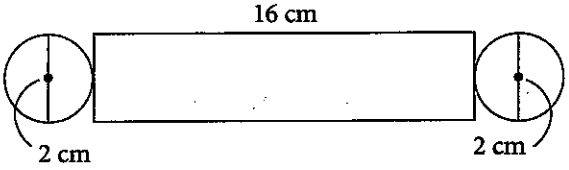
The area of the net is __402 cm^2__, to the nearest square centimetre.

b)



The area of the net is **107 cm^2**, to the nearest square centimetre.

c)



The diameter of each circle is **2 cm**, so the radius of each circle is **1 cm**.

The area of the net is **1407 cm^2** 107, to the nearest square centimetre.

2. Calculate the surface area of each cylinder, to the nearest square unit.

a) radius 8 cm, height 12 cm

Surface area of cylinder = 2 × area of one circle + area of the rectangle

$= 2 \times \pi r^2 + 2\pi r \times h$

$= 2 \times \pi \times \underline{\mathbf{8}}^2 + 2 \times \pi \times \underline{\mathbf{8}} \times \underline{\mathbf{12}}$

$\doteq \underline{\mathbf{1005.3}}$

The surface area is **1005 cm^2**, to the nearest square **centimetre**.

b) diameter 9 m, height 6.8 m

The diameter of each circle is **9 m**, so the radius of each circle is **4.5 m**.

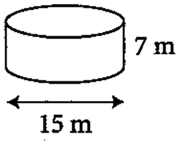
The surface area is **319 m^2**, to the nearest square **metre**.

c) diameter 7.2 cm, height 9.3 cm

The surface area is **292 cm^2**, to the nearest square **centimetre**.

3. Calculate the outside surface area each cylinder, to one decimal place. The cylinders are open at one end.

a)



The diameter is **15 m**, so the radius is **7.5 m**.

Surface area of cylinder = area of circle + area of the rectangle

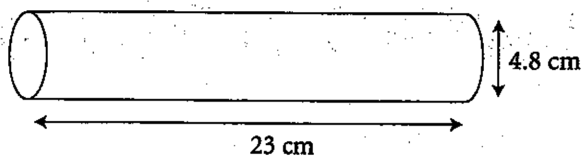
$= \pi r^2 + 2\pi r \times h$

$= \pi \times \underline{\mathbf{7.5}}^2 + 2 \times \pi \times \underline{\mathbf{7.5}} \times \underline{\mathbf{12}}$

$\doteq \underline{\mathbf{506.6}}$

The surface area of the cylinder is **506.6 m^2**, to one decimal place.

b)



The diameter is **4.8 cm**, so the radius is **2.4 cm**.

The surface area of the cylinder is **364.9 cm²**, to one decimal place.

4. Cylindrical rollers are used in a steel mill. One roller has diameter 1.8 m and length 2.6 m. What is the area of the curved surface of the roller?

The diameter is **1.8 m**, so the radius is **0.9 m**

Curved surface area of roller = area of the rectangle

$$= 2\pi r \times h$$

$$= 2 \times \pi \times \underline{\mathbf{1.8}} \times \underline{\mathbf{2.6}}$$

$$\doteq \underline{\mathbf{29.4}}$$

The area of the curved surface of the roller is **29.4 m²**, to one decimal place.

5. A cylinder with no top and no bottom has an outside surface area of 377 cm². The height of the cylinder is 10 cm.

a) What is the circumference of the base of the cylinder?

Curved surface area of cylinder = circumference × height

$$\underline{\mathbf{377}} = \text{circumference} \times \underline{\mathbf{10}}$$

$$\underline{\mathbf{37.7}} = \text{circumference}$$

The circumference of the base is **37.7 cm**.

b) What is the radius of the base of the cylinder?

Circumference of base = $2\pi \times r$

$$\underline{\mathbf{37.7}} = 2\pi \times r$$

$$r = \frac{37.7}{2\pi}$$

$$r \doteq \underline{\mathbf{6.00}}$$

The radius of the base is **6 cm**.

4.8

Volume of a Right Cylinder

Quick Review

➤ Calculate the volume of a cylinder with base area 312 m^2 and height 9 m.

Volume of a cylinder = base area × height
= 312 × 9
= 2808

The volume of the cylinder is 2808 m^3.

➤ Calculate the volume of a cylinder with diameter 18 cm and height 15 cm.

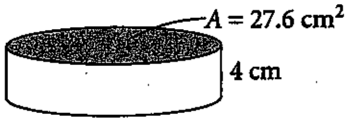
Use the formula for the volume of a cylinder:
$V = \pi r^2 h$
The diameter is 18 cm, so the radius is 9 cm.
Substitute $r = 9$ and $h = 15$.
$V = \pi \times 9^2 \times 15$
$\doteq 3817$

The volume of the cylinder is 3817 cm^3.

Practice

1. The base area and height of each cylinder are given. Calculate the volume, to the nearest cubic unit.

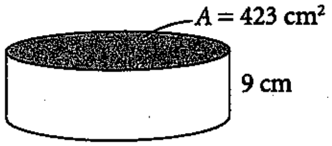
a)



Volume of a cylinder = base area × height
= **27.6** × **4**
= **110.4**

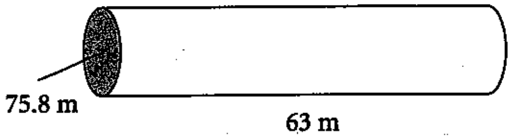
The volume of the cylinder is **110 cm^3**, to the nearest cubic **centimetre**.

b)



The volume of the cylinder is **3807 cm^3**, to the nearest cubic **centimetre**.

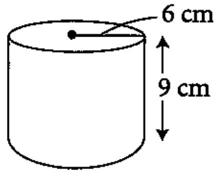
c)



The volume of the cylinder is **4775 m^3**, to the nearest cubic **metre**.

2. Calculate the volume of each cylinder, to the nearest cubic unit.

a)



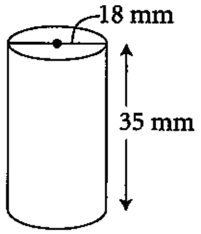
$V = \pi r^2 h$

$= \pi \times \underline{3^2} \times \underline{9}$

$\doteq \underline{254.47}$

The volume of the cylinder is **254 cm^3**, to the nearest cubic **centimetre**.

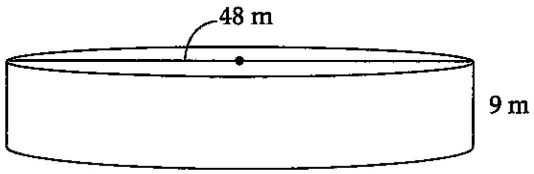
b)



The diameter is **18 mm**, so the radius is **9 mm**

The volume of the cylinder is **8906 mm^3**, to the nearest cubic **millimetre**.

c)



The diameter is **48 m**, so the radius is **24 m**.

The volume of the cylinder is **16 286 m^3**, to the nearest cubic **metre**.

3. Calculate the volume of each cylinder, to one decimal place.

a) radius 12 cm, height 12 cm
The volume of the cylinder is **5428.7 cm^3**, to one decimal place.

b) diameter 16.8 m, height 5.4 m
The diameter is **16.8 m**, so the radius is **8.4 m**.
The volume of the cylinder is **1197.0 m^3**, to one decimal place.

4. Which of the following cylinders has the greater volume? By how much?
A a cylinder with radius 6.4 cm, height 3.2 cm
B a cylinder with radius 4.3 cm, height 7.2 cm
Cylinder A has volume **411.8** cm^3 and cylinder B has volume **418.2** cm^3, so cylinder **B** has the greater volume by **6.4** cm^3.

5. a) Calculate the volume of a cylinder with radius 5 cm and height 10 cm, to one decimal place.
The volume is **785.4 cm^3**.

b) What happens to the volume of the cylinder in part a) if the radius is doubled?
Double the radius is **10 cm**.
The new volume is **3141.6 cm^3**, which is **4** times the original volume.

c) What happens to the volume of the cylinder in part a) if the height is doubled?
Double the height is **20 cm**.
The new volume is **1570.8 cm^3**, which is **2** times the original volume.

In Your Words

Here are some of the important mathematical words of this unit.
Build your own glossary by recording definitions and examples here. The first one is done for you.

net *a pattern that can be folded to make a solid*

polyhedron *an object whose faces are polygons*

regular prism *an object with 2 congruent faces that are regular polygons, and with remaining faces that are rectangles*

regular pyramid *an object whose base is a regular polygon and whose other faces are triangles*

surface area *the total area of the surface of an object*

volume *the amount of space occupied by an object*

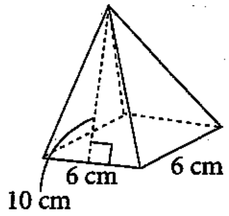
List other mathematical words you need to know.

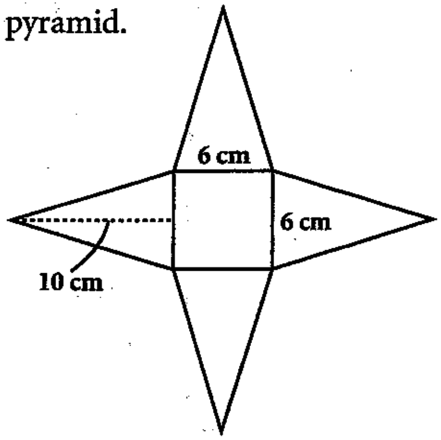
Sample Answers: regular dodecagon, capacity

Unit Review

LESSON

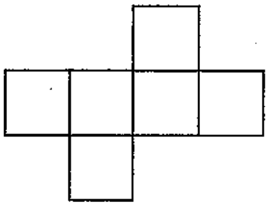
4.1 **1.** Sketch a net of the square pyramid.



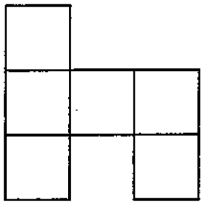


2. Which of the following is **not** the net of a cube?

A



B



C

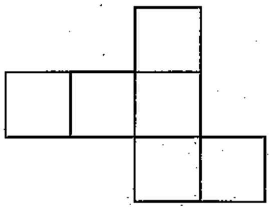
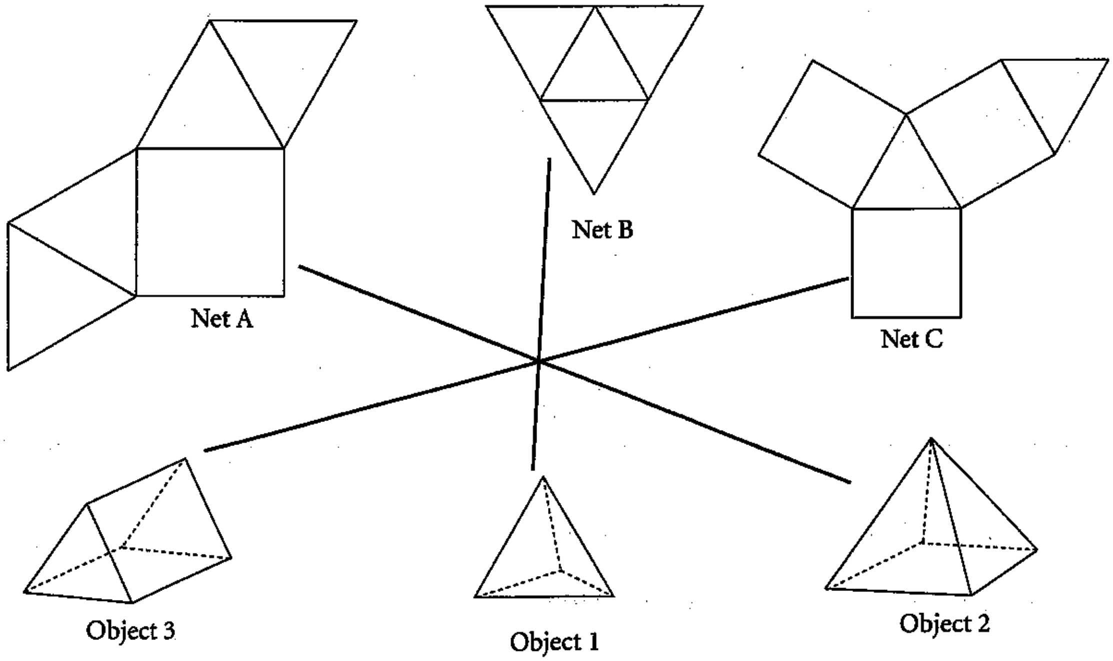


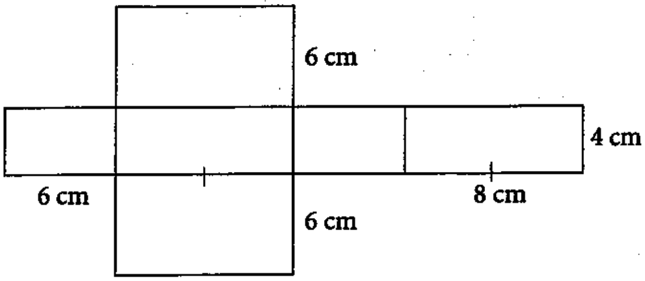
Figure B is not the net of a cube.

4.2 **3.** Match each net with the corresponding object.



LESSON

4.3 **4.** Calculate the area of the net of the right rectangular prism.



The area of the net is **208 cm^2**.

4.3 4.5 **5.** A cube has a surface area of 384 cm^2.

a) What is the length of one edge of the cube?

The area of one face of the cube is $384\text{ cm}^2 \div$ **6** = **64 cm^2**.

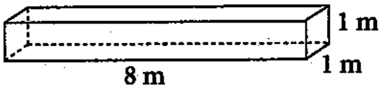
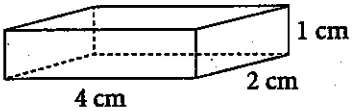
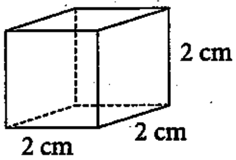
Thus, the length of one edge of the cube is **8 cm**.

b) What is the volume of the cube?

The volume of the cube is **512 cm^3**.

6. **a)** Sketch all possible right rectangular prisms with volume 8 cm^3, where each edge length must be a whole number of centimetres. State the dimensions of each.

Record your results in this table.

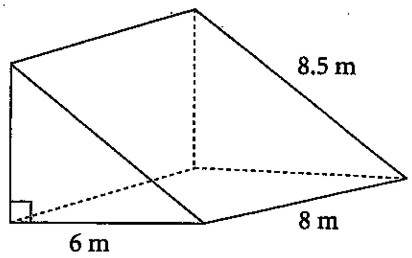
Length	Width	Height	Sketch
8	1	1	
4	2	1	
2	2	2	

b) Calculate the surface area of each prism in the table.

34 cm^2, **28 cm^2**, **24 cm^2**

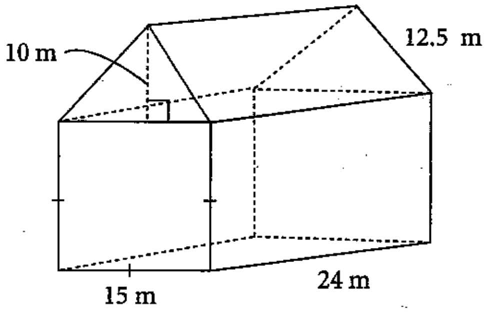
LESSON

4.4 **7.** Calculate the surface area of the prism.



The surface area is **200 m²**.

4.5
4.6 **8.** Calculate the volume of the object.

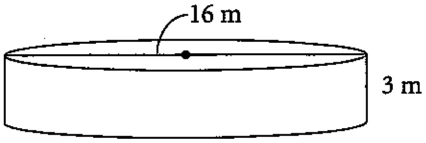


The volume of the triangular prism is **1800 m³**.

The volume of the rectangular prism is **5400 m³**.

The total volume is **7200 m³**.

4.7
4.8 **9.** A cylindrical water tank is open at the top.



a) Calculate the volume of the tank, to the nearest cubic metre.

The diameter is **16 m**, so the radius is **8 m**.

The volume of the tank is **603 m³**, to the nearest cubic metre.

b) If the inside of the tank is to be painted, including the floor, what is the area to be painted, to the nearest square metre?

The area to be painted is **352 m²**, to the nearest square metre.

UNIT 5

Percent, Ratio, and Rate

Just for Fun

One at a Time

Change TWO into TEN by changing one letter at a time. Each step must be a real word.

T	W	O
T	O	O
T	O	N
T	E	N

This one might look easy. But, can it be done? If you say no, show why not.

R	A	T	I	O
P	A	T	I	O
R	A	T	E	D
R	A	T	E	S

Sample Answer: 1st step: you get RADIO or PATIO. Working backward from RATES, words, such as RATED, require more than 2 steps to get to RADIO or PATIO.

Decimal

Make as many words as you can from the letters of the word DECIMAL.
Sample Answer:

2-letter words: **ad, am, id, me**

3-letter words: **dim, led, lad, die, dam, mad, cad, lam, lid**

4-letter words: **lame, lace, male, dame, dime, mice, mace, dale, dial, dice, made**

5-letter words: **laced, lamed**

6-letter words: **malice**

7-letter words: **claimed, declaim, medical**

A Game for [illegible] to 4

Make a Pair

Get a deck of playing cards and remove the face cards.

Deal each player 6 cards. Put the remaining cards face down in a stack.

The first player turns over one card. If she holds a number that divides evenly into the number showing, that is a "pair." She takes the card and puts the pair face down.

The next player can take the card showing if the previous player could not make a pair, but he can. Then he turns over one more card from the stack for an extra turn.

Continue until one player is out of cards. The player who made the most pairs wins.

Variation: Use the face cards as well. A jack represents 12, a queen represents 15, and a king represents 20.

Activating Prior Knowledge

Relating Fractions, Decimals, and Percents

To write a fraction as a decimal, divide: $\frac{4}{5} = 4 \div 5 = 0.8$

To write a decimal as a percent, multiply by 100%: $0.8 \times 100\% = 80\%$

To write a percent as a decimal, divide by 100%: $80\% \div 100\% = 0.8$

✓ Check

1. Write each fraction as a decimal and a percent.

a) $\frac{3}{50} = 3 \div$ **50**
= **0.06**
= **6** %

b) $\frac{7}{25} =$ **7 ÷ 25**
= **0.28**
= **28** %

c) $\frac{3}{5} =$ **3 ÷ 5**
= **0.6**
= **60** %

2. Write each percent as a decimal and as a fraction.

a) 36% = **0.36** = $\frac{9}{25}$

b) 5% = **0.05** = $\frac{1}{20}$

c) 44% = **0.44** = $\frac{11}{25}$

d) 86% = **0.86** = $\frac{43}{50}$

Finding Common Multiples

➤ To find the common multiples of 2, 4 and 5:

List the multiples of each number.

The multiples of 2 are 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, …

The multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, …

The multiples of 5 are 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, …

The common multiples of 2, 4, and 5 are 20, 40, 60, …

✓ Check

3. Find the first 5 multiples of each number.

a) 6: 6, 12, **18, 24, 30**

b) 9: 9, **18, 27, 36, 45**

c) 15: **15, 30, 45, 60, 75**

4. Find two common multiples for each set of numbers.

a) 15, 25

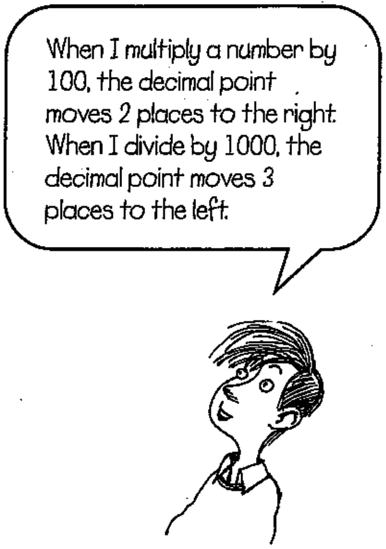
Multiples of 15 are **15, 30, 45, 60, 75, 90, 105, 120, 135, 150, ...**

Multiples of 25 are **25, 50, 75, 100, 125, 150, ...**

Two common multiples of 15 and 25 are **75** and **150**.

b) 6, 10 **30, 60**

c) 4, 10, 15 **60, 120**



Converting between Metric Units

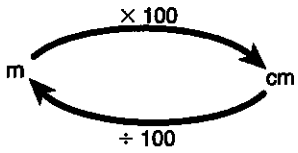
1 m = 100 cm

1 km = 1000 m

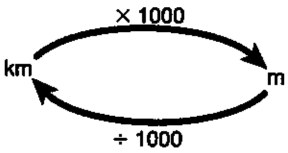
1 kg = 1000 g

1 L = 1000 mL

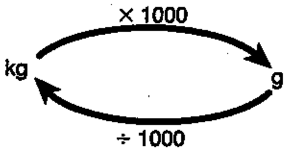
- ➤ To convert to a smaller unit, multiply.
- ➤ To convert to a larger unit, divide.



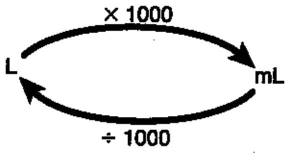
2.75 m = 2.75 × 100 cm
= 275 cm



2450 m = $\frac{2450}{1000}$ km
= 2.45 km



425 g = $\frac{425}{1000}$ kg
= 0.425 kg



3.4 L = 3.4 × 1000 mL
= 3400 mL

✓ Check

5. Convert. Show your work.

a) 3650 cm to metres

3650 cm = $\frac{3650}{100}$ m

= **36.5** m

b) 5260 mL to litres

5260 mL = $\frac{5260}{1000}$ L

= **5.26** L

c) 17 kg to grams

17 kg = **17 × 1000** g

= **17 000** g

d) 75 km to metres

75 km = **75 × 1000** m

= **75 000** m

5.1 Relating Fractions, Decimals, and Percents

Quick Review

➤ Sophia and Jacob are in a basketball free-throw competition. Sophia makes 11 out of her 20 free throws and Jacob makes 10 out of 16 of his free throws. Who has the best free-throw percentage?

Sophia makes 11 out of 20 free throws, which can be expressed as the fraction $\frac{11}{20}$.

Percent means per hundred. To write the fraction as a percent, write the fraction with a denominator of 100: $\frac{11 \times 5}{20 \times 5} \times \frac{55}{100}$

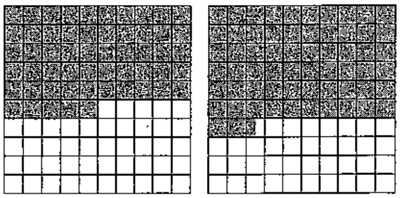
$\frac{55}{100}$ can be expressed as the decimal 0.55 or as the percent 55%.

Jacob made 10 out of 16 free throws, which can be expressed as the fraction $\frac{10}{16}$, or $\frac{5}{8}$.

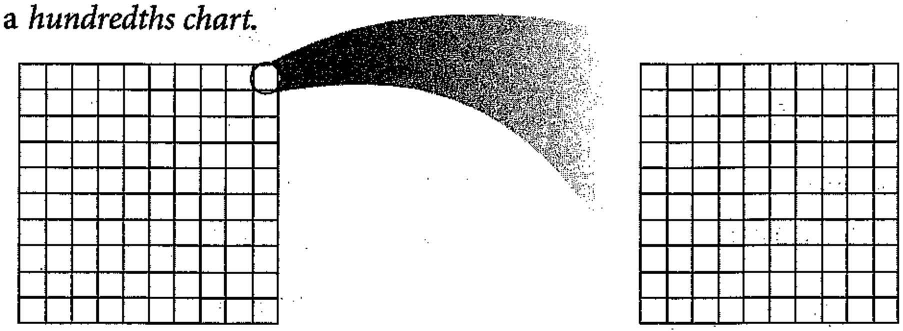
Express $\frac{5}{8}$ with a denominator of 1000.

$$\begin{aligned}\frac{5 \times 125}{8 \times 125} &= \frac{625}{1000}\\ &= \frac{625 \div 10}{1000 \div 10}\\ &= \frac{62.5}{100}\\ &= 62.5\%\end{aligned}$$

Shade hundred charts to represent Sophia's and Jacob's percentage of successful free throws. Sophia made 55% of her free throws, so 55 squares on the hundred chart are shaded. Jacob made 62.5% of his free throws, so $62\frac{1}{2}$ squares on the hundred chart are shaded. Jacob had the better free throw percentage.



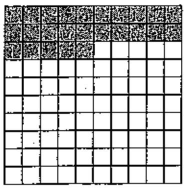
One small square on a hundred chart can be enlarged to show 100 squares. This is called a *hundredths chart.*



Each small square of a hundredths chart represents $\frac{1}{100}$ of 1%, or $\frac{1}{100}$%, or 0.01%.

$\frac{1}{4}$ of 1% or $\frac{1}{4}$% can be represented on the hundredths chart by shading $\frac{1}{4}$ of the hundredths chart, which is 25 squares.

$\frac{1}{4}$% = 0.25%



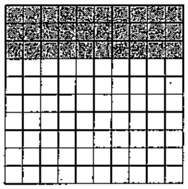
$\frac{1}{4}$ % can be written as a decimal.

$\frac{1}{4}\% = \frac{0.25}{100} = \frac{25}{10\,000} = 0.0025$

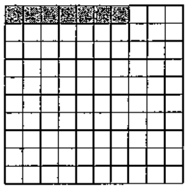
Practice

1. Each hundred chart represents 100%. Shade the chart to represent the given percent.

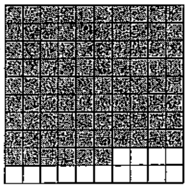
a) 30%



b) 7%



c) 86%



2. Write each percent as a fraction and as a decimal.

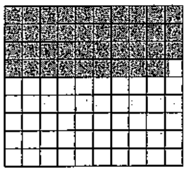
a) 6% = $\frac{6}{100}$ = **0.06**

b) 87% = $\frac{87}{100}$ = **0.87**

c) 48% = $\frac{48}{100}$ = **0.48**

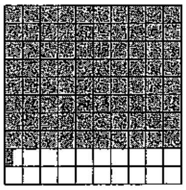
3. Each hundred chart represents 100%. What fraction is shaded? Write each fraction as a decimal and as a percent.

a)



$\frac{39.25}{100} = \frac{3925}{10\,000}$ = **0.3925** = **39.25%**

b)



$\frac{80.5}{100} = \frac{805}{1000}$ = **0.805** = **80.5%**

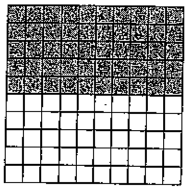
4. Write each percent as a fraction and as a decimal.

a) 48.5% = $\frac{48.5}{100} = \frac{485}{1000}$ = **0.485**

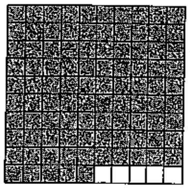
b) 10.75% = $\frac{10.75}{100} = \frac{107.5}{1000} = \frac{1075}{10\,000}$ = **0.1075**

5. Use a hundred chart to represent 1%. Shade the chart to represent each percent.

a) 0.5%



b) 0.95%



6. Write each percent as a fraction and as a decimal.

a) 0.75% = $\frac{\mathbf{0.75}}{100}$ = $\frac{\mathbf{75}}{\mathbf{10\,000}}$ = **0.0075**

b) 0.4% = $\frac{\mathbf{0.4}}{100}$ = $\frac{\mathbf{4}}{\mathbf{1000}}$ = **0.004**

7. Write each fraction as a decimal and as a percent. Use a calculator if necessary.

a) $\frac{10}{500}$ = **0.02** = **2%**

b) $\frac{15}{200}$ = **0.075** = **7.5%**

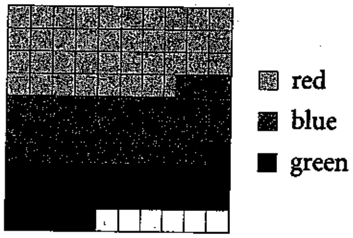
c) $\frac{17}{400}$ = **0.0425** = **4.25%**

8. Use coloured pencils to shade the hundred chart.

Shade $\frac{3}{8}$ of the grid squares in the rectangle red.

Shade 25% of the grid squares green.

Shade 0.315 of the grid squares blue.



a) Explain how you decided on the number of squares to shade each colour.

Explanations may vary.

b) What fraction of the hundred chart is not shaded? $\frac{\mathbf{6}}{\mathbf{100}}$ **or** $\frac{\mathbf{3}}{\mathbf{50}}$

c) Write the fraction that is not shaded as a decimal and as a percent.

0.06 or 6%

9. In a parking lot, 19 out of 40 cars are hybrids. Express the number of cars in the parking lot as a fraction and as a percent.

In the parking lot, $\frac{\mathbf{19}}{\mathbf{40}}$ or **47.5%** of the cars are hybrids.

10. Milo will pay $8\frac{1}{4}$% on a loan from a bank. Express the interest rate as a fraction and as a decimal.

$8\frac{1}{4}$% is the same as 8.**25**%.

The interest rate is $\frac{\mathbf{825}}{\mathbf{10\,000}}$ or **0.0825**.

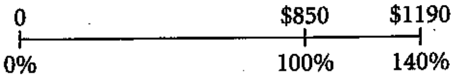
5.2 Calculating Percents

Quick Review

To calculate a percent of a quantity, first write the percent as a decimal. Then calculate the decimal value of the quantity.

To find 140% of \$850, write 140% as a decimal. $140\% = \frac{140}{100} = 1.40$
Then, 140% of \$850 = 1.40 × \$850
= \$1190

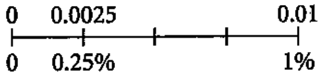
This answer can be illustrated on a number line.



Percents that are less than 1% can also be illustrated on a number line.
$1\% = \frac{1}{100} = 0.01$

Use this pattern:
100% = 1.0 10% = 0.10 1% = 0.01

0.25% = 0.0025



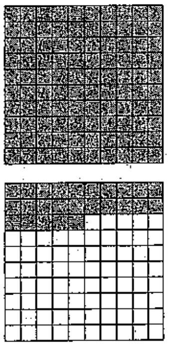
HINT

To change a percent to a decimal, move the decimal point 2 places to the left.

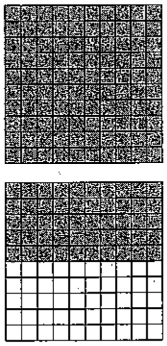
Practice

1. One hundred chart represents 100%. Shade hundred charts to show each percent.

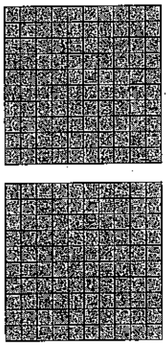
a) 125%



b) 150%

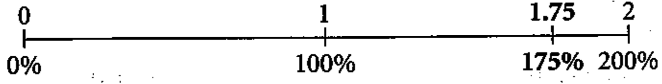


c) 200%



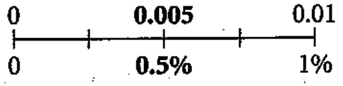
2. a) Write 175% as a decimal and draw a number line to show this percent.

175% = **1.75**



b) Write 0.5% as a decimal and draw a number line to show this percent.

0.5% = **0.005**



3. Write each percent as a decimal.

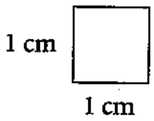
a) 230% **2.3**
b) 185% **1.85**
c) 324% **3.24**
d) 0.74% **0.0074**
e) 0.7% **0.007**
f) 0.09% **0.0009**

4. Write each fraction as a percent.

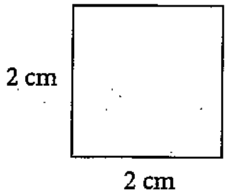
a) $\frac{1}{2}$ **50%** **b)** $\frac{3}{2}$ **150%** **c)** $\frac{5}{2}$ **250%**

d) $\frac{1}{100}$ **1%** **e)** $\frac{1}{200}$ **0.5%** **f)** $\frac{3}{200}$ **1.5%**

5. a) Draw a square with sides 1 cm long.

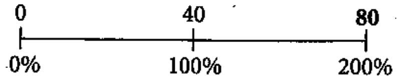


b) Redraw the square so the sides are 200% of the original length.
The new square has sides of length **2** cm.

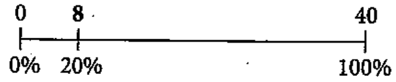


6. a) Find the percent of each number. Draw number line to illustrate each answer.

i) 200% of 40



ii) 20% of 40



iii) 2% of 40

b) Describe the pattern in the answers to part a).

Each answer is one-tenth the size of the previous answer.

c) Use the pattern in part a) to find each percent of 40.

i) 2000% of 40 = **800** **ii)** 0.2% of 40 = **0.08**.

Tip
Extend the pattern both ways—increase and decrease by a factor of 10.

7. A total of 45 412 runners participated in the Vancouver Sun Run. Of these runners, 0.85% completed the run in under 40 min.

How many runners completed the run in under 40 min? **0.0085 × 45 412 = 386**

In fact, 0.13% of the runners completed the run in less than 34 min.

How many runners were in this group? **0.0013 × 45 412 = 59**

8. Which is the greater amount of money, 120% of 0.3% of $1000 or 120.3% of $1000? Explain.

120% of 0.3% of $1000 = 1.2 × 0.003 × $1000 = $3.60
120.3% of $1000 = 1.203 × $1000 = $1203

120.3% of $1000 is the greater amount of money.

5.3 Solving Percent Problems

Quick Review

Several hundred students were surveyed. 160 students were from one school. These students represent 40% of those surveyed.

To find how many students were surveyed, follow these steps:
40% of those surveyed is 160.
1% of those surveyed is $\frac{160}{40} = 4$
100% of those surveyed is $4 \times 100 = 400$

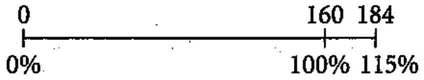
In the next survey, 15% more students were surveyed from the same school. To find the number of students surveyed, use the original number, 160, as 1 whole.

Method 1: The increase was 15%.
The new number is: 100% + 15% = 115%
115% of 160 = $1.15 \times 160 = 184$

Tip
Choose the method you feel comfortable using.

Method 2: The increase was 15%. 160 is 100%.
15% of 160 = $0.15 \times 160 = 24$
The new number is: 160 + 24 = 184

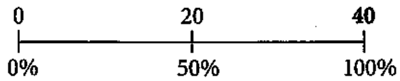
Both methods show that the new number of students surveyed is 184. This result can be illustrated on a number line.



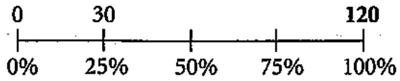
Practice

1. Use a number line to find each number.

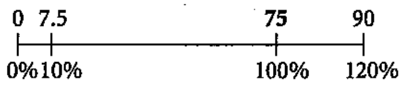
a) 50% of a number is 20



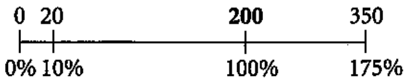
b) 25% of a number is 30



c) 120% of a number is 90



d) 175% of a number is 350



2. Find the number in each case.

a) 6% of a number is 9.

6% = 9

1% = **1.5**

100% = **150**

b) 28% of a number is 56.

28% = **56**

1% = **2**

100% = **200**

c) 150% of a number is 36.

150% = **36**

1% = **0.24**

100% = **24**

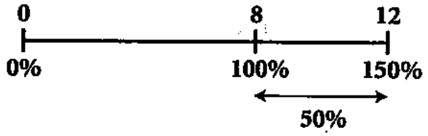
3. Write each increase as a percent. Illustrate each answer on a number line.

a) The width of the rectangle increased from 8 cm to 12 cm.

Increase = 12 cm – 8 cm = **4 cm**

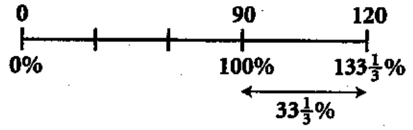
Increase as a fraction of the original = $\frac{4\text{ cm}}{8\text{ cm}} = \frac{1}{2}$

Percent increase = $\frac{1}{2}$ × 100% = **50%**.



b) The price of a hotel room increased from $90.00 to $120.00.

Percent increase = **$33\frac{1}{3}$%**



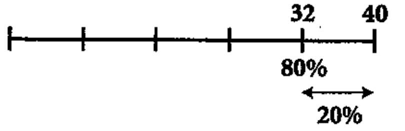
4. Write each decrease as a percent. Illustrate each answer on a number line.

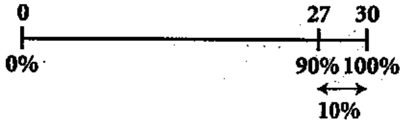
a) The volume of water in the tank decreased from 40 L to 32 L.

Decrease = 40 L – 32 L = **8 L**

Decrease as a fraction of the original = $\frac{8\text{ L}}{40\text{ L}} = \frac{1}{5}$

Percent decrease = $\frac{1}{5}$ × 100% = **20%**.





b) The number of students in the class decreased from 30 to 27.

Percent decrease = **10%**

5. In a batch of eggs, 3% were broken. There were 18 broken eggs.

How many eggs were there in the batch?

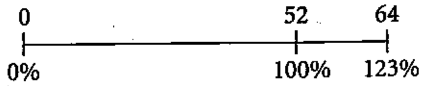
There were 600 eggs in the batch.

Tip
Identify which number represents 1 whole, or 100%.

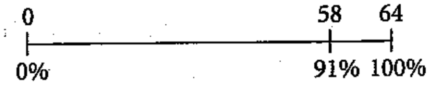
6. The prices for a day pass for skiing are:

Low Season: $52 High Season: $64 Spring Season: $58

a) Write the increase in cost from Low Season to High Season. **$12**
Illustrate the percent increase on a number line.



b) Write the decrease in cost from High Season to Spring Season. **$6**
Illustrate the percent decrease on a number line.



7. a) The rural population of Quebec is about 1 650 000.
This represents 22% of the population of Quebec. Estimate the population of Quebec.

The population of Quebec is about 7 500 000.

b) The population of Yukon Territory is about 31 400. Of these, 18 840 live in urban areas.
What percent of the population of Yukon Territory lives in rural areas?

About 40% of the population of Yukon Territory lives in rural areas.

8. A fish tank contains 24 L of water. Water is added to increase the volume by 12.5%.
What is the new volume of water in the tank?

The new volume of water is 27 L.

9. Thirty-six percent of a number is 63. Find 124% of the number.

The result is 217.

10. A factory produces 900 items per week at a unit cost of $75. New equipment is installed that increases the productivity by 12% and reduces the unit cost by 16%.

a) What is the new production rate?

The new production rate is 1008 items/week.

b) What is the new unit cost?

The new unit cost is $63.

KEY TO SUCCESS

Problems can always be solved in more than one way. If you cannot solve a problem by one method, look at the problem from another view for an alternative method.

5.4 Sales Tax and Discount

Quick Review

When an item is sold at a reduced price, we say there is a **discount.**
In many provinces, taxes are added to the selling price.

Corey works in a shoe store. She has to calculate the cost of a pair of running shoes priced at $129 that is on sale at 20% off.

A discount of 20% means the sale price is:
100% – 20% = 80% of the regular price
80% of $129 = 0.8 × $129 = $103.20
The total sales tax in Corey's province is 13%. The tax is:
13% of $103.20 = 0.13 × $103.20 = $13.42
So, the cost of the running shoes is:
$103.20 + $13.42 = $116.62

Tip

Always round money amounts to the nearest hundredth of a dollar.

This can be calculated directly as: 113% of $103.20 = 1.13 × $103.20 = $116.62

Practice

1. Calculate a 14% tax on each item.

a) $288

0.14 × $288

= **$40.32**

b) $36.50

0.14 × **$36.50**

= **$5.11**

c) $149.99

0.14 × **$149.99**

= **$21.00**

2. Calculate the cost, including 15% total sales tax, for each item.

a) $2.40

1.15 × $2.40

= **$2.76**

b) $3428

1.15 × **$3428**

= **$3942.20**

c) $128.79

1.15 × **$128.79**

= **$148.11**

3. Calculate each discount and the sale price before tax.

a) $92 watch, 30% off

Discount: **0.3 × $92 = $27.60**

Sale price: **$92 – $27.60 = $64.40**

b) $476 TV, 15% off

Discount: **0.15 × $476 = $71.40**

Sale price: **$476 – $71.40 = $404.60**

4. Calculate the discount, sale price before taxes, and sale price including 13% total tax.

a) $28.95 book at 10% off

Discount: **$2.90**

Sale price: **$26.05**

13% tax: **$3.26**

Total cost: **$29.31**

b) $239 coat at 25% off

Discount: **$59.75**

Sale price: **$179.25**

13% tax: **$22.41**

Total cost: **$201.66**

5. The cost of a ticket for a CFL game 3 years ago was $36.00. The cost of the ticket has increased by 25%. Calculate the new cost of the ticket.

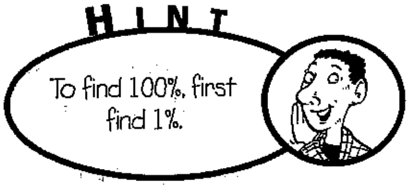
Increase in price: **$9.00** Total cost: **$45.00**

6. Store A offers successive discounts of 10% one week and 20% the second week. Store B offers a one-time discount of 25% the second week.

Which store offers the greater discount?
Assume $100 sales. Find the selling price:
Store A: 0.9 × 0.8 × $100 = $72 **Store B: 0.75 × $100 = $75**

Store **A** offers the greater discount.

7. At a discount of 25%, skateboards are on sale for $135. What is the original price?


HINT
To find 100%, first find 1%.

The original price is **$180**.

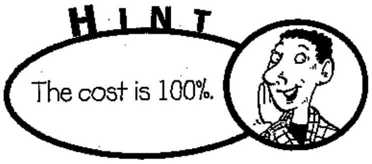
8. A TV set, regularly priced at $256, is offered for sale at 25% off. Sales tax is 15%.

a) Calculate the sale price at a 25% discount and then add 15% sales tax to it.
75% of $256 = 0.75 × $256 = $192
115% of $192 = 1.15 × $192 = $220.80

b) Add 15% tax to the original price and then calculate the sale price at a 25% discount.
115% of $256 = 1.15 × $256 = $294.40
75% of $294.40 = 0.75 × $294.40 = $220.80

Which calculation results in the greater discount? **Discounts are the same for both.**

9. The sales tax in Ontario is 13%.
Janis pays a total of $32.77 for a fishing pole.
Find the cost of the fishing pole before sales tax.


HINT
The cost is 100%.

113% is $32.77.

1% is: $\frac{\$32.77}{113} = \$0.29$

100% is: $0.29 × 100 = $29.00

So, the fishing pole cost $29.00 before sales tax.

5.5 Exploring Ratios

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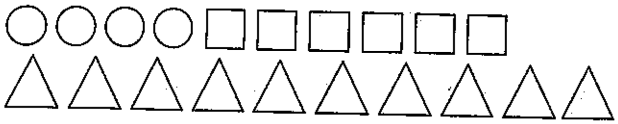
Quick Review

The picture shows 4 circles, 6 squares, and 10 triangles.



Here are some ways you can use ratios, fractions, and percents to compare the shapes.

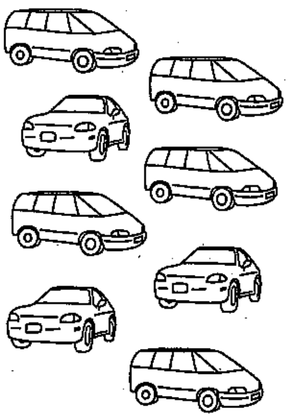
- **Part-to-Whole Ratios**
  The ratio of circles to all of the shapes is 4 to 20 or 4:20.
  This part-to-whole ratio can be written as the fraction $\frac{4}{20}$ or $\frac{1}{5}$.
  It can also be written as a percent. $\frac{4}{20} = \frac{20}{100} = 20\%$
  20% of the shapes are circles.
- **Part-to-Part Ratios**
  The ratio of circles to squares is 4 to 6 or 4:6. 4 and 6 are the terms of the ratio.
  The ratio of circles to squares to triangles is 4 to 6 to 10 or 4:6:10.
  This is called a three-term ratio.
  A part-to-part ratio cannot be written in fraction or percent form, as it is not comparing one part to the whole.

Practice

1. Write each ratio.

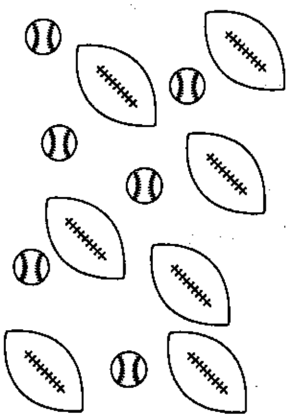
/3

a) cars to vans



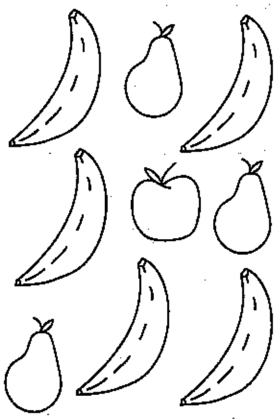
3 : 5

b) footballs to baseballs



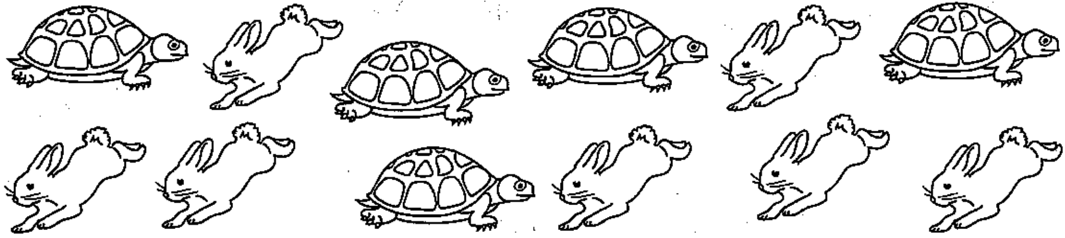
7:6

c) bananas to fruit



5:9

2. Write each part-to-whole ratio as a ratio, a fraction, and a percent. Round percents to two decimal places.



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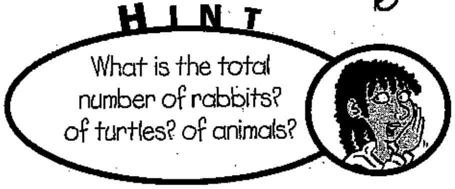
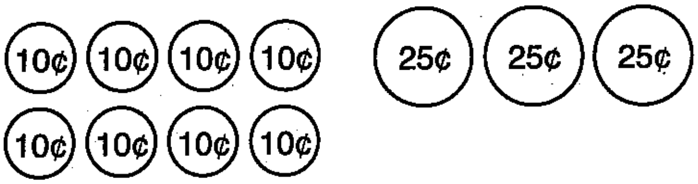


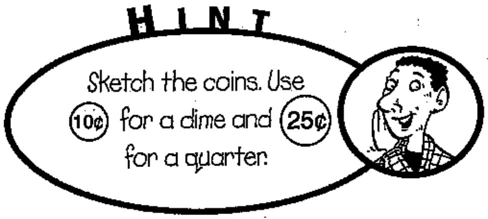
What is the total number of rabbits? of turtles? of animals?

a) turtles to total animals 5 : 12, $\frac{5}{12}$, 41.67%

b) rabbits to total animals 7 : 12, $\frac{7}{12}$, 58.33%

3. Franny has only dimes and quarters in her pocket. The ratio of dimes to total coins is 8 to 11.





Sketch the coins. Use (10¢) for a dime and (25¢) for a quarter.

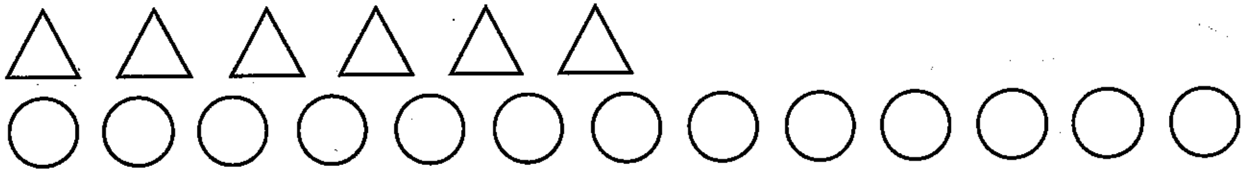
a) How many quarters might be in Franny's pocket? 3

b) What is the ratio of dimes to quarters? 8:3

c) What is the ratio of quarters to the total number of coins? 3:11

/3

4. Make a sketch to show that the ratio of triangles to circles is 6:13. Write 3 ratios to compare the figures.



a) circles to triangles 13:6

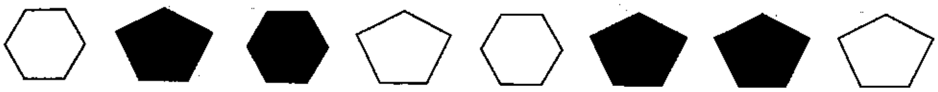
b) circles to total figures 13:19

c) triangles to total figures 6:19

/4

/13

5. Write each ratio.



a) hexagons to pentagons 3:5

b) pentagons to hexagons 5:3

/6

c) hexagons to total shapes 3:8

d) pentagons to total shapes 5:8

e) black figures to white shapes 4:4

f) white hexagons to black hexagons to white pentagons 2:1:2

6. What objects are being compared in each ratio?

a) 7:15 carrots to total vegetables

b) 2:7 tomatoes to carrots

c) 2:7:6 tomatoes to carrots to broccoli cauliflowers

/6

d) 6:7 broccoli cauliflowers to carrots

e) $\frac{2}{15}$ tomatoes to total vegetables

f) $\frac{6}{15}$ cauliflowers broccoli to total vegetables

7. A pencil case contains 7 yellow, 3 red, 1 black, and 5 green pencil crayons.

a) Write each ratio.

/5

- red:green 3:5
- yellow:red 7:3
- black:total pencil crayons 1:16
- yellow:total pencil crayons 7:16
- yellow:red:green 7:3:5

/3

b) What is the ratio of yellow and red pencil crayons to total pencil crayons? 10:16

What percent of all the pencil crayons are red or yellow? 62.5%

c) What is the ratio of green pencil crayons to black and red pencil crayons? 5:4

d) Suppose 2 yellow and 2 green pencil crayons are lost.
Rewrite the ratios in part a).

/5

- red:green 3:3
- yellow:red 5:3
- black:total pencil crayons 1:12
- yellow:total pencil crayons 5:12
- yellow:red:green 5:3:3

/25

5.6 Equivalent Ratios

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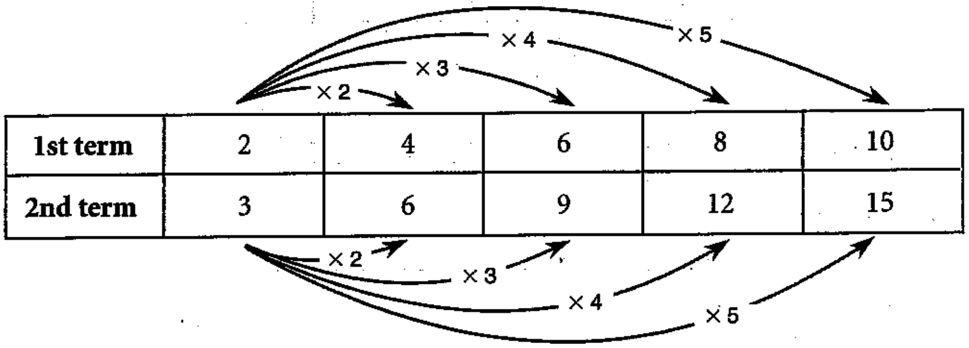
At Home At School

Quick Review

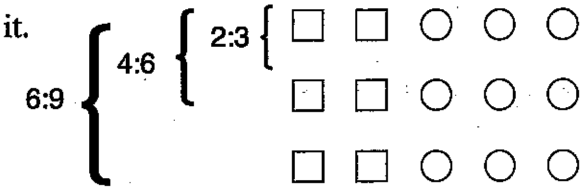
- ➤ You can find **equivalent ratios** by multiplying. Multiply the terms by the same number.



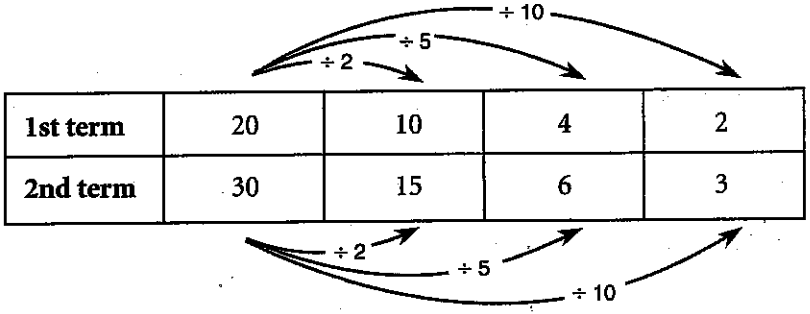
1st term	2	4	6	8	10
2nd term	3	6	9	12	15

Four equivalent ratios of 2:3 are: 4:6, 6:9, 8:12, and 10:15.

Picture it.



- ➤ You can also find equivalent ratios by dividing. Divide the terms by the same number.



1st term	20	10	4	2
2nd term	30	15	6	3

Three equivalent ratios of 20:30 are: 10:15, 4:6, and 2:3.

- ➤ To write a ratio in its simplest form, divide the terms by their GCF.

$$21:14 = (21 \div 7):(14 \div 7)$$
$$= 3:2$$

HINT

The GCF of 21 and 14 is 7. Divide by 7.

/37

1. Write three ratios that are equivalent to each ratio.

a) 4:5 **Sample Answer:**

× 3

× 2

1st term	4	8	12	16
2nd term	5	10	15	20

× 2

× 3

/3

Three ratios equivalent to 4:5 are 8:10, **12:15**, and **16:20**.

b) 32:24 **Sample Answer:**

÷ 2

1st term	32	16	8	4
2nd term	24	12	6	3

÷ 2

/4

Three ratios equivalent to 32:24 are **16:12**, **8:6**, and **4:3**.

c) 16:28 **Sample Answer:**

1st term	16	4	8	32
2nd term	28	7	14	56

/5

Three ratios equivalent to 16:28 are 4:7,

8:14, and 32:56.

HINT

Multiply or divide the terms by the same number.

2. Write two ratios that are equivalent to each ratio.

a) 8:5:2

Sample Answer:

16 : 10 : 4, 24 : 15 : 6

/4

b) 24:16:12

Sample Answer:

12 : 8 : 6, 6 : 4 : 3

/16

/21

3. Write each ratio in simplest form.

/6

a) 10:4

GCF of 10 and 4 is 2.

10:4 = (10 ÷ 2):(4 ÷ 2)

= 5: **2**

b) 6:15

GCF of 6 and 15 is **3**.

6:15 = (6 ÷ **3**):(15 ÷ **3**)

= **2** : **5**

c) 14:28

1:2

d) 25:10

5:2

Tip
Divide the terms by the GCF.

4. a) Match the pairs of equivalent ratios.

/7

i)

5:6	1:2
18:3	15:18
9:18	8:40
4:20	6:1

ii)

1:8	1:9
3:27	1:3
12:36	9:1
18:2	2:16

b) How do you know that 12:36 and 1:3 are equivalent?

12:36 and 1:3 are equivalent because 12 ÷ 12 = 1 and 36 ÷ 12 = 3.

5. The ratio of cats to dogs at the animal shelter is 4 to 5.
How many cats could there be? How many dogs?
Write six different answers.

/6

Sample Answer:

4 cats and 5 dogs

8 cats and **10** dogs

12 cats and **15** dogs

16 cats and 20 dogs

20 cats and 25 dogs

32 cats and 40 dogs

Tip
Multiply each term by the same number.

6. The length-to-width ratio of Colby's poster is 3:2.
The poster is 90 cm long. How wide is it?

/2

HINT

Find a ratio equivalent to 3:2 in which the first term is 90.

$$\underset{\times 30}{\overset{\times 30}{3 : 2 = 90 : \mathbf{60}}}$$

The poster is **60** cm wide.

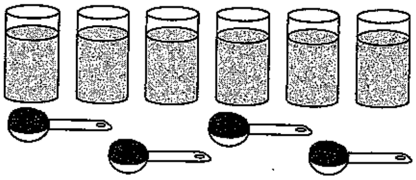
5.7 Comparing Ratios

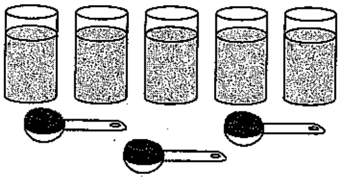
Quick Review

34.

You can use equivalent ratios to compare ratios.

Joe and Petra make orange punch with different ratios of crystals to water.





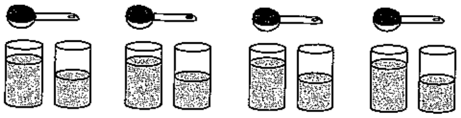
Joe makes orange punch with 4 scoops of crystals and 6 cups of water.

Petra makes orange punch with 3 scoops of crystals and 5 cups of water.

➤ **Method 1**

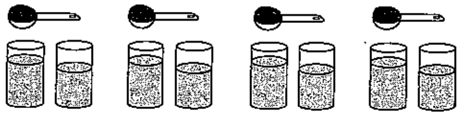
Draw a picture. Find out how much water for 1 scoop of orange crystals.

Joe



Orange crystals to water: $1:1\frac{1}{2}$

Petra



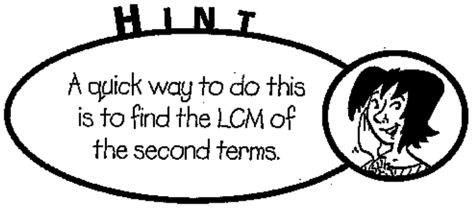
Orange crystals to water: $1:1\frac{2}{3}$

➤ **Method 2**

Equivalent Ratios

To find out whose orange punch is stronger:

- Write each mixture as a ratio.
- Write each ratio with the same second term.
- Compare the first terms.



Joe 4:6

Petra 3:5

The LCM of 6 and 5 is 30.

Use equivalent ratios.

4:6 = (4 × 5):(6 × 5)
= 20:30

3:5 = (3 × 6):(5 × 6)
= 18:30

Joe uses 20 scoops of crystals with 30 cups of water.

Petra uses 18 scoops of crystals with 30 cups of water.

20 > 18, so Joe's orange punch is stronger.

Method 3

Write each ratio with a second term of 1.

For each ratio, divide each term by the second term.

Joe

$4{:}6 = \frac{4}{6} : \frac{6}{6}$

$= 0.\overline{6} : 1$

Petra

$3{:}5 = \frac{3}{5} : \frac{5}{5}$

$= 0.6 : 1$

Since $0.\overline{6} > 0.6$, Joe's orange punch is stronger.

Method 4

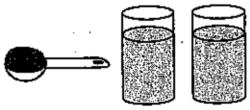
Compare the part-to-whole ratios and change both ratios to percents.

	Joe	**Petra**
Ratio (part to whole)	4:10	3:8
Ratio expressed as a fraction	$\frac{4}{10}$	$\frac{3}{8}$
Percent	40%	37.5%

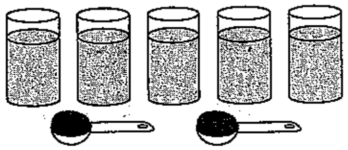
Joe has a higher percent of orange crystals in his punch, so his orange punch is stronger.

Practice

1.



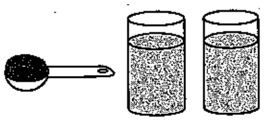
A



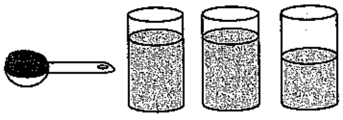
B

Which mixture is stronger, A or B?

a) Draw a picture to show how much water is used for each scoop of powder in Mixture A and Mixture B.



A



B

4

b) Which mixture is stronger? Explain how you know.

Mixture **A** is stronger because **it has less water for every scoop of powder.**

4

2. Two cages contain white mice and brown mice.
In one cage, the ratio of white mice to brown mice is 2:3.
In the other cage, the ratio is 3:1.
The cages contain the same number of mice.

HINT

Multiply to find equivalent ratios. Add to find the total numbers of mice. Keep going until you get two totals that match.

a) What could the total number of mice be?

Cage A ①

White	Brown	Total
2	3	5
4	6	10
6	9	15
8	12	20

Cage B ①

White	Brown	Total
3	1	4
6	2	8
9	3	12
12	4	16
15	5	20

/7

The number of mice in each cage could be 20 .①

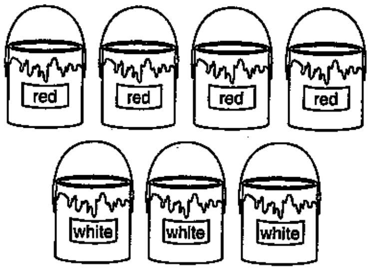
The total number of mice could be 40 ①

b) Which cage contains more white mice?

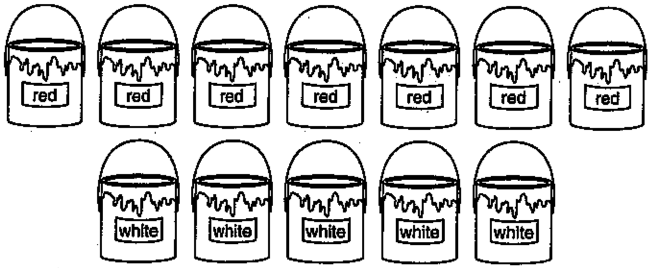
Number of white mice in A: 8 ① Number of white mice in B: 15 ①

Cage B ① contains more white mice.

3.



A



B

The red paint and white paint in each picture will be mixed.

Write the ratio of red paint to white paint. A 4:3 B 7:5

Write each ratio with the same second term. A 20:15 B 21:15

Compare the first terms. 20 < 21

Which mixture will give a darker shade of red? Mixture B

Tip

Use the LCM of the second terms to write equivalent ratios.

13

/6

4. The ratio of computers to students at Jan's school is 3:5.
The ratio of computers to students at Karl's school is 2:3.
Both schools have the same number of students.
Which school has more computers? Show your work.

Jan's School	**Karl's School**
3:5	2:3
= (3 × 3):(5 × **3**)	= (2 × 5):(3 × **5**)
= **9** : **15**	= **10** : **15**
9 computers to **15** students	**10** computers to **15** students

7

Karl's school has more computers.

5. Hamid jogs 5 laps in 6 min.
Amelia jogs 8 laps in 11 min.
Which person jogs faster? Show your work.

Hamid	**Amelia**
5:6	**8:11**
= (5 × 11):(6 × 11)	**= (8 × 6):(11 × 6)**
= 55:66	**= 48:66**
55 laps in 66 minutes	**48 laps in 66 minutes**

5

Hamid jogs faster.

6. The Rebels hockey team has won 9 of its first 15 games. No game was tied. The Sabres' record is 7 wins and 5 losses.

Which team has the better record? Show your work.

$\frac{9}{15}$ = 0.6 = 60% **$\frac{7}{12}$ ≐ 0.58 = 58%**

5

The Rebels have won 60% of their games, and the Sabres have won about 58% of their games. The Rebels have the better record.

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5.8 Solving Ratio Problems

/20

Quick Review

You can often solve a problem involving ratios by setting up a proportion.
A **proportion** is a statement that two ratios are equal.

In a box of red and blue marbles, the ratio of red marbles to blue marbles is 3:4.
If there are 48 blue marbles, you can find the number of red marbles using a proportion.

Let r represent the number of red marbles.
Then: $r:48 = 3:4$
In fraction form: $\frac{r}{48} = \frac{3}{4}$
To find the value of r, first isolate r by multiplying each side of the proportion by 48.

$$48 \times \frac{r}{48} = 48 \times \frac{3}{4}$$
$$r = \frac{144}{4}$$
$$= 36$$

There are 36 red marbles.

Practice

1. State the number you would multiply each side of the proportion by to isolate the variable.

/3

a) $\frac{r}{6} = \frac{5}{6}$ **6**
b) $\frac{t}{15} = \frac{2}{5}$ **15**
c) $\frac{v}{3} = \frac{5}{6}$ **3**

2. Find each missing term.

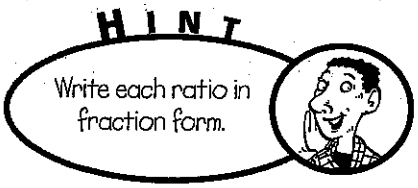
a) $p:4 = 9:12$

$\frac{p}{4} =$ $\mathbf{\frac{9}{12}}$

/1

4 $\times \frac{p}{4} =$ $\mathbf{\frac{9}{12} \times 4}$

$p =$ **3**



HINT

Write each ratio in fraction form.

b) $c:12 = 5:6$ **$c = 10$**

c) $3:14 = t:70$ **$t = 15$**

/2

~~6~~

3. Find each missing term.

a) $\frac{f}{10} = \frac{4}{5}$ **8** **b)** $\frac{h}{8} = \frac{12}{3}$ **32** **c)** $\frac{w}{11} = \frac{6}{33}$ **2**

d) $x{:}6 = 12{:}9$ **8** **e)** $m{:}4 = 9{:}6$ **6** **f)** $x{:}16 = 5{:}4$ **20**

6

4. In a bag of coloured cubes, the ratio of green cubes to purple cubes is 5:7. If there are 70 green cubes, how many purple cubes are there?

Let p represent the number of purple cubes. Write a proportion:

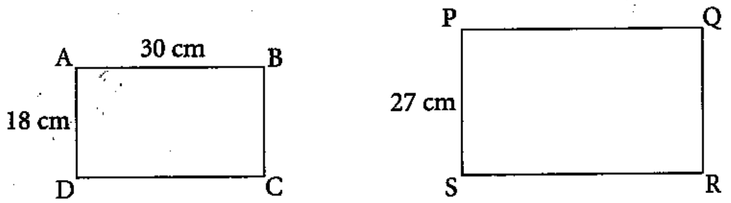
p: **70** = **7** : **5**

5

There are **98** purple cubes.

Tip
Writing the variable as the first term in the ratio makes it easier to solve the proportion.

5. Rectangles ABCD and PQRS have the same length-to-width ratio. Calculate the length of rectangle PQRS.



3/3

The length of rectangle PQRS is **45 cm**.

6. The length of a bug is 6.4 cm in a drawing. The drawing was made using a scale of 4:1. What is the actual length of the bug?
Let the actual length of the bug be l centimetres.
Length of bug:length of drawing = 1:4

$\frac{l}{6.4} =$ $\frac{1}{4}$

6.4 $\times \frac{l}{6.4} =$ $\frac{1}{4} \times 6.4$

$l =$ **1.6**

The actual length of the bug is **1.6** cm.

3

omit

7. On a school trip, the ratio of teachers to students is 2:21. The ratio of boys to girls is 4:3.

If there are 18 girls on the trip, how many boys are there? **24**

How many teachers? **4**

4

5.9 Exploring Rates

Quick Review

/36

- A **rate** is a comparison of two quantities measured in different units.

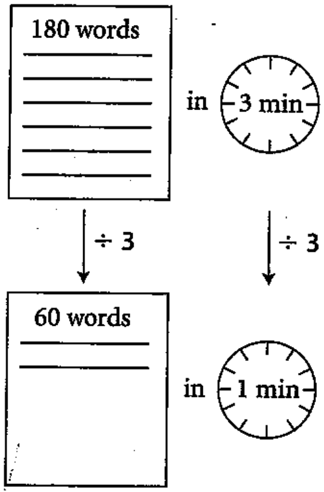
  Leo types 180 words in 3 min.
  180 words in 3 min is a **rate**.

  This means Leo types 60 words in 1 min.
  Leo's rate of typing is 60 words per minute.
  You can write this as 60 words/min.

  60 words/min is a **unit rate.**
  It compares a quantity (60 words) to 1 unit (1 min).

- To find a unit rate, you can use a diagram, a table, or a graph.
  In 3 min, Leo types 180 words.
  In 1 min, Leo types 60 words.

A Diagram



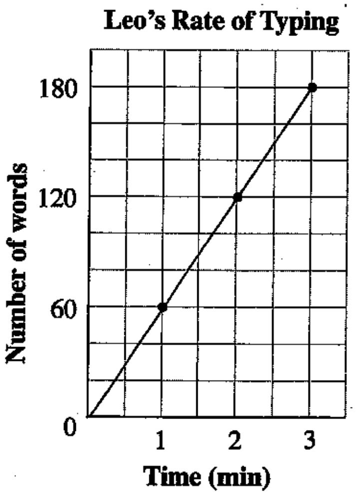
A Table

÷ 3

Minutes	3	2	1
Words	180	120	60

÷ 3

A Graph



1. Express as a unit rate.

a) Serena walks 4 km in 1 h. **4 km/h**

b) Sanjit reads 3 books in 1 week. **3 books/week**

/3

c) The tap drips 25 drops in 1 min. **25 drops/min**

2. Express as a unit rate. Show your work.

a) Betty drives her car 150 km in 2 h.

Tip
Express each rate in kilometres per hour (km/h).

150 km ÷ 2 = **75** km

/2

Betty's average driving speed is **75** km/h.

b) The helicopter travels 180 km in 3 h.

180 km ÷ 3 = 60 km

The helicopter's rate of travelling (average speed) is 60 km/h.

/2

c) Gerald walks 1 km in 15 min.

Distance (km)	1	2	3	4
Time (min)	15	30	45	60

Tip
1 h = 60 min

/5

Gerald's rate of walking is **4** km/h.

3. Determine whether the sentence expresses a ratio or a rate. Write the rate or ratio for each.

a) The cost of pecans is $10.89 for each kilogram. ratio/[✓ rate] **$10.89** per kg

b) Three out of every seven people are wearing glasses. [✓ ratio]/rate **3:7**

c) Mr. Thompson travelled 620 km in 6 h. ratio/[✓ rate] **620 km per 6 h** 103/hr.
~~118/hr~~.

/10

d) Each block of a quilt has 5 red patches, 4 yellow patches, and 6 blue patches.
[✓ ratio]/rate **5:4:6**

e) In 7 games, the team scored a total of 23 points. ratio/[✓ rate] **23 points per 7 games**
3.29 pp/g.

4. Maria charges $15 for 3 h of babysitting.

a) What is Maria's rate per hour? **Maria charges $5/h.**

/3

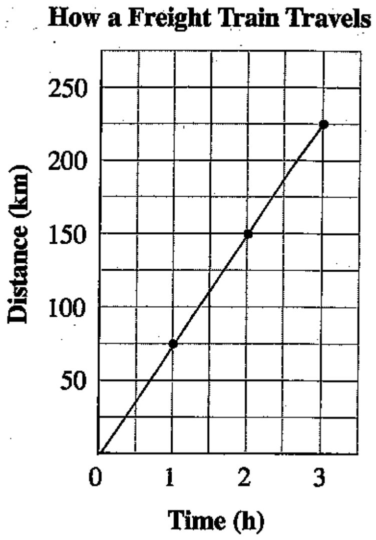
b) How much does Maria charge for 5 h of babysitting? **Maria charges $25.**

c) How many hours does Maria have to babysit to earn $50?

Maria has to babysit **10 h** to earn $50.

/25

5. The graph shows the distance a freight train travels in 3 h.



a) How far does the train travel in 1 h?

75 km

/2

b) What is the average speed of the train?

75 km/h

6. Frozen fruit bars cost $3.95 for 5 bars.
Find how many you can buy with $12. Show your work.

$3.95 ÷ 5 = $0.79

~~**$0.79 × 15 = $11.85**~~ 12 ÷ .79 = 15

/3

I can buy 15 bars.

7. Terence came to Canada shopping on a long weekend.
The exchange rate for his US money was $1.00 US to $1.05 Canadian.

a) How many Canadian dollars would Terence get for $500.00 US?

$525.00 Can

/4

b) Terence spent $504.90 Canadian altogether during his 3-day stay in Canada.
What was his average spending per day?

$168.30 Can/day

c) A jacket he purchased cost $39.95 Canadian.
What is this value in US dollars?

$38.05 US

/11

5.10 Comparing Rates

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Quick Review

➤ To compare different rates, you need to calculate their unit rates.

A case of 12 cartons of juice costs \$11.76.
A packet of 3 cartons of the same juice costs \$2.88.
To find which juice is the better buy, compare the unit costs of the 2 packages.
The unit cost of the case of 12 cartons is: \$11.76 ÷ 12 = \$0.98
The unit cost of the packet of 3 cartons is: \$2.88 ÷ 3 = \$0.96
So, the packet of 3 cartons is the better buy.

The unit cost can be written as a unit rate:
The unit rate of the better-buy juice is \$0.96/carton.

To find the unit rate of a 450-g packet of cereal that costs \$3.96,
use the cost of 100 g as the unit cost.
The cost of 100 g of the 450-g packet is: $\frac{\$3.96}{450} \times 100 = \$0.88$
So, the unit rate of the 450-g packet of cereal is \$0.88/100 g.

Practice

1. Write a unit rate for each.

a) 6 bottles of juice for \$3.96

$\frac{\$3.96}{6}$ = **\$0.66/bottle**

b) 840 words typed in 12 min

70 words/min

4

c) \$564 earned in 4 weeks

\$141/week

d) 130 mL of toothpaste for \$1.69

\$1.30/100 mL

Tip
A unit rate can be a rate for a quantity greater than 1.

2. Which is the better buy? Explain.

a) 475 g of cereal for \$3.80 or 750 g for \$6.30

$\frac{\$3.80}{475\text{ g}}$ **= \$0.80/100 g;** $\frac{\$6.30}{750\text{ g}}$ **= \$0.84/100 g; first one is the better buy.**

6

b) 385 mL of shampoo for \$5.39 or 400 mL for \$5.72

$\frac{\$5.39}{385\text{ mL}}$ **= \$1.40/100 mL;** $\frac{\$5.72}{400\text{ mL}}$ **= \$1.43/100 mL; first one is the better buy.**

/10

3. Find the average speed of each.

a) 242 km in 4 h	**b)** 372 km in 6 h	**c)** 309 km in 5 h
60.5 km/h	**62 km/h**	**61.8 km/h**

/4

Which is the greatest average speed? **62 km/h**

4. Shamar types 279 words in 4.5 min, Tasha types 320 words in 5 min, and Cody types 341 words in 5.5 min. Who has the greatest average typing speed?

/1

Tasha has the greatest average typing speed.

5. In the first 6 games of the basketball season, Lucinda scored 87 points.

/2

a) What was her average number of points scored per game? **14.5 points/game**

b) At this rate, how many points will Lucinda score in 26 games? **377 points**

6. Which is the better buy?
Twelve 710-mL bottles of water for $6.60 or twenty-four 500-mL bottles for $9.18

/1

Twenty-four 500-mL bottles for $9.18 is the better buy.

7. Population density is defined as the average number of people per square kilometre.
The population density of Canada is approximately 3.5 people/km^2.
Use the data in the table. Name a province or territory that has:

a) A population density closest to that of Canada **British Columbia**

b) A population density about half of that of Canada **Saskatchewan**

/4

c) A population density about 4 times that of Canada **Ontario**

d) A population density about 300 times that of Nunavut **British Columbia**

Province/Territory	**Population**	**Area (km^2)**
Ontario	12 393 000	917 700
Saskatchewan	995 000	591 700
British Columbia	4 196 000	925 200
Nunavut	29 600	1 939 000

/12

In Your Words

Here are some of the important mathematical words of this unit.
Build your own glossary by recording definitions and examples here. The first one is done for you.

discount *the amount that a price is reduced due to a sale*

sales tax *the tax based on a percent of the selling price that is set by the government*

It is calculated at the place of sale, collected by the retailer, and passed on to the government.

ratio *a comparison between two quantities measured in the same unit*

For example, the number of circles to the number of squares is 3 to 2. The ratio is 3:2.

○ ○ ○ □ □

equivalent ratios *ratios that are equal*

For example, 8:6 and 4:3 are equivalent ratios because you can multiply each term in the ratio 4:3 by 2 to get 8:6.

rate *a comparison of two quantities with different units*

For example, 10 km travelled in 2 h is a rate. You compare distance (10 km) with time (2 h).

proportion *a statement that two ratios are equal*

For example, $\frac{x}{5} = \frac{12}{15}$. *The value of an unknown x can be found by solving the proportion.*

List other mathematical words you need to know:

percent increase, percent decrease, two-term ratio, three-term ratio, part-to-whole ratio, part-to-part ratio, unit rate

Unit Review

LESSON

5.1 **1.** Write each decimal as a fraction and as a percent.

a) $0.15 = \frac{15}{100} = $ **15%**

b) $0.4 = \frac{40}{100} = $ **40%**

c) $0.875 = \frac{875}{1000} = \frac{87.5}{100} = $ **87.5%**

d) $0.003 = \frac{3}{1000} = \frac{0.3}{100} = $ **0.3%**

2. In Carmela's class, 61% of the students are girls, while in Analise's class, 20 out of 32 students are girls. Which class has a greater ratio of girls to students? Explain how you found out.

$\frac{20}{32}$ = 62.5%; 62.5% > 61%, so Analise's class has a greater ratio of girls to students.

5.2 **3.** Write each percent as a fraction and as a decimal.

a) $85\% = \frac{85}{100} = $ **0.85**

b) $0.7\% = \frac{0.7}{100} = \frac{7}{1000} = $ **0.007**

c) $139\% = \frac{139}{100} = $ **1.39**

d) $412\% = \frac{412}{100} = $ **4.12**

4. Write each fraction as a decimal and as a percent.

a) $\frac{4}{5} = $ **0.8**
$= $ **80%**

b) $\frac{8}{5} = $ **1.6**
$= $ **160%**

c) $\frac{3}{1000} = $ **0.003**
$= $ **0.3%**

d) $\frac{15}{6000} = $ **0.0025**
$= $ **0.25%**

HINT

To convert a decimal to a percent, move the decimal point 2 places to the right or multiply by 100.

5. In 1895, the population of a small town was 2120. By 1905, the population increased to 115% of the 1895 figure.

a) What was the population in 1905?

1.15 × 2120 = 2438; the population was 2438 in 1905.

b) Find the increase in population from 1895 to 1905.

2438 – 2120 = 318; the increase in population was 318.

LESSON

5.3 **6.** Find the amount in each case.

a) 8% is 56 kg.	b) 125% is 85 cm.	c) 0.48% is 84 L.
700 kg	**68 cm**	**17 500 L**

7. In a sponsored walk for charity, 560 students participated.
Of these, 0.72% completed the 15-km walk. How many students completed this distance?

Four students completed this distance.

8. Write each increase or decrease as a percent.

a) The price of gasoline rose from 132.5¢/L to 137.8¢/L.

Percent increase = **4%**

b) The number of trucks crossing the border fell from 3240 to 2673.

Percent decrease = **17.5%**

9. A water tank is filled with 1500 L of water. In 1 h, the tank loses 5.4% of the water due to leakage. What is the volume of water in the tank after 1 h?

The volume of water in the tank after 1 h is 1419 L.

5.4 **10.** The tax rate is 12%. Calculate the selling price of each item before and after tax.

a) $125 item at 10% off	b) $1820 item at 25% off	c) $6.80 item at 15% off
Before: **$112.50**	Before: **$1365**	Before: **$5.78**
After: **$126.00**	After: **$1528.80**	After: **$6.47**

11. The sale price of a computer at 15% off is $746.30. What is the regular price?

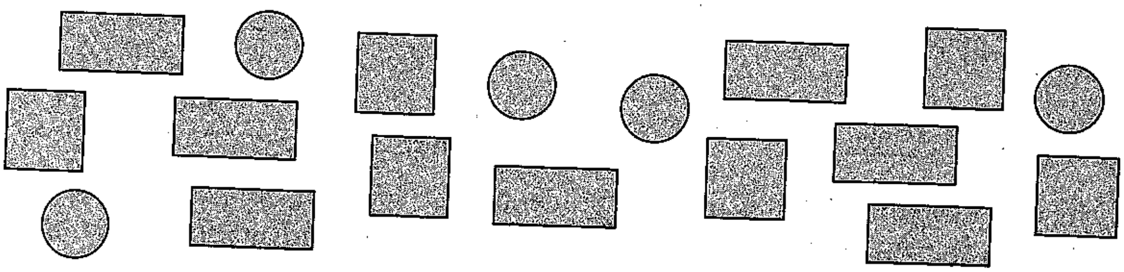
The regular price is $878.

12. A store owner buys coats for $56 each. She adds 30% to the cost and sells the coats at 15% off. Find the selling price of each coat.

$56 × 1.3 × 0.85 = $61.88; the selling price is $61.88.

LESSON

5.5 **13.** Write each ratio.



a) squares to circles **6:5**

b) rectangles and circles to squares **12:6**

c) circles to total figures **5:18**

5.6 **14.** a) Write three ratios equivalent to 2:5. Show your work.

◆ ◆ ◇ ◇ ◇ ◇ ◇

HINT

Multiply or divide each term by the same number.

Sample Answers:

2:5 = (2 × **2**):(5 × **2**)
= **4** : **10**

2:5 = **(2 × 3):(5 × 3)**
= **6:15**

2:5 = **(2 × 4):(5 × 4)**
= **8:20**

b) Write three ratios equivalent to 36:18. Show your work.
Sample Answers:

36:18 = **(36 ÷ 18):(18 ÷ 18)**
2:1

36:18 = **(36 × 2):(18 × 2)**
72:36

36:18 = **(36 ÷ 6):(18 ÷ 6)**
6:3

LESSON

15. Write each ratio in simplest form.

a) 25:15 = (25 ÷ 5):(15 ÷ 5)

= 5 : 3

b) 28:35 = **(28 ÷ 7):(35 ÷ 7)**

= **4:5**

c) 45:72 = **(45 ÷ 9):(72 ÷ 9)**

= **5:8**

5.7 **16.** Class 8B has 3 globes for every 7 students.
Class 8D has 2 globes for every 5 students.
Each class has the same number of students.
Which class has more globes? Explain.

Tip
Write each ratio with the same second term.

3:7 = (3 × 5):(7 × 5) = 15:35 2:5 = (2 × 7):(5 × 7) = 14:35

Class 8B has more globes.

5.8 **17.** At a summer camp, for every 3 students who sailed, 5 kayaked.
Forty-five students kayaked. How many students sailed?

Let *s* be the number of students who sailed. Write a proportion.

Tip
Writing the variable as the first term in the ratio makes it easier to solve the proportion.

s: 45 = 3 : 5

27 students sailed.

18. In a bag of coloured cubes, the ratio of red cubes to total number of cubes is 5:7.
If there are 105 cubes in the bag, how many cubes are red?

75 cubes are red.

LESSON

19. The scale of a map is 1:6 000 000.

HINT

1 km = 1000 m
1 m = 100 cm

a) The distance between 2 towns on the map is 8.7 cm. What is the actual distance?

1 cm on the map represents **6 000 000** cm of actual distance.

The actual distance between the 2 towns is:

8.7 × **6 000 000** cm = **52 200 000** cm = **522** km

b) The distance between 2 other towns is 1248 km. What is the distance on the map?

1248 km = 124 800 000 cm

124 800 000 ÷ 6 000 000 = 20.8

The distance between the 2 towns on the map is 20.8 cm.

5.9 **20.** Express as a unit rate.

Tip

Express in kilometres per hour (km/h).

a) The van travels 280 km in 4 h.

The van travels at an average speed of 70 km/h.

b) Mikki jogs 2 km in 20 min.

Mikki jogs at an average speed of 6 km/h.

5.10 **21.** Which is the better buy?

2.9 L of detergent for $4.56 or 3.8 L for $5.78

3.8 L of detergent for $5.78 is the better buy.

22. A cruise ship travelled 84 km in 3.5 h.
At this rate, how long will it take to travel 1050 km?

It will take 43.75 h to travel 1050 km.

23. Which country has the greater population density? Write its population density.
The United Kingdom has about 60 million people and an area of 244 800 km^2, and China has about 1806 million people and an area of 9 590 000 km^2.

The United Kingdom, with a population density of about 245 people/km^2.

UNIT 6

Linear Equations and Graphing

Just for Fun

Date Palindrome

A number palindrome is a number that reads the same backward as forward. 13631 is a number palindrome.

In this century, February 20, 2002 is a date palindrome when it is written in the day/month/year short form without slashes (DDMMYYYY). Write this date palindrome.

20022002

Write two other date palindromes for this century.

Sample Answers: 17022071 (Feb 17, 2071),

14022041 (Feb 14, 2041)

Will you have a birthday that is a date palindrome? If so, what is it?

Sample Answer: 21022012 (Feb 21, 2012)

Word Scramble

Unscramble the letters in each row to form a word in mathematics.

ILLTUMPY	**MULTIPLY**
BRATTCUS	**SUBTRACT**
RAILBAVE	**VARIABLE**
NERPECT	**PERCENT**
COFTRAIN	**FRACTION**
LOVES	**SOLVE**
GREENTI	**INTEGER**

Make up your own scrambled words in mathematics for your friends to unscramble.

Four Fours

Use exactly four 4s and any mathematical symbols you know to make up as many expressions as you can with whole-number values between 1 and 20.
You may use symbols such as (), +, –, ×, ÷, and the decimal point. For example: $44 \div 44 = 1$

Sample Answers: $4 - (4 + 4) \div 4 = 2$, $(4 + 4 + 4) \div 4 = 3$

Variation: Work with a friend. Make this activity more challenging by trying whole number values between 1 and 100.

Activating Prior Knowledge

Graphing Ordered Pairs

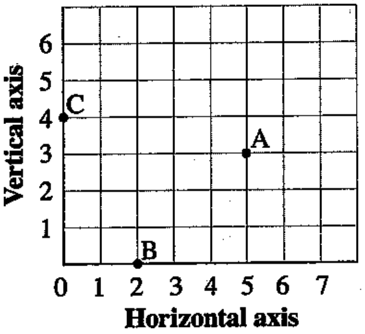
An ordered pair, such as (5, 3), tells you the position of a point on a grid.
The first number is the horizontal distance from the origin, O.
The second number is the vertical distance from the origin, O.
The numbers of an ordered pair are also called the **coordinates** of a point.

➤ To graph the points A(5, 3), B(2, 0), and C(0, 4) on a grid:

To plot point A, start at 5 on the horizontal axis, then move up 3.

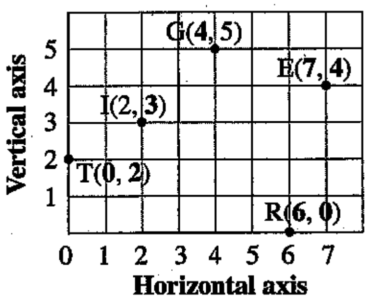
To plot point B, start at 2 on the horizontal axis, then move up 0. Point B is on the horizontal axis.

To plot point C, start at 0 on the horizontal axis, then move up 4. Point C is on the vertical axis.

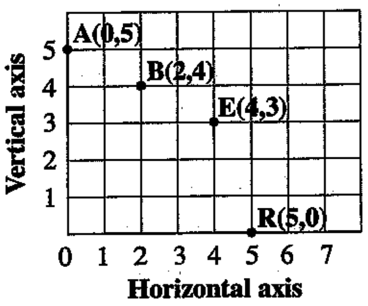


Checking

1. Write the ordered pair for each point on the grid.



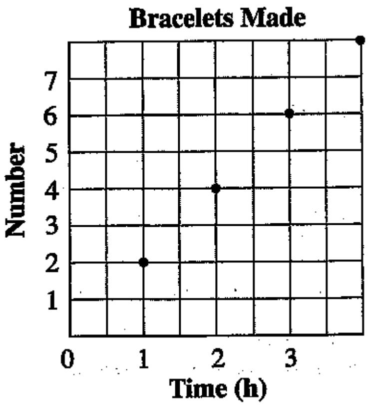
2. Plot and label these points:
A(0, 5), B(2, 4), E(4, 3), R(5, 0)



3. The graph shows the number of bracelets Jan can make over time.

a) How many bracelets can Jan make in 3 h? **6 bracelets**

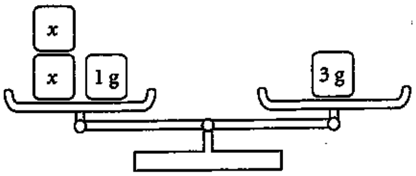
b) How long will it take to make 10 bracelets? **5 h**

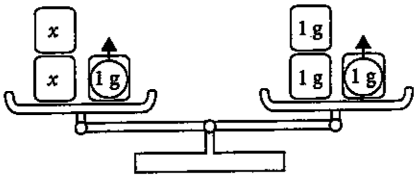
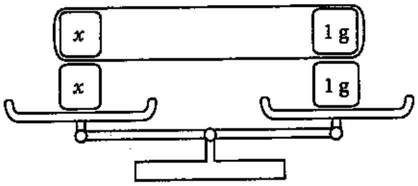


Preserving Equality

When you perform the same operation on both sides of an equation, the solution to the equation does not change. This is how the algebraic method of solving equations works.

Consider the equation $2x + 1 = 3$.



Subtract 1 from both sides.	$2x + 1 - 1 = 3 - 1$ $2x = 2$	
Divide both sides by 2.	$2x \div 2 = 2 \div 2$ $x = 1$	

To show that the solution did not change, check it in the original equation.
Substitute $x = 1$ into $2x + 1 = 3$.

Left side $= 2x + 1$ Right side $= 3$

$$= 2(1) + 1$$
$$= 2 + 1$$
$$= 3$$

Since the left side equals the right side, $x = 1$ is the correct solution to $2x + 1 = 3$.

✓ Checking

4. Write the operations you can perform, in the correct order, so that the solution to the equation does not change.

a) $3a - 2 = 4$

Add 2 to both sides.

Divide both sides by 3.

b) $\frac{c}{2} + 3 = -2$

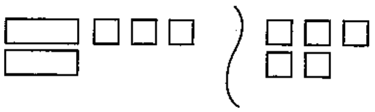
Subtract 3 from both sides.

Multiply both sides by 2.

6.1 Solving Equations Using Models

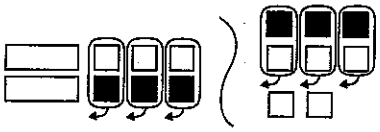
Quick Review

Algebra tiles and balance scales can both be used to model and solve equations. To solve the equation $2x + 3 = 5$:

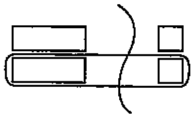


What you do to one side of the equation, you also do to the other side.

Isolate the x-tiles by adding 3 black tiles to make zero pairs. Then remove the zero pairs.



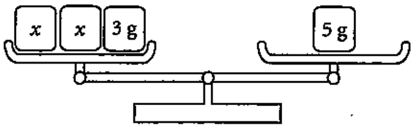
Arrange the tiles on each side into 2 equal groups. Compare groups.



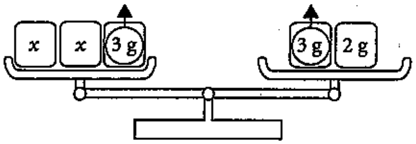
One x-tile equals 1 white tile.
So, $x = 1$.

Tip

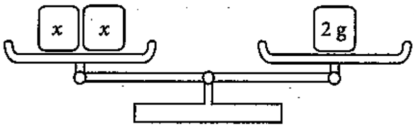
A white square tile models +1 and a black square tile models –1. These are called unit tiles. White rectangular tiles model variable tiles, or x-tiles. One white unit tile and one black unit tile form a ***zero pair.***



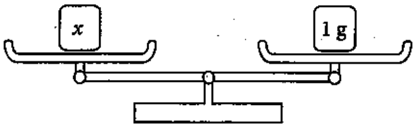
Replace 5 g in the right pan with 3 g and 2 g. Then remove 3 g from each pan.



The unknown masses are isolated in the left pan, and 2 g is left in the right pan.



The two unknown masses balance 2 g. So, each unknown mass is 1 g.



So, $x = 1$.

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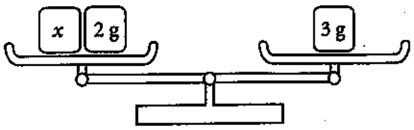
Practise

1. Write the equation modelled by each of the following.

a)

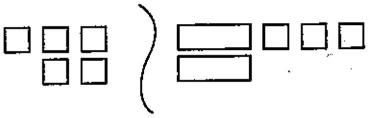


Tip

To isolate the x-tile or mass, make zero pairs.

$x + 2 = 3; x = 1$

b)



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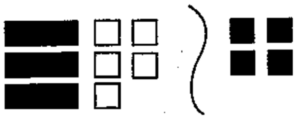
$5 = 2x + 3; x = 1$

c)

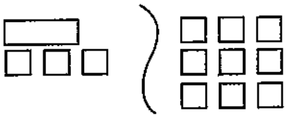


$-3x + 5 = -4; x = 3$

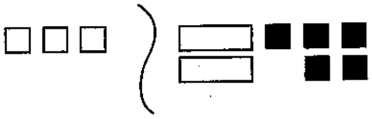
2. Construct a model to represent each equation. Then solve the equation using your model. Verify the solution.

Models may vary.

a) $x + 3 = 9$ **$x = 6$**



b) $3 = 2x - 5$ **$x = 4$**



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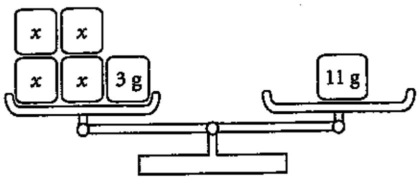
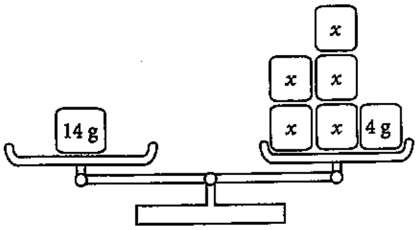
c) $4x + 3 = 11$ **$x = 2$**



d) $14 = 5x + 4$ **$x = 2$**

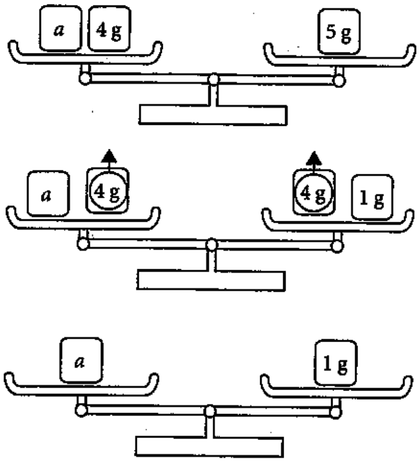


24.

3. Draw a model for each equation and the steps of its solution. Verify the solution.
Models may vary.

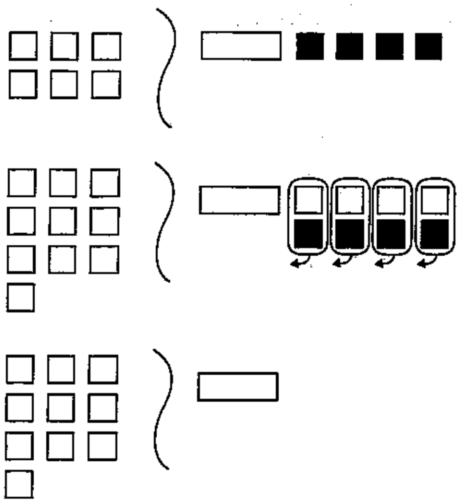
Al. Tiles

a) $a + 4 = 5$



$a = 1$

b) $6 = c - 4$



$c = 10$

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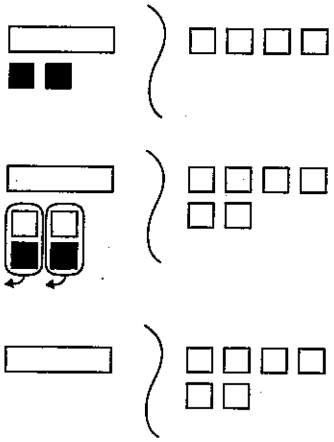
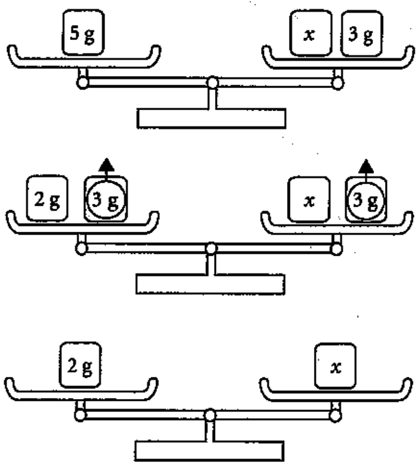
c) $y - 2 = 4$



$y = 6$

d) $5 = x + 3$



$x = 2$

4. Draw a model for each equation and the steps of its solution.
Verify the solution.
Models may vary.

HINT

Line up each variable tile or mass with the same quantity of number tiles or masses in your model.

BS -

a) $2v = 6$

b) $4n = -8$

$v = 3$

$n = -2$

/12

c) $5 = 5y$

d) $-6 = 3r$

$y = 1$

$r = -2$

/18

5. Draw a model to represent the steps you took to solve each equation. Verify the solution.
Models may vary.

a) $3x + 2 = 11$

b) $-5 = 5 + 2y$

/8

$\mathbf{x = 3}$

$\mathbf{y = -5}$

6. Five more than twice a number is seven. Let n represent the number.

a) Write an equation you can use to solve for n.

$\mathbf{2n + 5 = 7}$

b) Represent the equation for this problem with a model. Use the model to solve the equation.
Models may vary.

/6

$\mathbf{n = 1}$

c) Verify the solution and write a concluding statement.

Answers may vary. The number is 1.

7. One less than three times a number is eleven. Write an equation and use a model to solve the problem. Verify the solution and write a concluding statement.
Models may vary.

/4

$\mathbf{3n - 1 = 11; n = 4}$

6.2 Solving Equations Using Algebra

Quick Review

In Section 6.1, you solved the equation $2x - 3 = 1$ using algebra tiles. You are going to solve the same equation using algebra and compare it to the algebra tile model.

HINT

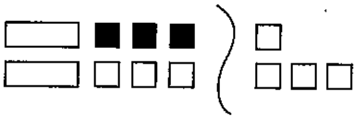
There are two main ideas:
1. Do *opposite* operations.
2. Do them to *both* sides.

Algebra tile model

Algebra steps

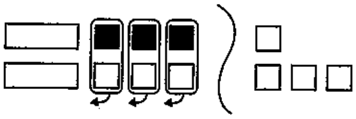
$$2x - 3 = 1$$

Isolate the *x*-tiles by adding +3 to both sides



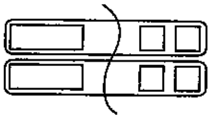
$$2x - 3 + 3 = 1 + 3$$

Remove zero pairs.



$$2x = 4$$

Arrange the tiles on each side into 2 equal groups.



$x = 2$

Divide both sides by 2 to isolate the *x*-variable:

$$\frac{2x}{2} = \frac{4}{2}$$

$$x = 2$$

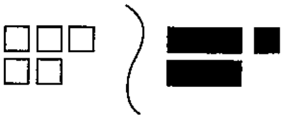
Practice

1. Write the equation modelled by each set of algebra tiles. Then solve the equation using both the algebra tile method and the algebra method.

a)

$3x + 1 = -5; x = -2$

b)



$5 = -2x - 1; x = -3$

2. Sketch algebra tiles to represent each equation. Then solve the equation using both the algebra tile method and the algebra method.

Models may vary.

a) $2y - 1 = 7$

$\mathbf{y = 4}$

b) $-4 = 2 + 3a$

$\mathbf{a = -2}$

3. Use algebra to solve each equation. Verify the solution.

a) $6m + 5 = -7$

$$\mathbf{6m + 5 = -7}$$
$$\mathbf{6m + 5 - 5 = -7 - 5}$$
$$\mathbf{6m = -12}$$
$$\mathbf{\frac{6m}{6} = \frac{-12}{6}}$$
$$\mathbf{m = -2}$$

The solution is $\mathbf{m = -2}$.

b) $3c - 2 = 2$

The solution is $\mathbf{c = \frac{4}{3}}$.

c) $2 + 5y = 2$

The solution is $\mathbf{y = 0}$.

d) $4 - 3x = -5$

The solution is $\mathbf{x = 3}$.

4. Each solution has an error. Check the solution and show that it is incorrect. Then show a correct solution.

a)

$$3y - 4 = 8$$
$$3y - 4 + 4 = 8 + 4$$
$$3y = 12$$
$$\frac{3y}{3} \cancel{\times 3} = \frac{12}{3} \cancel{\times 3}$$
$$y = \cancel{9}\ \mathbf{4}$$

The solution is $\mathbf{y = 4}$.

b)

$$9 = 6 - 2x$$
$$9 \cancel{+} - 6 = 6 - 6 - 2x$$
$$\cancel{15}\ \mathbf{3} = -2x$$
$$\frac{\cancel{15}\ \mathbf{3}}{-2} = \frac{-2x}{-2}$$
$$\mathbf{1}\cancel{7}\mathbf{\tfrac{1}{2}} = x$$

The solution is $\mathbf{x = -1\frac{1}{2}}$.

5. For each part below, let the number be *n*. Write an equation and solve it algebraically, verify the solution, and then write a concluding statement.

a) Four less than three times a number is fourteen.

$\mathbf{3n - 4 = 14;\ n = 6}$

b) The sum of twelve and twice a number is forty-four.

$\mathbf{2n + 12 = 44;\ n = 16}$

6.3 Solving Equations Involving Fractions

Quick Review

Remember the two basic concepts in solving an equation:

1. Isolate the variable by using *opposite operations.*
2. Do operations to *both sides* to keep the equations in balance.

The opposite operation of addition is subtraction.
The opposite operation of multiplication is division.
The opposite operation of division is multiplication.

You can solve $2x = 10$ by dividing both sides by 2 because dividing is the opposite operation of multiplication.

The solution looks like this:

$$2x = 10$$

$$\frac{2x}{2} = \frac{10}{2}$$

$$x = 5$$

You can solve $\frac{a}{3} = 6$ by multiplying both sides by 3 because multiplication is the opposite operation of division.

The solution looks like this:

$$\frac{a}{3} = 6$$

$$\frac{a}{3} \times 3 = 6 \times 3$$

$$a = 18$$

Solve the equation $-10 + \frac{m}{5} = -14$

Remember, first you isolate the variable by doing opposite operations.

$$-10 + \frac{m}{5} = -14$$

$$-10 + \frac{m}{5} + 10 = -14 + 10$$

$$\frac{m}{5} = -4$$

$$\frac{m}{5} \times 5 = -4 \times 5$$

$$m = -20$$

To verify the solution, substitute $m = -20$ into $10 + \frac{m}{5} = -14$.

Left side $= -10 + \frac{m}{5}$

$= -10 + \frac{(-20)}{5}$

$= -10 + (-4)$

$= -14$

Right side $= -14$

Since the left side equals the right side, $m = -20$ is correct.

1. For each equation, describe the opposite operation required to isolate the variable.

a) $2x = 14$

division

b) $\frac{y}{3} = 20$

multiplication

c) $m - 3 = 15$

addition

d) $21 = \frac{a}{7}$

multiplication

2. Solve each equation and verify the results.

a) $\frac{c}{6} = -3$

$\frac{c}{6} = -3$

$\frac{c}{6} \times 6 = -3 \times 6$

$c = -18$

b) $\frac{n}{2} = 14$

$n = 28$

c) $16 = -5y$

$y = -\frac{16}{5}$

d) $9 = \frac{a}{4}$

$a = 36$

3. The senior girls basketball team took one-third of the basketballs to their game. They took 7 balls to their game. How many basketballs were there altogether?

HINT

Finding one-third of a quantity is the same as dividing by three.

a) Let b be the total number of balls. Write an equation you can use to solve this problem.

$\frac{b}{3} = 7$

b) Solve the equation.

$b = 21$

c) Verify the solution and write a concluding statement.

Left side $= \frac{b}{3}$

$= \frac{21}{3}$

$= 7$

Right side $= 7$

There were 21 basketballs altogether.

4. Bob ate 22 jellybeans. His mom says that he ate one-quarter of the bag. How many jellybeans were in the bag to start with?

a) Set up an equation to solve this problem. Let j be the number of jellybeans in the bag.

$\frac{j}{4} = 22$

b) Solve the equation and verify the result.

$j = 88$

c) Write a concluding statement.

There were 88 jellybeans in the bag.

5. Solve each equation. Verify the results.

a) $\frac{w}{3} + 6 = 2$

$\frac{w}{3} + 6 = 2$

$\frac{w}{3} + 6 - 6 = 2 - 6$

$\frac{w}{3} = -4$

$\frac{w}{3} \times 3 = -4 \times 3$

$w = -12$

b) $-1 = \frac{y}{4} + 3$

$y = -16$

c) $\frac{x}{5} - 2 = -10$

$x = -40$

d) $4 + \frac{c}{10} = 8$

$c = 40$

6. The solution to this problem has an error it. Find the error, and then show a correct solution and verify your answer.

$$6 + \frac{w}{3} = 2$$

$$6 - 6 + \frac{w}{3} = 2 - 6$$

$$\frac{w}{3} = -4$$

$$\frac{w}{3} \cancel{\div} \times 3 = -4 \cancel{\div} \times 3$$

$$w = -\cancel{\frac{4}{3}}\ 12$$

7. Maya took one-fifth of the cookies out of the cookie jar and ate them. She took out an additional 4 to give to her brother. If 9 cookies in total were taken out of the jar, how many were in the jar at the start?

a) Write an equation to solve this problem.

Let j represent **the number of cookies that were in the jar to start**.

$$\frac{j}{4} + 4 = 9$$

b) Solve your equation and verify the solution.

$j = 25$

c) Write a concluding statement to the problem.

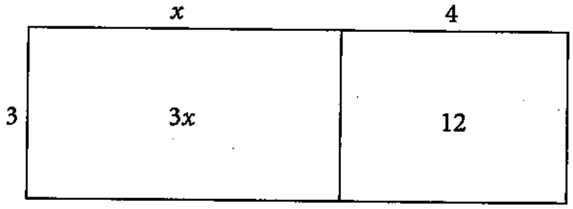
There were 25 cookies in the jar to start.

6.4 The Distributive Property

Quick Review

To multiply $3(x + 4)$:

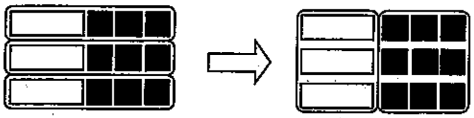
Draw a diagram.



$3(x + 4) = 3x + 12$

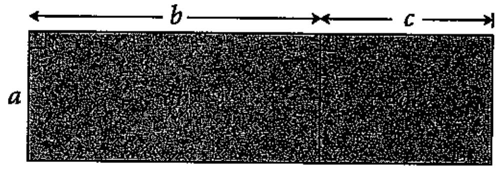
To multiply $2(x - 3)$:

Use algebra tiles.



$2(x - 3) = 2x - 6$

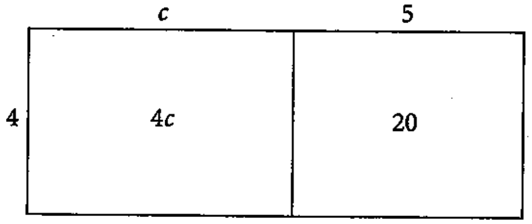
The distributive property says that $a(b + c) = ab + ac$



Practice

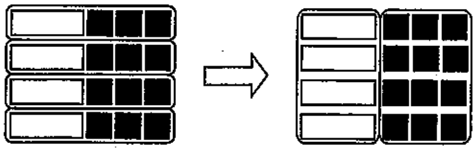
1. Write the equation represented by each model.

a)



$4(c + 5) = 4c + 20$

b)

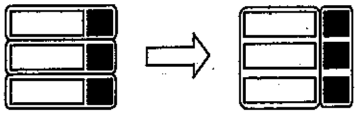


$4(a - 3) = 4a - 9$

2. Use algebra tiles to model each equation.

a) $2(y + 5) = 2y + 10$

b) $3(w - 1) = 3w - 3$



3. Use the distributive property to expand each expression.

a) $3(u - 6) =$ **$3u - 18$** **b)** $2(5 + q) =$ **$10 + 2q$**

c) $5(r + 1) =$ **$5r + 5$** **d)** $7(3 - p) =$ **$21 - 7p$**

4. Expand.

a) $-6(a - 7) =$ **$-6a + 42$** **b)** $4(-5 - w) =$ **$-20 - 4w$**

c) $-2(x - 20) =$ **$-2x + 40$** **d)** $-1(b + 8) =$ **$-b - 8$**

5. Vanessa expanded $3(y - 2)$ below. Did she make an error? [✓ YES]/NO

$3(y - 2) = 3y - 5$

If so, write the correct solution.

$3(y - 2) = 3y - 6$

6. Hazar is having 4 friends over to play video games. Each person will spend \$6 on game rental and \$4 on drinks and snacks.

a) Write two expressions for the total cost for Hazar and his friends.

$5(6 + 4); 5 \times 6 + 5 \times 4$

b) Evaluate each of the expressions.

50; 50

c) Show how the distributive property is being illustrated in this question.

$5(6 + 4) = 5 \times 6 + 5 \times 4$

6.5 Solving Equations Involving the Distributive Property

Quick Review

Francis thought of his favourite number.
He subtracted 9.
Then he multiplied the difference by –2.
The product was 10.
What is Francis's favourite number?

Let n represent Francis's favourite number. Write an equation to solve for n.

Start with n.	n
Subtract 9.	$n - 9$
Multiply the difference by –2.	$-2(n - 9)$
The product is 10.	$-2(n - 9) = 10$

Solve the equation.

$$-2(n - 9) = 10$$
$$-2n + 18 = 10$$
$$-2n + 18 - 18 = 10 - 18$$
$$-2n = -8$$
$$\frac{-2n}{-2} = \frac{-8}{-2}$$
$$n = 4$$

Francis's favourite number is 4.

Practice

1. Solve each equation using the distributive property. Verify the results.

a) $4(r + 3) = -8$

$$\mathbf{4(r + 3) = -8}$$
$$\mathbf{4r + 12 = -8}$$
$$\mathbf{4r + 12 - 12 = 8 - 12}$$
$$\mathbf{4r = -20}$$
$$\mathbf{\frac{4r}{25} = \frac{-20}{4}}$$
$$\underline{\mathbf{r = 5}}$$

b) $15 = 3(p - 7)$

$$\underline{\mathbf{p = 12}}$$

c) $-3(m-2)=21$

d) $3=5(x+7)$

$m=-5$

$x=-6\frac{2}{5}$

e) $-6(7+r)=30$

f) $0=2(-2+h)$

$r=-12$

$h=2$

2. Brittany has some cookies. She gave four of them to friends. If she doubles the number that she has left, she will have 12 cookies.

a) Choose a variable to represent the number of cookies Brittany had at the start.
Sample Answer:

c

b) Write an algebraic expression to represent how many she would have if she gave four of them away to friends.

$c-4$

c) Now write an expression to double what you wrote in part b).

$2(c-4)$

d) Write an equation for this problem and solve it.

$2(c-4)=12$; $c=10$

e) Verify your answer and write a concluding statement.

Brittany had 10 cookies to start.

6.6 Creating a Table of Values

Quick Review

If a relation is represented by the equation $y = 2x + 1$, you can write a table of values as:

x	1	2	3	4	5	6	7
y	3	5	7	9	11	13	15

A related pair of x and y values is called an **ordered pair.**
Some ordered pairs for this relation are:
(1, 3), (2, 5), (3, 7), (4, 9), (5, 11), (6, 13), (7, 15), (x, y)

A one-scoop ice-cream cone costs $3.00 plus $0.50 for each topping. An equation for this relation is $c = 3 + \frac{t}{2}$, where t represents the number of toppings and c represents the cost of the ice-cream cone in dollars.

Tip

$0.50 is the same as $\frac{1}{2}$ of $1, so $\frac{t}{2}$ represents the cost of the toppings.

Use different values of t to complete a table of values.

$t = 0$

$$\begin{aligned} c &= 3 + \tfrac{t}{2} \\ &= 3 + \tfrac{0}{2} \\ &= 3 + 0 \\ &= 3 \end{aligned}$$

$t = 1$

$$\begin{aligned} c &= 3 + \tfrac{t}{2} \\ &= 3 + \tfrac{1}{2} \\ &= 3 + 0.5 \\ &= 3.5 \end{aligned}$$

$t = 2$

$$\begin{aligned} c &= 3 + \tfrac{t}{2} \\ &= 3 + \tfrac{2}{2} \\ &= 3 + 1 \\ &= 4 \end{aligned}$$

$t = 3$

$$\begin{aligned} c &= 3 + \tfrac{t}{2} \\ &= 3 + \tfrac{3}{2} \\ &= 3 + 1.5 \\ &= 4.5 \end{aligned}$$

A table of values is:

t	c
0	3
1	3.5
2	4
3	4.5

To find the cost of an ice-cream cone with 5 toppings, substitute $t = 5$ into the equation.

$$\begin{aligned} c &= 3 + \tfrac{t}{2} \\ &= 3 + \tfrac{5}{2} \\ &= 3 + 2.5 \\ &= 5.5 \end{aligned}$$

An ice-cream cone with 5 toppings costs $5.50.

To find how many toppings are on a crazy ice-cream cone that costs \$7.50, substitute $c = 7.5$ into the equation.

$$7.5 = 3 + \frac{t}{2}$$

$$7.5 - 3 = 3 + \frac{t}{2} - 3$$

$$4.5 = \frac{t}{2}$$

$$4.5 \times 2 = \frac{t}{2} \times 2$$

$$9 = t$$

A crazy ice-cream cone that costs \$7.50 has 9 toppings!

1. Copy and complete each table of values.

a) $y = x - 7$

x	y
−3	−10
−2	−9
−1	−8
0	−7
1	−6
2	−5
3	−4

b) $y = -x + 14$

x	y
−3	17
−2	16
−1	15
0	14
1	13
2	12
3	11

c) $y = -3x$

x	y
−3	9
−2	6
−1	3
0	0
1	−3
2	−6
3	−9

2. Make a table of values for each relation.

Tables may vary.

a) $y = x + 4$

x	y
−3	1
−2	2
−1	3
0	4
1	5
2	6
3	7

b) $y = -2x + 2$

x	y
−3	8
−2	6
−1	4
0	2
1	0
2	−2
3	−4

c) $y = 5 - x$

x	y
−3	8
−2	7
−1	6
0	5
1	4
2	3
3	2

3. The equation of a linear relation is: $w = 6r + 3$

a) Substitute 33 for w in the equation.
$\mathbf{33 = 6r + 3}$

b) Solve the equation to complete the ordered pair (**5**, 33) for this relation.

$$\begin{aligned} 33 &= 6r + 3 \\ 33 - 3 &= 6r + 3 - 3 \\ 30 &= 6r \\ \frac{30}{6} &= \frac{6r}{6} \\ 5 &= r \end{aligned}$$

4. Repeat the steps of question 3 to complete the following ordered pairs for the relation $w = 6r + 3$.

a) (**2**, 15)

b) (**–4**, –21)

5. The equation of a linear relation is: $d = 4t + 6$
Find the missing number in each ordered pair.

a) (2, **14**)

b) (**3**, 18)

c) (12, **54**)

d) (–4, **–10**)

6. Bergy's Hamburger Emporium sells its famous double-cheese mushroom burger for $4. The relation $c = 4n$ represents the cost, c, of n hamburgers.

a) Use the relation to complete the table of values.

n	1	2	3	4	5
c	**4**	**8**	**12**	**16**	**20**

b) How many hamburgers would have to be sold to have a cost of $28?

7 hamburgers would have to be sold.

6.7 Graphing Linear Relations

Quick Review

Daniel works at the local gas bar. He is paid \$5 per shift plus \$10 per hour for each hour that he works. David is only paid for whole hours. An equation that relates his earnings to the number of hours he works is $e = 5 + 10n$, where e represents his earnings for a shift that lasts n hours.

Substitute values for n to find corresponding values of e.

When $n = 0$, $e = 5 + 10(0)$
$= 5 + 0$
$= 5$

When $n = 1$, $e = 5 + 10(1)$
$= 5 + 10$
$= 15$

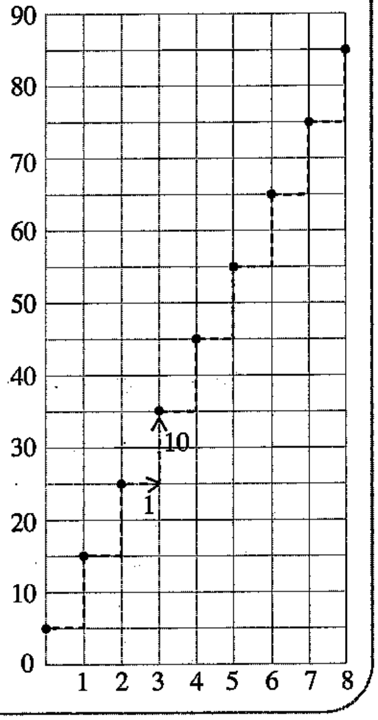
A table of values is:

n	0	1	2	3	4	5	6	7	8
e	5	14	25	35	45	55	65	75	85

To graph the relation, plot n along the horizontal axis and e along the vertical axis.
Label the axes and write the equation of the relation on the graph.
The points lie on a straight line, so the relation is linear.
Since Daniel only gets paid for whole numbers of hours, do not join the points. These data are **discrete**. This means that there are numbers between those given that are not meaningful in the context of the problem.

The graph shows that for every hour Daniel works, his pay increases by \$10. As the number of hours increases, so does his pay.

Graph of $e = 5 + 10\,n$

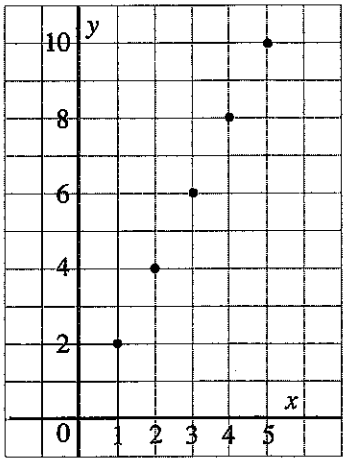


Practice

You will need grid paper.

1. **a)** Graph the table of values.

x	y
0	2
1	4
2	6
3	8
4	10

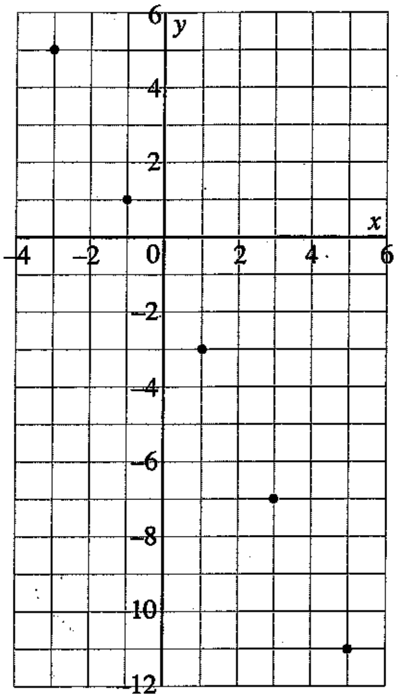


b) Describe the relationship between the variables in the graph.

When x increases by 1, y increases by 2. The y values start at 2.

2. **a)** Graph the table of values.

x	y
–3	5
–1	1
1	–3
3	–7
5	–11



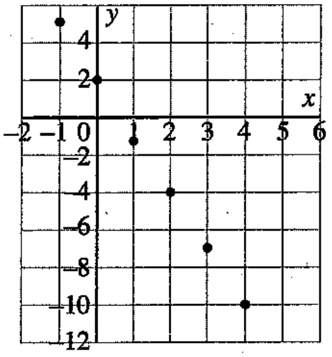
b) Describe the relationship between the variables in the graph.

When x increases by 2, y decreases by 4. The y values start at 5.

3. **a)** Complete the table of values for the relation with equation $y = -3x + 2$.

x	y
–1	5
0	2
1	**–1**
2	–4
3	–7
4	**–10**

b) Graph the ordered pairs.

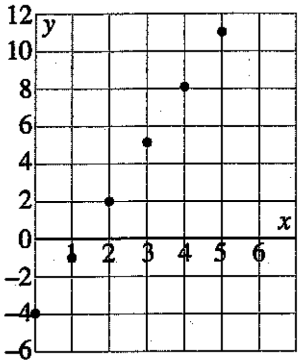


4. For $y = 3x - 4$:

a) Make a table of values using values of x from 0 to 5.

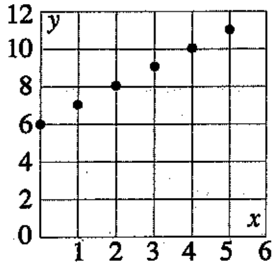
x	0	1	2	3	4	5
y	**–4**	**–1**	**2**	**5**	**8**	**11**

b) Graph the relation.

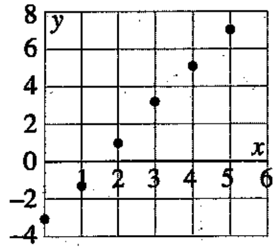


5. Graph each relation for integer values from 0 to 5.

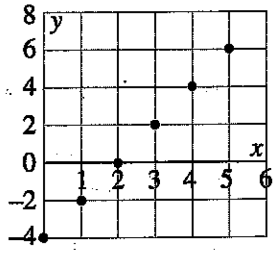
a) $y = x + 6$



b) $y = 2x - 3$



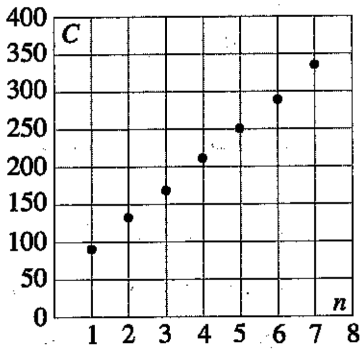
c) $y = -4 + 2x$



6. The snowboard club is planning a trip to a local hill. A bus company will charge them using the formula $C = 50 + 40n$, where C is the total cost for n people.

a) Make a table of values and draw a graph for the cost for 1 to 7 people.

n	1	2	3	4	5	6	7
C	**90**	**130**	**170**	**210**	**250**	**290**	**330**



b) A parent group is willing to give the club $410. How many people could go on the trip with that amount of money?

Substitute **410** for C in the equation and solve.

$$\mathbf{C = 50 + 40n}$$
$$\mathbf{410 = 50 + 40n}$$
$$\mathbf{410 - 50 = 50 + 40n - 50}$$
$$\mathbf{360 = 40n}$$
$$\mathbf{\frac{360}{40} = \frac{40n}{40}}$$
$$\mathbf{9 = n}$$

9 people could go on the trip with $410.

In Your Words

Here are some of the important mathematical words of this unit.
Build your own glossary by recording definitions and examples here. The first one is done for you.

distributive property multiplying a number by a sum of two numbers is the same as multiplying the first number by each number in the sum and then finding the sum of the products
For example, $5(a + b) = 5a + 5b$

opposite operation an operation that "undoes" a given operation
For example, multiplication and division are opposite operations, and addition and subtraction are opposite operations.

algebra tiles tiles that can be used to represent numbers and variables
For example, a small white square tile represents +1, a small black square tile represents −1, and a white rectangular tile represents x.

ordered pair two numbers in order; on a coordinate grid, the first number is the horizontal coordinate of a point, and the second number is the vertical coordinate of the point
For example, (2, 4) is an ordered pair.

table of values a way of organizing a relation in a table; ordered pairs can be read from a table of values

linear relation a relation that has a straight-line graph
For example, $y = 2x - 3$ is a linear relation.

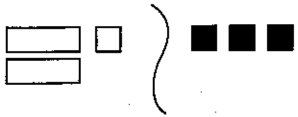
List other mathematical words you need to know.

Sample Answers: expand, discrete data, isolate, solve

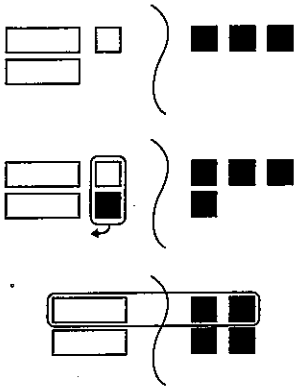
Unit Review

LESSON

6.1 **1.** Write the equation represented by the model. Then solve the equation using the model, showing your steps.



$2x + 1 = -3$



$x = -2$

2. Use a model to solve $4 + 3c = -5$.
Models may vary.

$c = -3$

6.2 **3.** Solve each equation algebraically and verify the result.

a) $4y - 7 = 13$

$$4y - 7 = 13$$
$$4y - 7 + 7 = 13 + 7$$
$$4y = 20$$
$$\frac{4y}{4} = \frac{20}{4}$$
$$y = 5$$

b) $-9 = 5 + 2m$

$m = -7$

4. Maria solved the equation $4 - 2p = 6$ using the steps below. Did Maria make an error? [✓ YES]/NO

If Maria made an error, correct it.

$$4 - 2p = 6$$
$$4 - 4 - 2p = 6 - 4$$
$$-2p = 2$$
$$\frac{-2p}{\cancel{2}-2} = \frac{2}{\cancel{2}-2}$$
$$p = \cancel{1}\ \mathbf{-1}$$

5. Rajinder collects hockey cards. He currently has 75. He has a plan to collect 12 more each week. After how many weeks will he have a total of 147?

a) Write an equation that you can use to solve this problem.

Let w represent the **number of weeks**.

$147 = 75 + 12w$

b) Solve the equation.

$$\mathbf{147 = 75 + 12w}$$
$$\mathbf{147 - 75 = 75 + 12w - 75}$$
$$\mathbf{72 = 12w}$$
$$\mathbf{\frac{72}{12} = \frac{12w}{12}}$$
$$\mathbf{6 = w}$$

c) Verify your result and write a concluding statement.

In 6 weeks, Rajinder will have 147 hockey cards.

6.3 **6.** Solve each of the following equations and verify the results.

a) $\frac{t}{2} = 4$

$$\mathbf{\frac{t}{2} = 4}$$
$$\mathbf{\frac{t}{2} \times 2 = 4 \times 2}$$

$t = 8$

b) $\frac{w}{3} + 4 = -2$

$$\mathbf{\frac{w}{3} + 4 = -2}$$
$$\mathbf{\frac{w}{3} + 4 - 4 = -2 - 4}$$
$$\mathbf{\frac{w}{3} = -6}$$
$$\mathbf{\frac{w}{3} \times 3 = -6 \times 3}$$

$w = -18$

c) $6 = 3 + \frac{x}{5}$

$x = 15$

6.4 **7.** Expand using the distributive property.

a) $6(v - 3)$

$6v - 18$

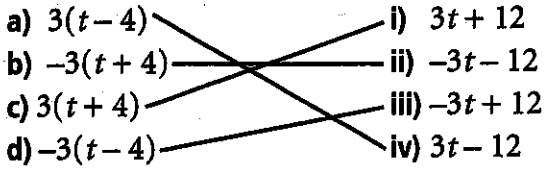
b) $-9(3 + p)$

$-27 - 9p$

c) $-1(-2 + w)$

$2 - w$

8. Match each expression in Column 1 with an equivalent expression in Column 2.



Column 1	**Column 2**
a) $3(t-4)$	i) $3t+12$
b) $-3(t+4)$	ii) $-3t-12$
c) $3(t+4)$	iii) $-3t+12$
d) $-3(t-4)$	iv) $3t-12$

6.5 **9.** Solve each equation and verify the results.

a) $5(a-3)=20$

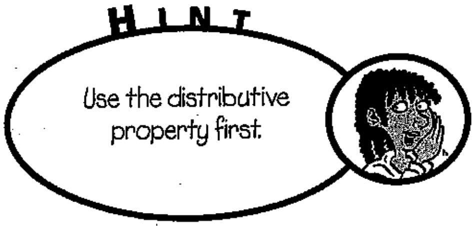
b) $-2(n+3)=-10$

$$\begin{aligned}5(a-3)&=20\\5a-15&=20\\5a-15+15&=20+5\\5a&=35\\\frac{5a}{5}&=\frac{35}{5}\\a&=7\end{aligned}$$

$\underline{a=7}$

$\underline{n=2}$

HINT



Use the distributive property first.

c) $7=4(2+y)$

d) $-2(x+3)=-6$

$\underline{y=-\frac{1}{4}}$

$\underline{x=0}$

6.6 **10.** Complete the table of values for each relation.

a) $y=x-4$

x	−2	−1	0	1	2
y	0	1	2	3	4

b) $y=-2x+5$

x	−2	−1	0	1	2
y	9	7	5	3	1

11. The equation of a linear relation is $y=4x-3$. Find the missing number in each ordered pair.

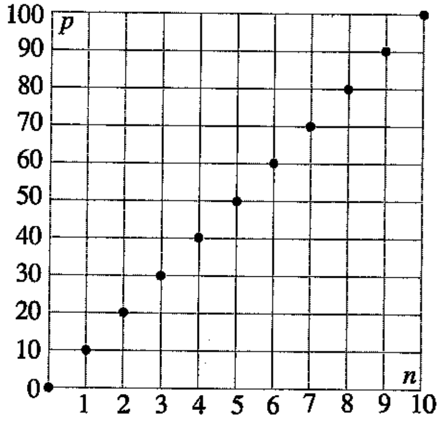
a) (2, __5__)

b) (__−2__, −11)

c) (__4__, 13)

LESSON 6.7

12. The graph below represents the relation of the percent score, p, on a math test and the number of questions, n, correct out of 10.
The equation for the relation is $p = 10n$.



a) State the ordered pair that represents the highest score.

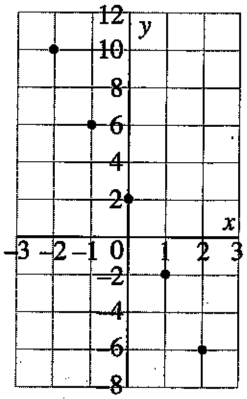
(10, 100)

b) Describe the relationship between the variables on the graph.

When n increases by 1, p increases by 10. The first p value is 0.

13. **a)** Draw a graph of the relation represented by the table of values.

x	y
–2	10
–1	6
0	2
1	–2
2	–6

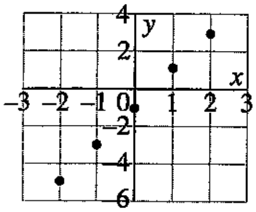


b) Describe how you know that this is a linear relation.

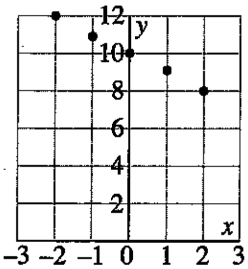
Its graph is a straight line.

14. On grid paper, draw the graph of each relation for integer x values from –2 to 2.

a) $y = 2x - 1$



b) $y = 10 - x$



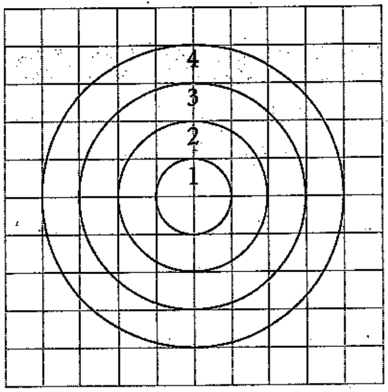
UNIT 7

Data Analysis and Probability

Just for Fun

Dartboard Design

The diagram shows a dartboard design drawn on grid paper. The side length of each square is 1 unit.



Use the formula for finding the area of a circle: $A = \pi r^2$, where r is the radius of the circle. Find the area of each region on the dartboard. Leave π in your answers.

Region 1: $\pi(1)^2 = \pi$ square units 3.14

Region 2: $\pi(2)^2 - \pi(1)^2 = 3\pi$ square units 9.42

Region 3: $\pi(3)^2 - \pi(2)^2 = 5\pi$ square units 15.7

Region 4: $\pi(4)^2 - \pi(3)^2 = 7\pi$ square units 21.98

Do the 4 areas form a number pattern? If so, write a pattern rule.

Sample Answer: Start at π square units. Add 2π square units each time.

Determine the probabilities that a dart thrown at random will land on the 4 regions.

Region 1: $\frac{1}{16}$ Region 2: $\frac{3}{16}$

Region 3: $\frac{5}{16}$ Region 4: $\frac{7}{16}$

A 5th concentric circle with radius 5 units is added to the design.

Use your pattern rule to find the area of the 5th region. 9π square units

Find the new probabilities that a dart thrown at random will land on the 5 regions.

Sample Answer: The new probabilities, in order of the regions, are:

$\frac{1}{25}, \frac{3}{25}, \frac{1}{5}, \frac{7}{25}$, **and** $\frac{9}{25}$.

Activating Prior Knowledge

Drawing Circle Graphs

A circle graph is used to graph data that represent parts of one whole.
Each piece of data is written as a fraction of the whole.

To find the angle of the sector in the circle that represents each piece of data, multiply the fraction by 360°.

To find the percent of the circle that represents the piece of data, multiply the fraction by 100%.

Example 1

The students of a Grade 8 class took a test.
This table shows their marks.

Draw a circle graph to display the data.

Test Results

Mark	Number of Students
A	5
B	15
C	3
D	2

Solution

Add the numbers in the table: 5 + 15 + 3 + 2 = 25
There are 25 students in the class.

Write each number as a fraction, then a percent.

A: $\frac{5}{25} = \frac{20}{100} = 0.2 = 20\%$

B: $\frac{15}{25} = \frac{60}{100} = 0.6 = 60\%$

C: $\frac{3}{25} = \frac{12}{100} = 0.12 = 12\%$

D: $\frac{2}{25} = \frac{8}{100} = 0.08 = 8\%$

A circle has central angle 360°. The angle of the sector representing each grade is:

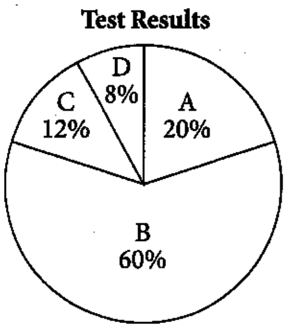
A: $0.2 \times 360° = 72°$

B: $0.6 \times 360° = 216°$

C: $0.12 \times 360° = 43.2° \doteq 43°$

D: $0.08 \times 360° = 28.8° \doteq 29°$

Draw a circle.
Use a protractor to measure each angle.
Label each sector.
Write a title for the graph.



✓ Check

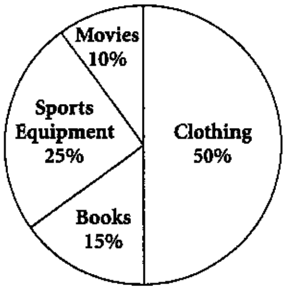
1. The table shows how Carl spends his earnings for this month.
   a) Complete the table for each item.

Item	Amount (5)	Sector Angle	Percent
Clothing	100	$\frac{100}{200} \times 360^\circ = 180^\circ$	$\frac{100}{200} \times 100^\circ = 50\%$
Books	30	$\frac{30}{200} \times 360^\circ = \underline{54}\,^\circ$	$\frac{30}{200} \times 100^\circ = \underline{15}\,\%$
Sports Equipment	50	$\frac{50}{200} \times 360^\circ = \underline{90}\,^\circ$	$\frac{50}{200} \times 100^\circ = \underline{25}\,\%$
Moview	20	$\frac{20}{200} \times 360^\circ = \underline{36}\,^\circ$	$\frac{20}{200} \times 100^\circ = \underline{10}\,\%$
Total	$\underline{200}$	$\underline{360}\,^\circ$	$\underline{100}\,\%$

   b) Draw a circle graph for the data.

Sample Answer:

How Carl Spends His Earnings



Tip

To make a sector, draw a radius, measure the angle with a protractor, and draw another radius.

Finding Probabilities Using Tables or Tree Diagrams

$$\text{The probability of an event} = \frac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}}$$

You can use a table or a tree diagram to show the outcomes of two independent events.

Example 2

The table shows the outcomes of tossing two coins.
Find the probability of tossing 1 head and 1 tail.

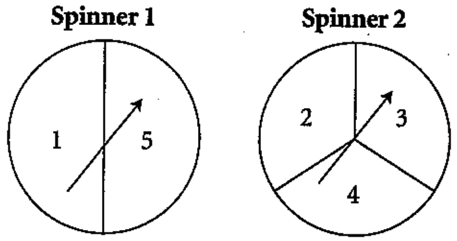
		2nd Coin	
		H	**T**
1st Coin	**H**	HH	HT
	T	TH	TT

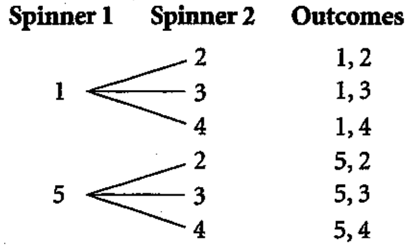
Solution

There are 4 possible outcomes.
Two outcomes have 1 head and 1 tail.
Probability of tossing 1 head and 1 tail = $\frac{2}{4}$, or $\frac{1}{2}$

Example 3

The tree diagram shows the possible outcomes of an experiment with spinning the pointers on these two spinners.





Find the probability of each event:

a) 5 on Spinner 1 and 3 on Spinner 2

b) a sum greater than 7

Solution

a) There are 6 possible outcomes.
One outcome is 5, 3.
The probability is: $\frac{1}{6}$

b) Two outcomes have a sum greater than 7:
5, 3 and 5, 4
The probability is: $\frac{2}{6} = \frac{1}{3}$

Check

2. **a)** Complete this table for the outcomes of the experiment in Example 3.

		Spinner 2		
		2	3	4
Spinner 1	1	1, 2	1, 3	1, 4
	5	5, 2	5, 3	5, 4

b) What is the probability of the pointers landing on two odd numbers?

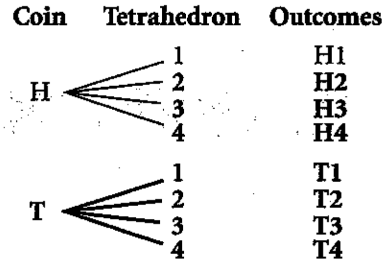
Two outcomes have 2 odd numbers: 1, 3 and 5, 3; probability = $\frac{2}{6} = \frac{1}{3}$

3. **a)** Complete this tree diagram for tossing a coin and rolling a tetrahedron labelled 1 to 4.

b) What is the probability of tossing tails and rolling an even number?

There are 2 outcomes: T2, T4

Probability = $\frac{2}{8} = \frac{1}{4}$



7.1 Choosing an Appropriate Graph

Quick Review

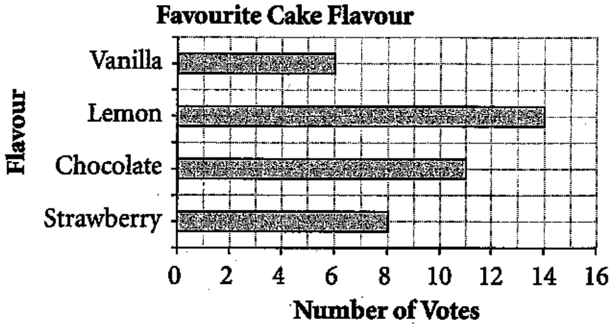
➤ Different types of graphs have different characteristics. It is important to choose an appropriate graph that best represents a set of data.

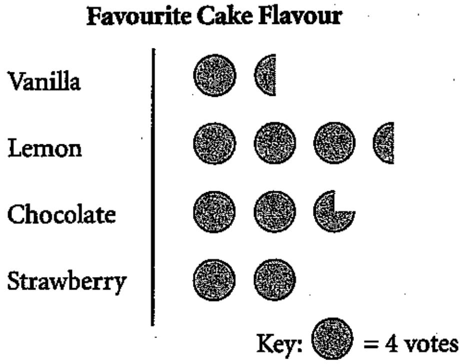
This table shows the strengths and limitations of five common types of graphs.

Type of Graph	Strengths	Limitations
Circle Graph	• Shows parts of a whole • Shows percents of the total • Sizes of sectors compare parts of the whole	• Does not show data values and the total • Difficult to draw accurately
Bar Graph	• Lengths of bars compare data values • Scale can be used to find the total • Easy to draw	• May be difficult to read depending on scale used • Does not show percents of the total for comparison
Line Graph	• Easy to draw and to read • Shows data changes over time • Can be used to estimate values between or beyond data points	• Does not show parts of a whole • Zig-zag pattern can be difficult to interpret
Pictograph	• Lengths of rows of symbols compare data values • Graph is visually appealing • Key can be used to find the total	• Large number of symbols make it difficult to read • Does not show parts of a whole • Difficult to draw
Double Bar Graph	• Directly compares two sets of data • Lengths of bars compare data values • Scale can be used to find the total of each data set • Easy to draw	• Can only be used to show discrete data • May be difficult to read depending on scale used • Two sets of data in one graph can be confusing

Practice

1. Rebecca's family hosted a party to gather opinions on the choice of flavour for her sister's wedding cakes. The guests' votes are displayed in these two graphs.





a) Which flavour is the most popular? **lemon** the least popular? **vanilla**

b) How many people voted at the party? 6 + 14 + **11** + **8** = **39**

c) From which graph is it easier to gather the information? Explain.

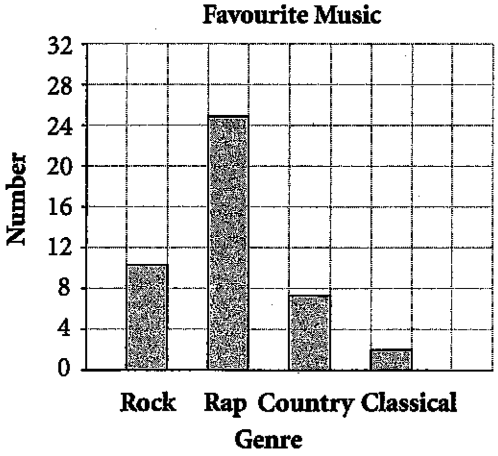
Sample Answer: It is easier to use the lengths of the bars in a bar graph to read data and compare data values.

2. This table shows the favourite types of music of the students in a Grade 8 class.

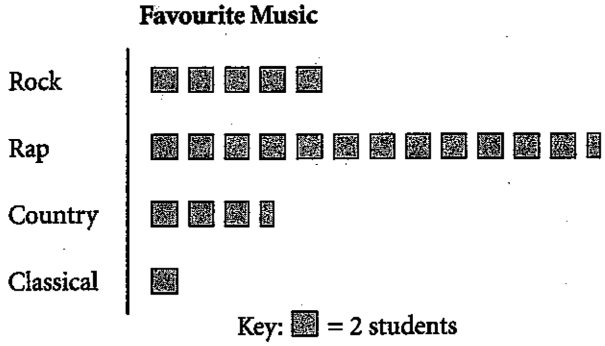
Favourite Music

Genre	Number
Rock	10
Rap	25
Country	7
Classical	2

a) Use a bar graph to display the data.



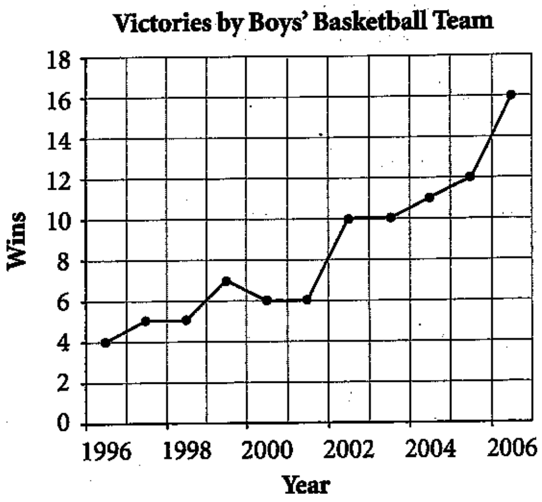
b) Use a pictograph to display the data.



c) Which graph was easier to draw? Justify your choice.

Sample Answer: It is easier draw the bar graph. It is more difficult to draw the many symbols and half of a symbol for the pictograph.

3. The graph shows the number of wins for the Boys' Basketball Team.



a) In what year did the team have the most wins? **2006**

b) In what year did the team have the least wins? **1996**

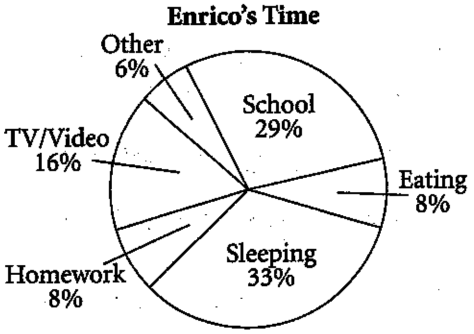
c) How well do you predict the team will do in the next year? Explain.

Sample Answer: The team would have more than 16 wins in the next year because the line graph shows an upward trend over the years.

d) Could you use a circle graph to display these data? Justify your answer.

Sample Answer: No. Since the data change over time, people would not be interested in knowing parts of the whole.

4. The circle graph shows the percents of time Enrico spends on different activities during an average day.



a) Why is a circle graph used?

Sample Answer: The data are parts of a whole. The sizes of sectors can be compared to make conclusions.

b) What percent of Enrico's time is spent at school and on sleeping?

29% + 33% = 62%

c) Could a bar graph be used to display the data? Would it more appropriate? Explain.

Sample Answer: Yes, you could use a bar graph, but it is not as appropriate as a circle graph, which is best at showing parts of a whole.

5. Match the most appropriate graph with the data.

Graph	**Data**
a) Circle graph	**i.** Change in your height over time
b) Line graph	**ii.** Number of shots by each starting player of the girls' and boys' basketball teams
c) Double bar graph	**iii.** Percent of each math topic on the final exam
d) Pictograph	**iv.** Number of students in a Grade 8 class from 5 different areas of origin
e) Bar graph	**v.** Number of food items donated by 4 Grade 8 classes

6. Suyama surveyed the clothes in her closet and counted these items:

Blue jeans:	6
Pants (not jeans):	2
Shorts:	5
Skirts:	4
Dresses:	3

a) List two graph types in question 5 you can use to display these data. Explain your choice.

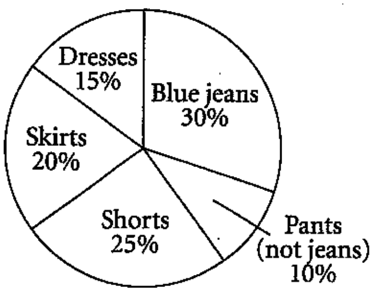
Sample Answer: I can use a circle graph or a bar graph since the data do not change over time. With the circle graph, I can compare parts of a whole.

b) List two graph types that may not be good choices. Justify your answer.

Sample Answer: A line graph and a double bar graph are not appropriate because the data do not change over time and there is only one set of data.

c) Pick one of your choices in part a) and draw the graph.
Sample Answer:

Suyama's Closet



7.2 Misrepresenting Data

Quick Review

➤ Graphs are a visual way of representing data. However, they can create false impressions by the way they are drawn.

There are many different ways in which graphs can be drawn to **misrepresent data.** Misinterpretation of the data may lead to incorrect conclusions or assumptions.

A wise consumer needs to become aware of the ways in which graphs can be misleading.

This table shows Tamara's math marks in the 4 report cards she has received for this year.

Report Card Marks

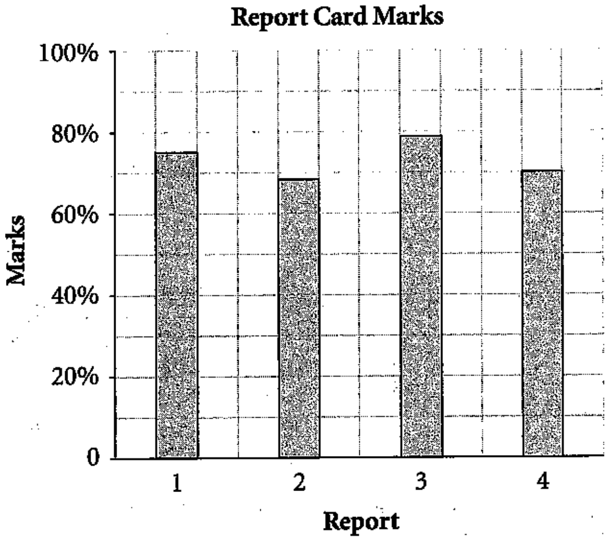
Report	Marks
1	75%
2	67%
3	78%
4	70%

Tamara's brother wants to show that Tamara's performance in math is not steady.

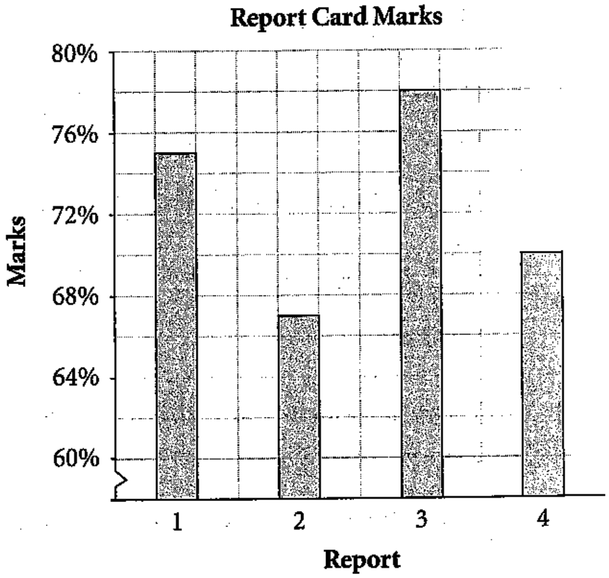
Tamara wants to show that this is not true.

They drew these graphs.

Tamara's Graph



Tamara's Brother's Graph

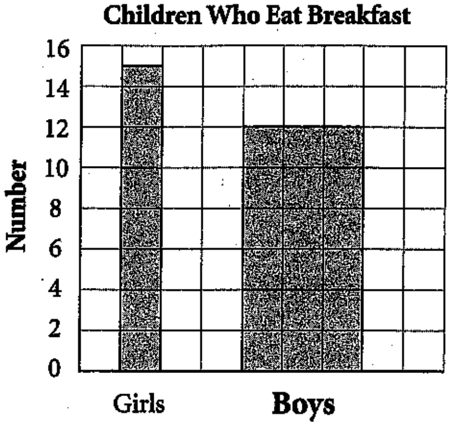


➤ These are some ways in which a graph can be drawn to misrepresent data.
- Start the scale on the vertical axis of a bar graph at a number other than 0, as in Tamara's brother's graph.
- Use different size symbols in a pictograph or bars of different widths in a bar graph to make one piece of data appear greater than another.
- Treat a part of a graph differently to draw people's attention to that piece of data.

Practice

1. This bar graph is drawn in a way to misrepresent the data.
   a) Which part of the graph draws your attention most? Why?

   Sample Answer: The bar for Boys because it is much wider and the type for "Boys" is larger.

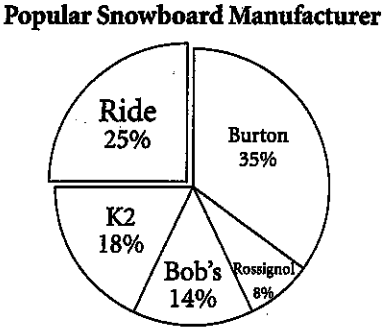


   b) What changes can you make to the graph to eliminate the false impression.

   Sample Answer: Make the 2 bars the same width and use the same type size for the 2 labels "Girls" and "Boys."

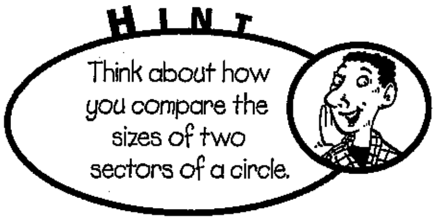
2. This circle graph is drawn in a way to misrepresent the data.
   a) Which part of the graph draws your attention most? Why?

   Sample Answer: The sector for "Ride" is separated from the rest of the graph and it has the largest type size.



   b) What changes can you make to the graph to eliminate the false impression.

   Sample Answer: Keep all sectors within the circle and keep the type size consistent for all similar labels.

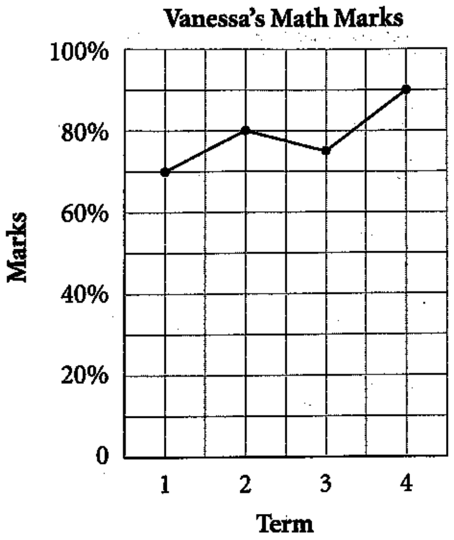


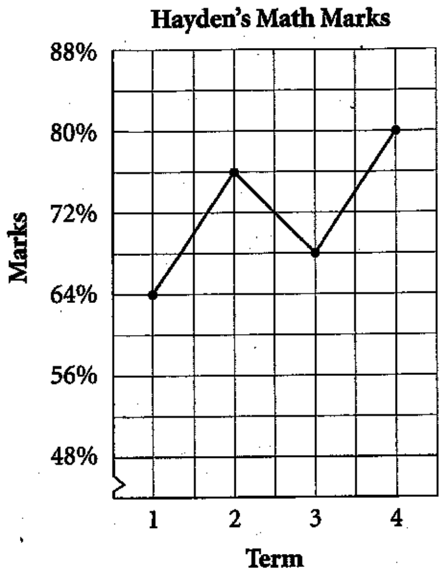
Think about how you compare the sizes of two sectors of a circle.

   c) Which snowboard manufacturer is the most popular? How do you know?

   Sample Answer: Burton is most popular. The sector representing Burton has the largest sector angle, and 35% is the greatest percent.

3. These line graphs display the math marks over 4 terms for 2 students, Vanessa and Hayden.





a) Which line graph appears to show a greater increase in marks from Term 3 to Term 4?

Hayden's graph appears to show a greater increase in marks.

b) Read the marks represented by the points on the line graph for Term 3 and Term 4. What is the actual increase in marks for each student?

Vanessa: **90% – 75% = 15%** Hayden: **80% – 68% = 12%**

c) How are the graphs misleading?

Sample Answer: The scale on the vertical axis of Hayden's graph starts at 48%, which exaggerates the increase in marks.

d) How can you change the graphs so that they do not misrepresent the data?

Sample Answer: To make a fair comparison, make the scales on the vertical axes of the two graphs the same.

e) If an award were to be given to the student with the most consistent marks over time, who would win the award? Explain your answer.

Sample Answer: Hayden is more consistent from term to term with a smaller range of marks between 64% and 80%.

4. Refer to the 2 graphs in question 3.

a) Which graph would you choose to show the most dramatic improvement? Explain.

Sample Answer: I would use Hayden's graph because he appears to have improved greatly based on the steepness of the lines on his graph.

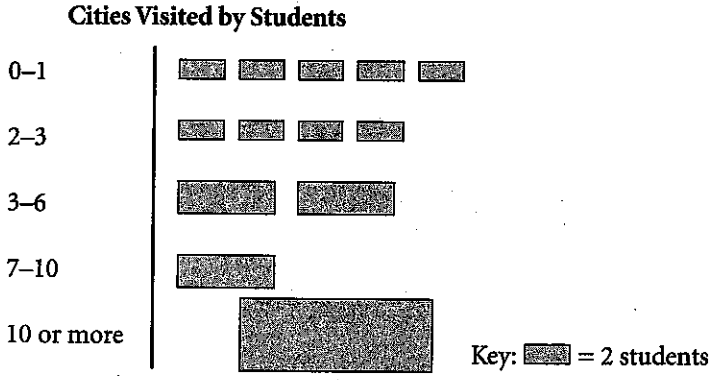
b) What is the overall increase in marks for each student for the 4 terms?

Vanessa: **90% – 70% = 20%** Hayden: **80% – 64% = 16%**

c) Does the result in part b) agree with the choice you made in part a)? Explain.

Sample Answer: The result is different from the impression that the graphs give me. So, my choice was incorrect.

5. The graph shows the number of cities visited by students surveyed.



Cities Visited by Students

a) What is the number of cities visited by the most students? **0–1 city**

b) Jean-Pierre estimates from the pictograph that about 8 students visited 10 or more cities. Why do you think Jean-Pierre made this estimate?

Sample Answer: The width of the symbol for 10 or more is about the same width as the row of symbols for 2–3 cities, which represents 8 students.

c) Marie says that only 2 students visited 10 or more cities. Explain who you agree with.

Sample Answer: I would agree with Marie because her number is based on the key of the pictograph—1 symbol represents 2 students.

d) How could the pictograph be changed to represent the data more accurately?

Sample Answer: Make all symbols the same size and line up all the rows on the left edge for easy counting of symbols and comparing lengths of rows.

Probability of Independent Events

At Home At School

Quick Review

- Two events are independent events when one event does not affect the other event.
  A coin is tossed and a regular die labelled 1 to 6 is rolled.

The table shows the possible outcomes.

		Die					
		1	2	3	4	5	6
Coin	H	H/1	H/2	H/3	H/4	H/5	H/6
	T	T/1	T/2	T/3	T/4	T/5	T/6

Whether the coin lands heads or tails has no effect on the outcome of rolling the die. So, the two events are independent.

- There are 12 possible outcomes.

Half of the outcomes have heads. The probability of tossing heads is $\frac{1}{2}$.

Two outcomes have a 4. The probability of rolling a 4 is $\frac{1}{6}$.

Only one outcome is H/4. The probability of tossing heads and rolling a 4 is $\frac{1}{12}$.

Note that $\frac{1}{12} = \frac{1}{2} \times \frac{1}{6}$

The above example illustrates the probability rule for two independent events:
P(A) and P(B) are the individual probabilities of the two events A and B.
P(A and B) is the probability of both A and B occurring.
If A and B are two independent events, P(A and B) = P(A) × P(B)

Practice

1. A quarter and a dime are both tossed. The table shows the possible outcomes.

		Quarter	
		H	T
Dime	H	HH	HT
	T	TH	TT

a) For a quarter, P(heads) = $\frac{1}{2}$

b) For a dime, P(heads) = $\frac{1}{2}$

c) How many outcomes in the table have 2 heads? 1

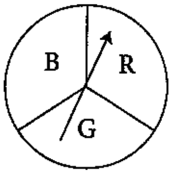
Write the probability of tossing 2 heads. P(2 heads) = $\frac{1}{4}$

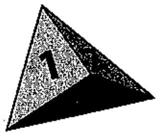
d) Find the product of the answers from parts a) and b). $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

e) Why do parts c) and d) have the same answer?

Sample Answer: The outcome of each coin showing heads is independent of the other coin. So, P(2 heads) = P(heads) × P(heads) = $\frac{1}{4}$

2. The pointer of a spinner with 3 congruent sectors is spun once and a tetrahedron labelled 1 to 4 is rolled.





The table shows the possible outcomes.

		Tetrahedron			
		1	2	3	4
Spinner	**R**	R/1	R/2	R/3	R/4
	G	G/1	G/2	G/3	G/4
	B	B/1	B/2	B/3	B/4

Find the probability of each event:

a) the pointer landing on red

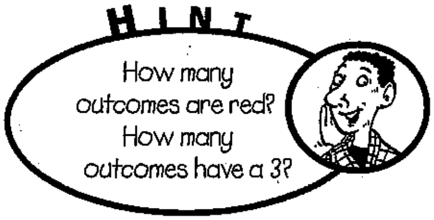
P(red) = $\frac{4}{12}$ = $\frac{1}{3}$

b) rolling a 3

P(3) = $\frac{3}{12}$ = $\frac{1}{4}$

c) red and a 3

P(R/3) = $\frac{1}{12}$



Compare the probability in part c) with the product of the probabilities in parts a) and b). What do you notice?

P(R/3) = P(red) × P(3) = $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

3. The pointer of the spinner in question 2 is spun and a regular die labelled 1 to 6 is rolled.

Find the probability of each event:

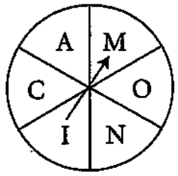
a) the pointer landing on green: **P(green) = $\frac{1}{3}$**

b) rolling a 1 **P(1) = $\frac{1}{3}$**

c) green and a 1

P(green and 1) = $\frac{1}{3}$ × $\frac{1}{6}$ = $\frac{1}{18}$

4. Monica writes each letter of her name on one of the equal sectors of this spinner.



She spins the pointer of the spinner and rolls the tetrahedron in question 2.

Find the probability of each event:

a) rolling an odd number **P(odd) = $\frac{2}{4} = \frac{1}{2}$**

b) an I and a 4 **P(I and 4) = P(I) × P(4) = $\frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$**

c) a vowel and a 2 **P(vowel and 2) = P(vowel) × P(2) = $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$**

5. Consider an experiment of rolling a red and a white regular die labelled 1 to 6.

a) What is the probability of rolling a 4 on the red die and a 5 on the white die?

P(red 4 and white 5) = P(red 4) × P(white 5) = $\frac{1}{6} \times \frac{1}{6} = \frac{1}{8}$

b) What is the probability of rolling a 5 on the red die and an even number on the white die?

P(5 and even) = P(5) × P(even) = $\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$

c) What is the probability of rolling a 3 on both dice?

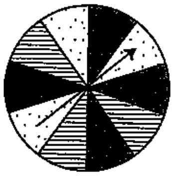
P(3 and 3) = P(3) × P(3) = $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

d) What is the probability of rolling a square number on the red die and a non-square number on the white die?

The square numbers are 1 and 4. P(1 or 4) = $\frac{2}{6} = \frac{1}{3}$; P(2, 3, 5, or 6) = $\frac{4}{6} = \frac{2}{3}$

P(square, non-square) = $\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$

6. The pointer of this spinner with 10 equal sectors is spun twice.



Find the probability of each event:

a) The pointer lands on a dotted sector and then a solid sector.

P(dotted) = $\frac{3}{10}$; P(solid) = $\frac{4}{10}$ = $\frac{2}{5}$

P(dotted, solid) = P(dotted) × P(solid) = $\frac{3}{10} \times \frac{2}{5} = \frac{3}{25}$

b) The pointer lands on a solid sector each time.

P(solid) = $\frac{2}{5}$

P(solid, solid) = P(solid) × P(solid) = $\frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$

c) The pointer does not land on a solid sector each time.

P(not solid) = P(dotted or striped) = $\frac{3}{10} + \frac{3}{10} = \frac{6}{10} = \frac{3}{5}$

P(not solid, not solid) = P(not solid) × P(not solid) = $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$

7. A bag contains 5 yellow balls, 3 red balls, and 2 green balls. A ball is removed from the bag without looking, the colour recorded, and then returned the bag. A second ball is drawn and its colour is also recorded.

a) What is the probability of a red ball followed by a green ball?

P(red) = $\frac{3}{10}$; P(green) = $\frac{2}{10} = \frac{1}{5}$

P(red, then green) = P(red) × P(green) = $\frac{3}{10} \times \frac{1}{5} = \frac{3}{50}$

b) What is the probability of both balls being yellow?

P(yellow) = $\frac{5}{10} = \frac{1}{2}$

P(yellow, yellow) = P(yellow) × P(yellow) = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

c) What would be the outcome of the 2 balls if it has a probability of $\frac{1}{10}$?

Sample Answer: P(1st ball) × P(2nd ball) = $\frac{1}{10}$

Since P(yellow) = $\frac{1}{2}$, P(green) = $\frac{1}{5}$, and $\frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$, the outcome is either a green ball followed by a yellow ball or a yellow ball followed by a green ball.

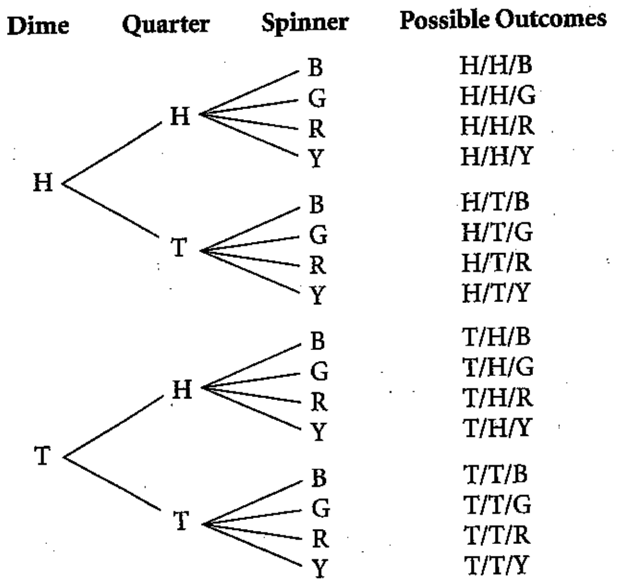
7.4 Solving Problems Involving Independent Events

Quick Review

➤ The rule for the probability of two independent events can be extended to three or more independent events.

Consider a case where there are more than two events.
You toss a dime and a quarter, and spin the pointer of a spinner with 4 equal sectors coloured blue, green, red, and yellow.

This tree diagram shows all possible outcomes:



There are 16 outcomes. One outcome is H/T/R.
So, the probability of tossing a head on the dime, tossing a tail on the quarter, and the pointer landing on red is $\frac{1}{16}$.

The probability of tossing a head on a dime is $\frac{1}{2}$ and tossing a tail on a quarter is $\frac{1}{2}$.

The probability of the pointer landing on red is $\frac{1}{4}$.

Note that $\frac{1}{16} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{4}$

The rule for the probabilities of independent events can be extended to:
The probability of 3 events A, B, and C occurring is P(A and B and C).
If A, B, and C are two independent events, P(A and B and C) = P(A) × P(B) × P(C)

1. Use the experiment in Quick Review.

a) Find the probability of each event:

i) a head on the dime **P(heads) = $\frac{1}{2}$** **ii)** a head on the quarter **P(heads) = $\frac{1}{2}$**

iii) the spinner landing on blue **P(blue) = $\frac{1}{4}$**

b) How many outcomes have 2 heads and a blue? **1**

c) The probability of 2 heads and a blue is:

P(H/H/B) = $\frac{1}{2}$ × $\frac{1}{2}$ × $\frac{1}{4}$ = $\frac{1}{16}$

d) How many outcomes have a head on the dime, a tail on the quarter, and a primary colour? **3**

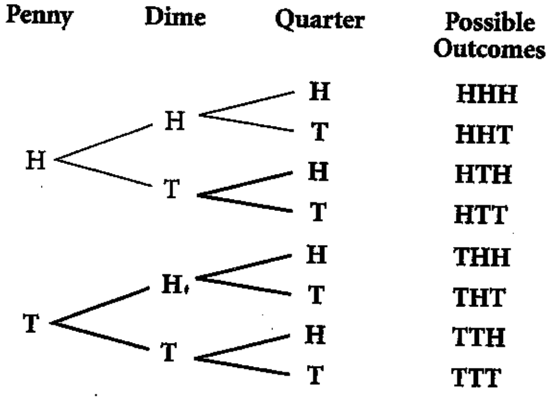
HINT

The primary colours are red, blue, and yellow.

e) Use the extended rule for the probabilities of independent events. Find the probability of a head on the dime, a tail on the quarter, and a primary colour.

P(H/H/R or B or Y) = P(H) × P(H) × P(R or B or Y) = $\frac{1}{2} \times \frac{1}{2} \times \frac{3}{4} = \frac{3}{16}$

2. Three coins—a penny, a dime, and a quarter—are tossed.
Draw a tree diagram to show the possible outcomes.



a) How many outcomes have 3 heads?

1

What is probability of 3 heads?

There are 8 possible outcomes.

P(H/H/H) = $\frac{1}{8}$

b) Use the individual probabilities for each coin and the probability of independent events.

What is the probability of all 3 coins showing heads?

P(H/H/H) = P(H) × P(H) × P(H) = $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

3. Diane, Claud, and Norma are playing a memory game.
They lay out 12 cards numbered 2 to 13 face down on a table.
Each player turns over a card and puts it back to its original position face down.
Find the probability of each event:

a) Each draws a prime. **P(2, 3, 5, 7, 11, or 13) = $\frac{1}{2}$; P(3 primes) = $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$**

b) Each draws a square number. **P(4 or 9) = $\frac{1}{6}$; P(3 squares) = $\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$**

c) Diane turns over a prime, Claud turns over a square number, and Norma turns over a number that is a factor of 17.

The probability is 0 since 17 is a prime number that has no factors other than itself and 1, and there are no cards labelled 1 or 17.

4. Robert has 25 songs in his MP3 player.
Ten of them are classic rock, 12 are country, and 3 are classical.
Robert sets the MP3 player to play the songs randomly.
Find the probability of each event:

a) The first song is classical, the second is country, and the third is classic rock.

P(classical) = $\frac{3}{25}$; P(country) = $\frac{12}{25}$; P(classic rock) = $\frac{10}{25} = \frac{2}{5}$

P(classical/country/classic rock) = $\frac{3}{25} \times \frac{12}{25} \times \frac{2}{5} = \frac{72}{3125}$

b) The first 2 songs are classic rock and the third is country.

P(2 classic rock/country) = $\frac{2}{5} \times \frac{2}{5} \times \frac{12}{25} = \frac{48}{625}$

5. To start a game of cards, each of 3 players cuts a card from a standard deck of 52 playing cards. Each time, the card is returned to the deck before the next player cuts.
Find the probability of each event:

a) Player 1 cuts a red card, player 2 cuts a jack, and player 3 cuts a spade.

P(red/jack/spades) = P(red) × P(jack) × P(spades) = $\frac{1}{2} \times \frac{1}{13} \times \frac{1}{4} = \frac{1}{104}$

b) Player 1 cuts a red ace, player 2 cuts a black card, and player 3 cuts the ace of clubs.

P(red ace/black/ace of clubs) = $\frac{1}{26} \times \frac{1}{2} \times \frac{1}{52} = \frac{1}{2704}$

c) Player 1 cuts an ace, player 2 cuts a heart, and player 3 cuts a card that is not a diamond.

P(not diamonds) = P(spades or hearts or clubs) = $\frac{3}{4}$

P(ace/hearts/not diamonds) = $\frac{1}{52} \times \frac{1}{4} \times \frac{3}{4} = \frac{3}{832}$

Tip
What are the suits that are not diamonds?

In Your Words

Here are some of the important mathematical words of this unit.
Build your own glossary by recording definitions and examples here. The first one is done for you.

appropriate graph graph with certain features that enables questions to be answered or information to be drawn easily from the displayed data For example, a circle graph shows the parts of a whole better than a line graph does.

misrepresentation of data display of data in a way that creates a false impression, leading to incorrect conclusions or decisions to be made For example, a wider bar in a bar graph creates the impression that the data value is greater than it actually is.

discrete data data that can be counted and do not change over time For example, the numbers of students in several classes are discrete data. The heights of a person are not discrete data since the height of the person changes over time.

possible outcomes possible results of an experiment or action For example, when a 6-sided die with a number on each face is rolled, there are 6 possible outcomes, even if some numbers on the die are the same.

probability of an event the number of outcomes of an event written as a fraction of all possible outcomes For example, there are 6 possible outcomes for rolling a regular die. The probability of rolling a 5 is $\frac{1}{6}$.

independent events two events in which the outcomes of one do not affect the outcomes of the other For example, the outcomes of tossing a coin will not affect the outcomes of rolling a die. Tossing a coin and rolling a die are independent events.

List other mathematical words you need to know.

Sample Answer: bar graph, line graph, circle graph, pictograph, double bar graph, scale, trend, tree diagram

Unit Review

LESSON

7.1 **1.** Each year, the Grade 8 classes collect food items for a charity. The table shows the number of items the classes collected for 6 years.

Nicolas wants to display the data on a line graph. Martina wants to use a circle graph, and Nicole is pushing for a bar graph.

Which graph do you think is a good choice? Explain your answer.

Food Drive Collection

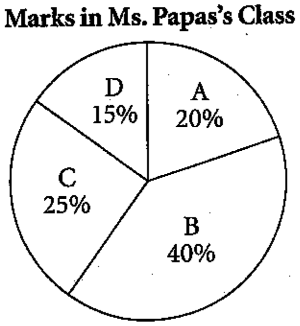
Year	Number of Items
2003	120
2004	155
2005	161
2006	180
2007	196
2008	210

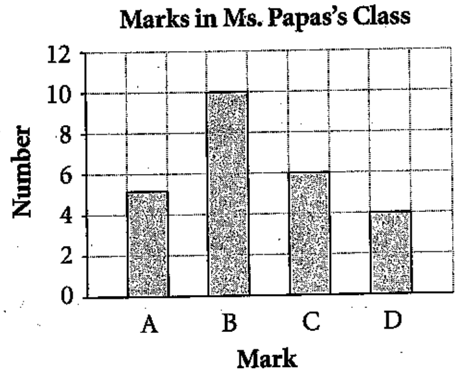
Sample Answer: The line graph is the best since the data change over time.

The circle graph is the least appropriate as the data are not parts of a whole.

The bar graph could work but it may not show the trend as well as a line graph.

2. These 2 graphs show the marks of students in Ms. Papas's class.





a) The principal asks Ms. Papas: "What fraction of your class got an A?" Which graph is easier to use to respond to this question? Explain.

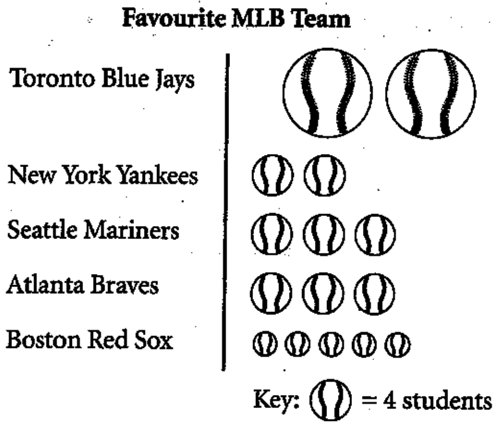
Sample Answer: The circle graph is easier to use. Ms. Papas can simply write the percent as a fraction. The bar graph requires an extra step of finding the total.

b) "How many students got a D?" Which graph can provide the answer? Explain.

Sample Answer: The height of the bar for D in the bar graph provides the answer. The circle graph does not have any actual numbers that can answer the question.

LESSON 7.2

3. Some Grade 8 students were asked to name their favourite major league baseball team. The graph shows the results.



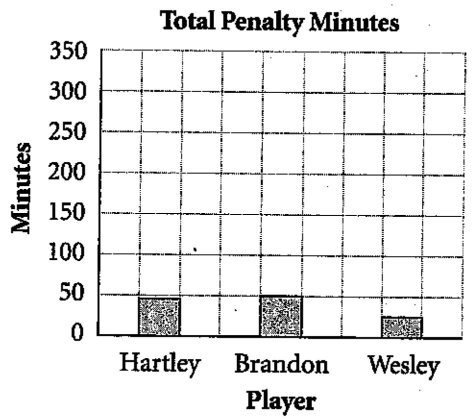
a) How many students named the Toronto Blue Jays? **8 students**

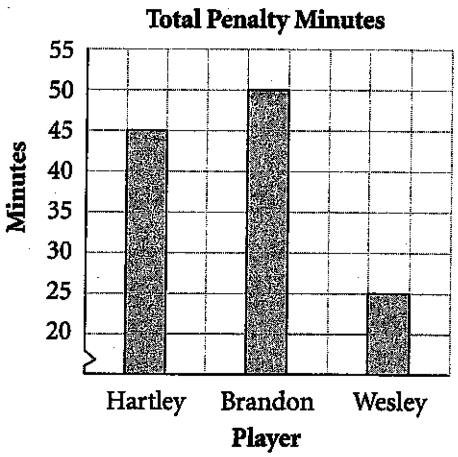
b) How many students named the Boston Red Sox? **20 students**

c) What aspect of the graph is misleading? Explain.

Sample Answer: The 2 large symbols for the Toronto Blue Jays give the false impression that the team is more popular than the Boston Red Sox.

4. These graphs show the penalty minutes in hockey for three players.





a) In the first graph, how do the total penalty minutes of the 3 players compare?

Sample Answer: The total times for the 3 players seem to be very close.

b) In the second graph, how do the total penalty minutes of the three players compare?

Sample Answer: The total time for Wesley seems to be much less than those for Hartley and Brandon.

c) The data in the two graphs are the same. What features of the graphs create the dramatic difference in their appearance?

Sample Answer: The second graph has its vertical scale not starting at 0 and uses a larger scale that exaggerates the difference among the penalty times.

5. The table shows the average salary of employees of a company over 8 years.

The employees are asking for a raise in salary for the next year.

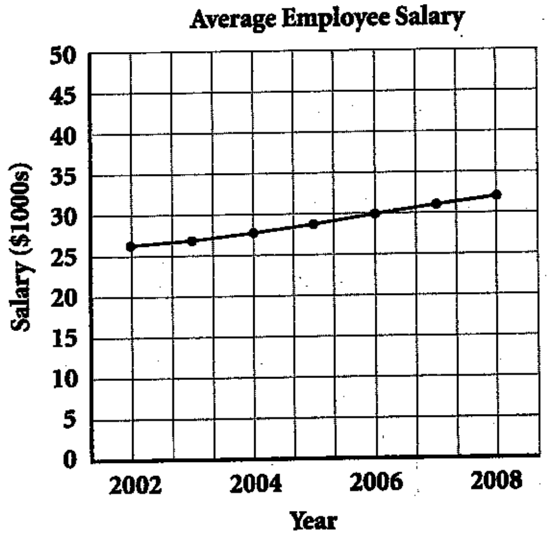
The manager wants to show that the raise should be lower. The employees want to show that the raise should be higher.

Construct two appropriate graphs for the manager and the employees.

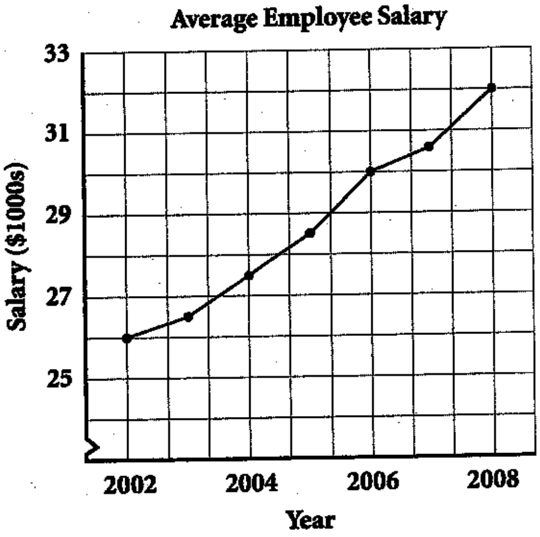
Average Employee Salary

Year	Salary
2002	$26 000
2003	$26 500
2004	$27 500
2005	$28 500
2006	$30 000
2007	$30 500
2008	$32 000

Sample Answer:
Manager's Graph



Employees' Graph



7.3 **6.** Identify which pair of events is a pair that are not independent. Explain your answer.

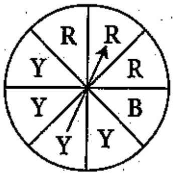
a) Roll two regular 6-sided dice.

b) Toss a penny and a nickel one after the other.

c) Remove 1 card from a standard deck of 52 cards and then remove a second card.

Sample Answer: The events in part c) are not independent events. Removing a second card from the remaining deck of 51 cards depends on what the first card removed from the original deck of 52 cards was.

7. The pointer on the spinner with 8 congruent sectors is spun twice.



Find the probability of each event:

a) a red and then a blue

P(red, then blue) = P(red) × P(blue) = $\frac{3}{8} \times \frac{1}{8} = \frac{3}{64}$

b) a yellow and then a red

P(yellow, then red) = P(yellow) × P(red) = $\frac{1}{2} \times \frac{3}{8} = \frac{3}{16}$

c) a blue on both spins

P(blue, then blue) = P(blue) × P(blue) = $\frac{1}{8} \times \frac{1}{8} = \frac{1}{64}$

8. A regular die labelled 1 to 6 is rolled and a quarter is tossed. Find the probability of each event:

a) an odd number and a tail **P(odd) × P(tails) = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$**

b) a number greater than 4 and a head **P(5 or 6) × P(heads) = $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$**

7.4 **9. a)** A quarter, a penny, and a dime are tossed. Find the probability of each event:

i) heads on the quarter and the penny and a tail on the dime

P(H/H/T) = P(H) × P(H) × P(T) = $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

ii) all 3 tails **P(T/T/T) = P(T) × P(T) × P(T) = $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$**

b) Explain why the probability of "all 3 tails" is the same as that of "all 3 heads."

Sample Answer: There is only 1 out of 8 possible outcomes that satisfies each event.

10. A box contains 26 cards, each with a letter of the alphabet on it. Stephanie draws a card from the box and puts it back in the box. She repeats this procedure 2 more times for a total of 3 draws.

Tip

Consider Y to be a consonant for this experiment.

Find the probability of each event:

a) an A, then an F, then a G **P(A) × P(F) × P(G) = $\frac{1}{26} \times \frac{1}{26} \times \frac{1}{26} = \frac{1}{17\,576}$**

b) a vowel, then 2 consonants **P(vowel) × P(con) × P(con) = $\frac{5}{26} \times \frac{21}{26} \times \frac{21}{26} = \frac{2205}{17\,576}$**

c) all vowels **P(vowel) × P(vowel) × P(vowel) = $\frac{5}{26} \times \frac{5}{26} \times \frac{5}{26} = \frac{125}{17\,576}$**

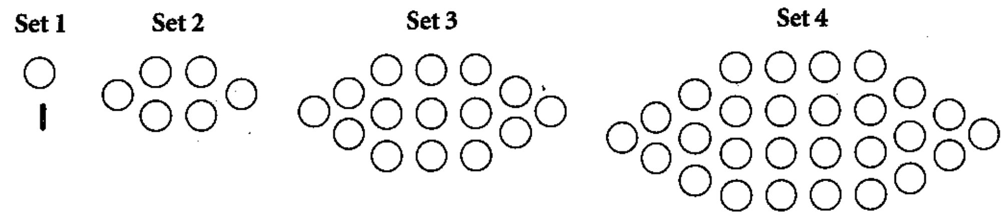
UNIT 8

Geometry

Just for Fun

Penny Patterns

This is a pattern of pennies.



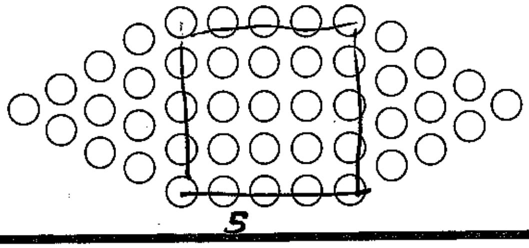
Complete the pattern for Set 5.

How many pennies do you need? 45

How many pennies do you need for Set n? $n(2n-1)$

Test out your rule for Set 5.

Set 5



Mental Squares

Here is a method for squaring 2-digit numbers ending in 5.
Impress your friends by squaring their 2-digit numbers ending in 5 by mental math.

Follow these steps: **Sample Answer:**

Start with a 2-digit number ending in 5. **85**

Multiply the first digit by the next higher digit. **$8 \times 9 = 72$**

This product forms the first part of the square number. The last 2 digits are always 25.

Combine the 2 parts to get the square number. **7225**

Try this method with 3-digit numbers ending in 5. Multiply the first 2 digits by the next higher number. Can you use the method to mentally square 3-digit numbers ending in 5?

No, the product of two 2-digit numbers is difficult to find using mental math.

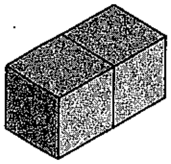
Activating Prior Knowledge

Using Isometric Dot Paper

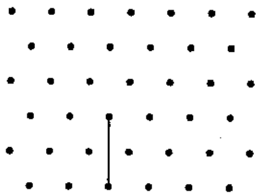
You can use isometric dot paper to represent a 3-dimensional object on a 2-dimensional drawing. Draw the parallel edges as parallel line segments on the isometric dot paper.

Example 1

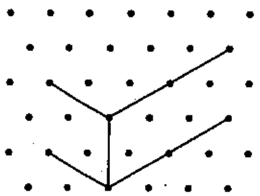
Draw this rectangular prism on isometric dot paper.



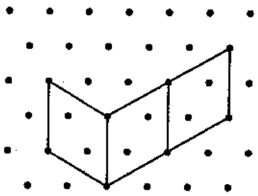
i) Start with one vertical edge.



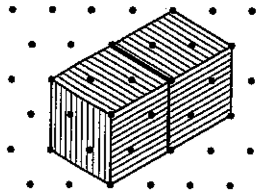
ii) Draw the adjacent horizontal edges that slant up to the left and to the right.



iii) Draw other vertical edges.



iv) Complete all edges and shade the visible faces to get a 3-D look.



✓ Check

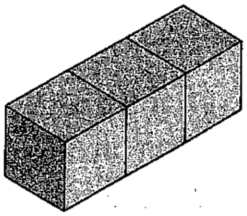
HINT

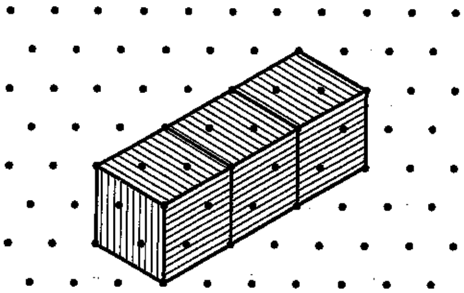
In an isometric drawing, the line segments joining adjacent dots are equal in length.

1. Complete the drawing of each object on isometric dot paper.

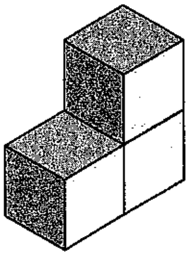
/6

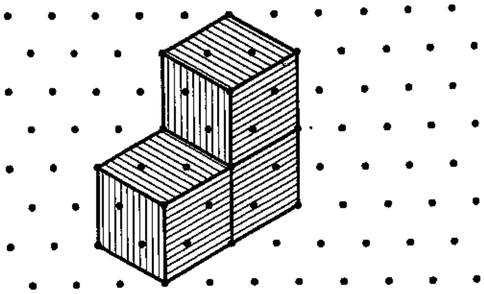
a)



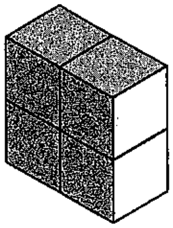


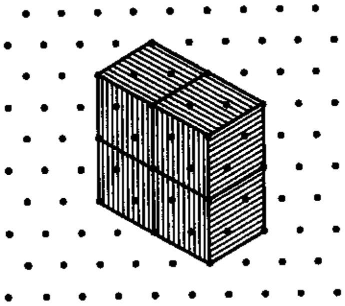
b)





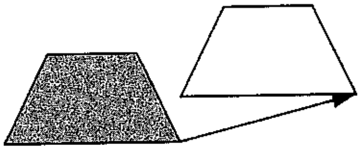
c)



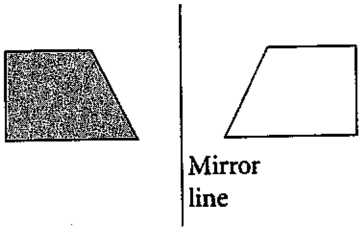


Transformations

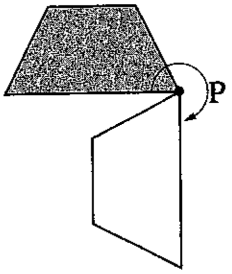
In this **translation**, the shaded shape is moved 4 units right and 1 unit up.



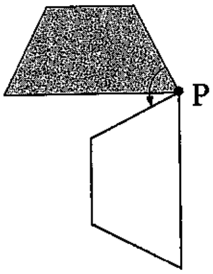
In this **reflection**, the shaded shape is reflected in a vertical line 1 unit to the right of the shape.



In this **rotation**, the shaded shape is rotated 270° clockwise about point P.



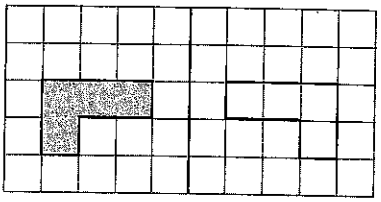
You get the same image if you rotate the shaded shape 90° counterclockwise about point P.



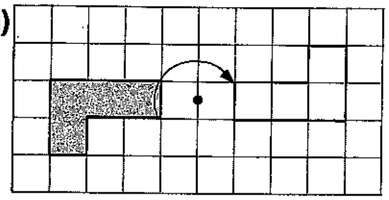
✓ Check

2. Each drawing shows a transformation. Match the transformation to the drawing.

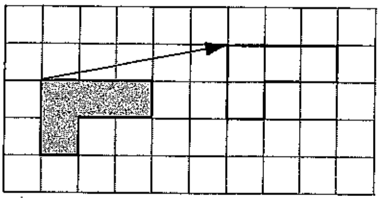
i)



ii)



iii)



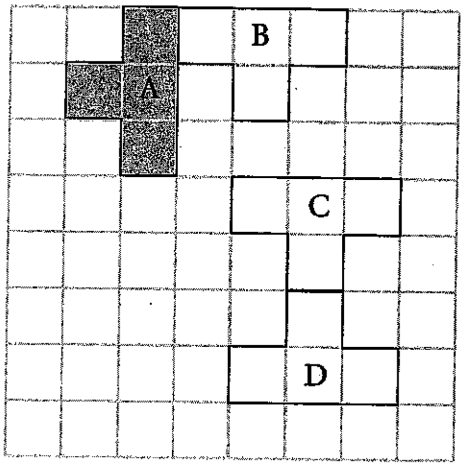
/3

a) translation **iii)**

b) reflection **i)**

c) rotation **ii)**

3. Identify each transformation.



a) Shape B is an image of Shape A.

rotation

b) Shape C is an image of Shape B.

/3

translation

c) Shape D is an image of Shape C.

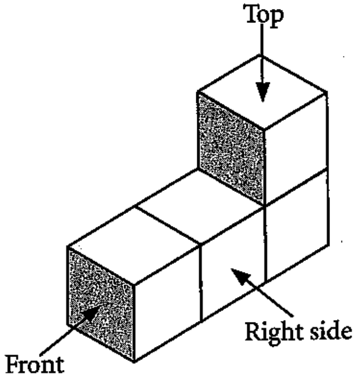
reflection

8.1 Sketching Views of Objects

Quick Review

➤ The front, top, and side views of an object can be drawn by looking at a model or an **isometric drawing** of the object.

The front, top, and side views of this model can be drawn by rotating the model in order to look at the views directly.



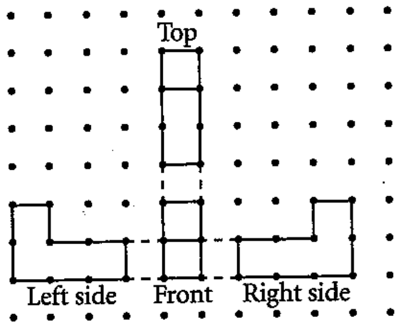
When you draw the different views of the object, draw the front view first.

Place the top view above the front view, and the side views beside the front view.

Broken lines show how the views align.

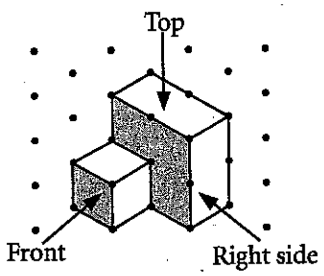
Internal line segments are used to show changes in depth or thickness.

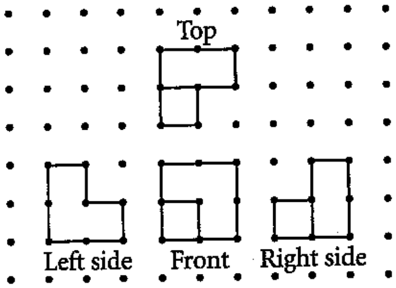
Notice the internal line segments on the front and top views that show the changes in depth. Since there are no changes in depth on the two side views, there are no internal line segments.



➤ When an isometric drawing of an object is given, you can build a model, and then draw the different views.

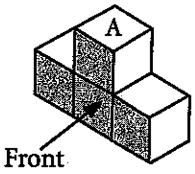
For example, the object with this isometric drawing has the front, top, and side views shown.

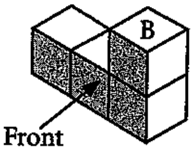




Practice

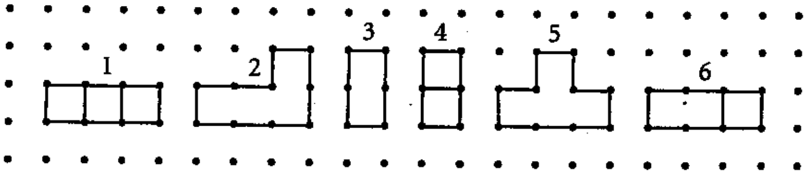
1. Figures 1 to 6 are views of objects A and B. Match each view (1 to 6) to each object A or B. A numbered view may be used more than once.





HINT

Use linking cubes to build a model for each object. Then rotate the model to see the different views.

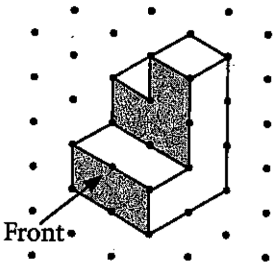


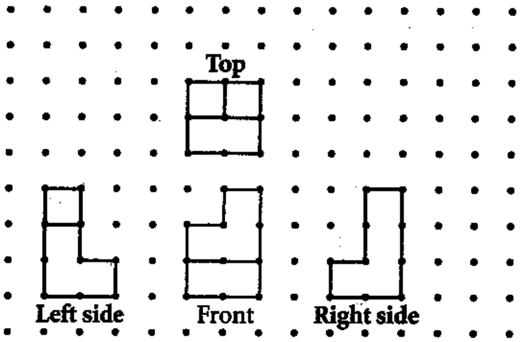
/8

Object	Front View	Top View	Left Side View	Right Side View
A	5	1	4	4
B	2	6	4	3

2. The front view of this object is given. Sketch the top and side views.

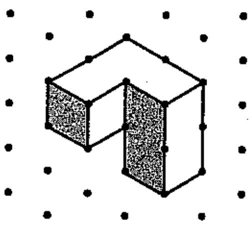
/3

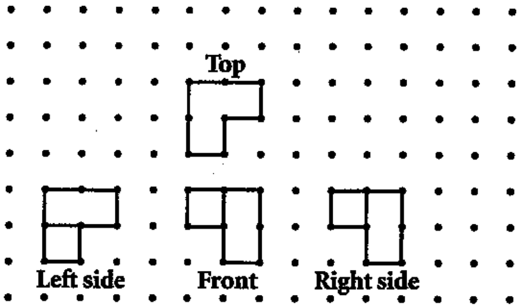




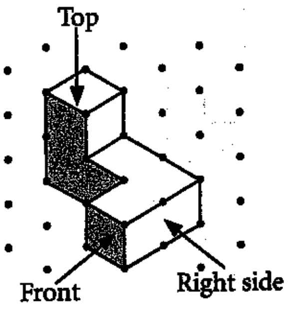
3. Sketch the front, top, and side views of this object.

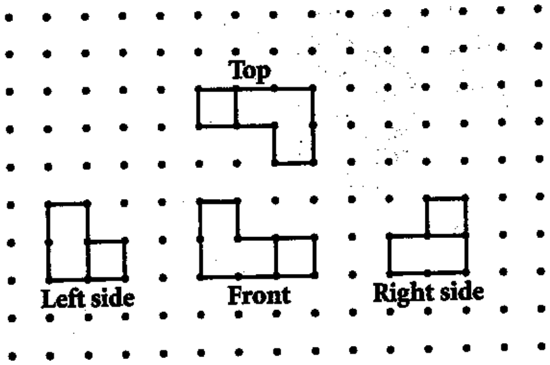
/4





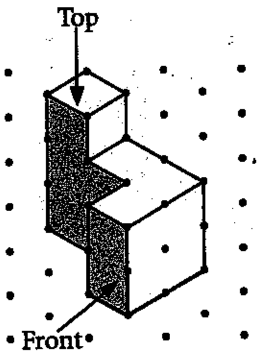
4. Sketch the front, top, and side views of this object.

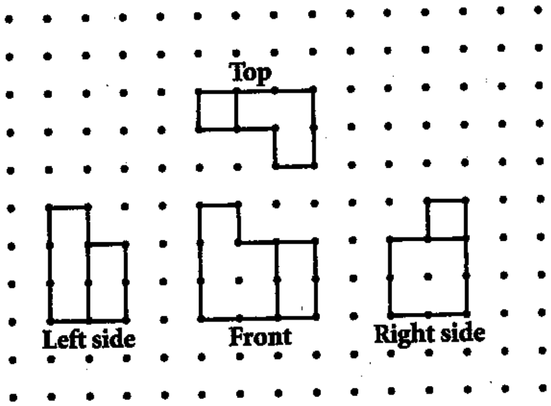




/4

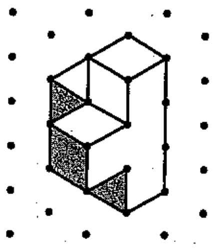
5. Use linking cubes to build a model of the object in the isometric drawing. Then draw the front, top, and side views of the object.

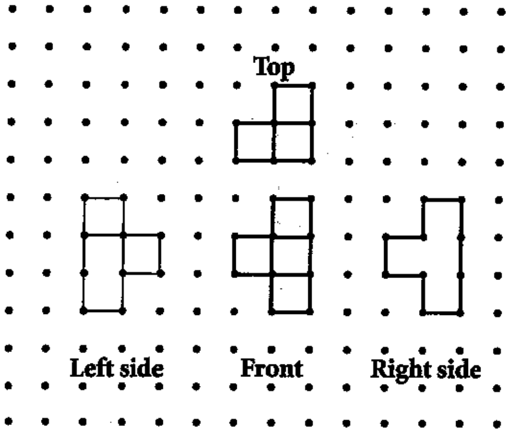




/4

6. Use linking cubes to build a model of the object in the isometric drawing.
Then draw the front, top, and side views of the object. The left side is done for you.



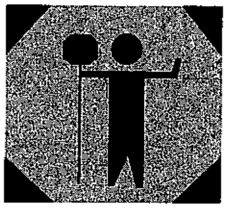


/3

7. Many signs have views of objects.
Identify the view (front, top, or side) of the object on each sign.

a) construction worker

front view



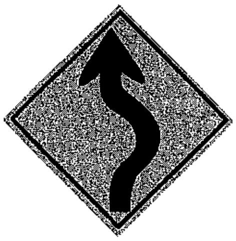
/3

b) school children

side view

c) curved road

top view

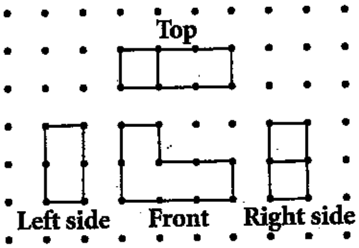


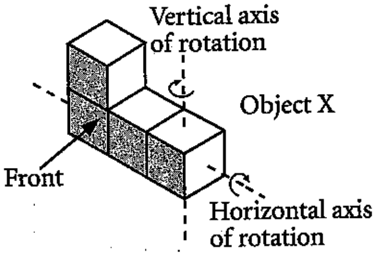
8.2 Drawing Views of Rotated Objects

Quick Review

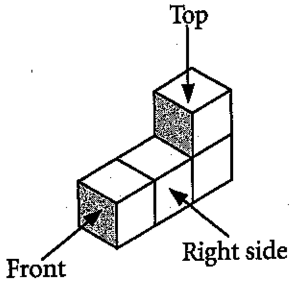
- ➤ An object can be rotated horizontally about a vertical **axis of rotation.** The rotation can be clockwise or counterclockwise.
- ➤ An object can also be rotated vertically about a horizontal axis of rotation. The rotation can be toward you or away from you.

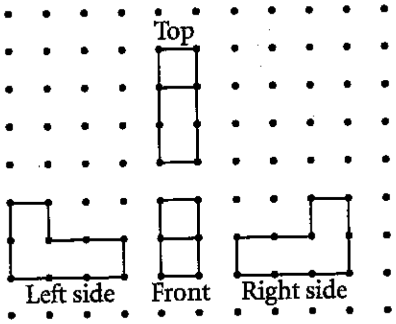
Object X has these views.



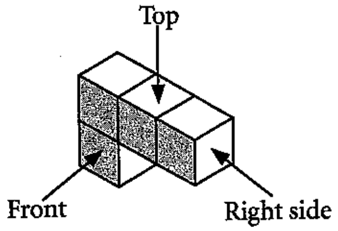


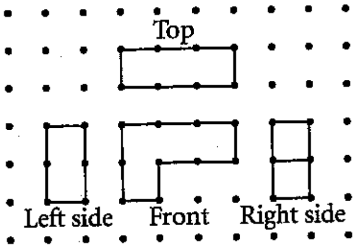
Object X is rotated horizontally 90° clockwise about a vertical axis. Here are the object's new orientation and views.





Object X is rotated vertically 180° about a horizontal axis away from you. Here are the object's new orientation and views.

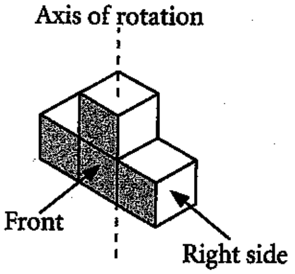




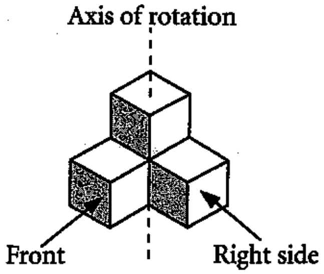
- ➤ A 180° clockwise rotation is the same as a 180° counterclockwise rotation.
  A 90° clockwise rotation is the same as a 270° counterclockwise rotation.
  A 270° clockwise rotation is the same as a 90° counterclockwise rotation.

Practice

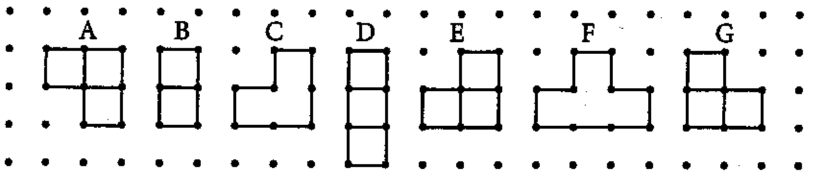
1. **a)** Build each object. Rotate each object horizontally 90° clockwise about the axis of rotation shown. Match each view (A to G) to the front, top, and side views of each rotated object. A lettered view may be used more than once.



Object P



Object Q



Object	Front View	Top View	Left Side View	Right Side View
P	B	D	F	F
Q	E	A	G	C

b) Which object, P or Q, has the left side view the same as the right side view after the rotation? **Object P**

2. The objects P and Q in question 1 are rotated horizontally 270° counterclockwise about the axis of rotation shown.

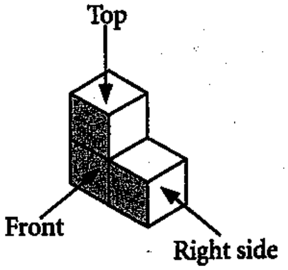
a) Which view (A to G) is the top view of the rotated Object P? **D**

b) Which view (A to G) is the front view of the rotated Object Q? **E**

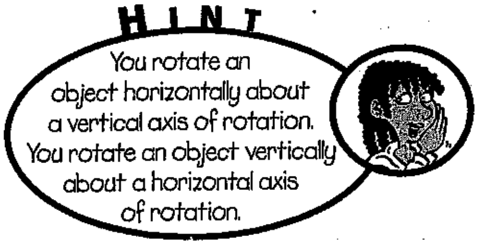
c) Explain your answers to parts a) and b).

Sample Answer: The top view of Object P and the front view of Object Q after the rotation are the same as those in question 1 because a 90° clockwise rotation is the same as a 270° counterclockwise rotation.

3. Use linking cubes to build Object Y. Match each rotation description i) to iv) to the drawing of the rotated Object Y in a) to d).

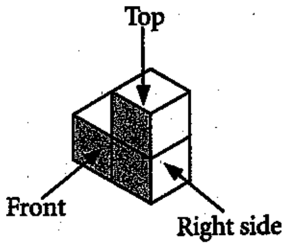


Object Y



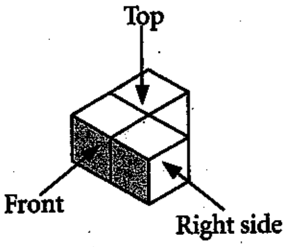
You rotate an object horizontally about a vertical axis of rotation. You rotate an object vertically about a horizontal axis of rotation.

a)



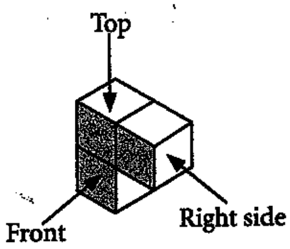
Rotation: iii)

b)



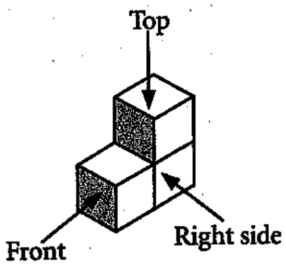
Rotation: iv)

c)



Rotation: ii)

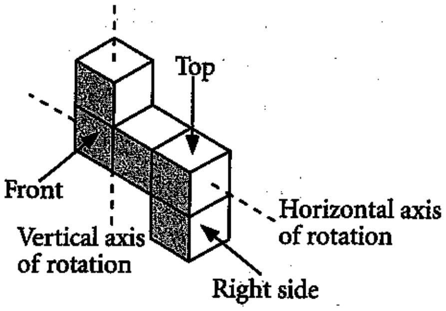
d)



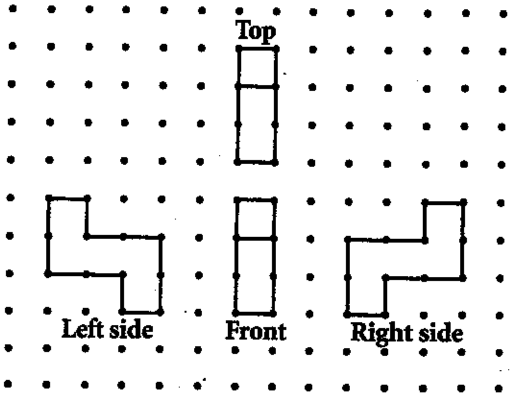
Rotation: i)

i) a rotation of 270° counterclockwise about a vertical axis
ii) a rotation of 180° about a horizontal axis
iii) a rotation of 180° about a vertical axis
iv) a rotation of 90° about a horizontal axis away from you

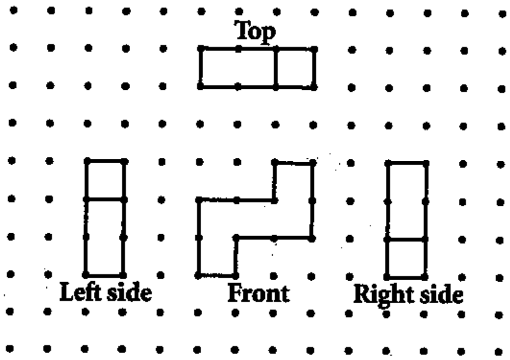
4. Use linking cubes to build this object. Draw the front, top, and side views after each horizontal rotation about the vertical axis shown.



a) 90° clockwise

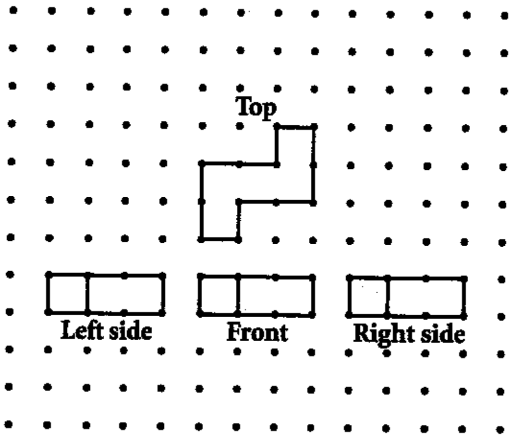


b) 180°

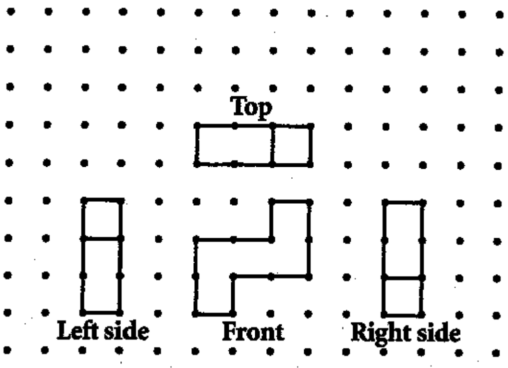


5. Draw the front, top, and side views of the object in question 4 after each vertical rotation about the horizontal axis shown.

a) 90° toward you



b) 180°



6. Use linking cubes to build an object that has the same front, top, and side views after each of the horizontal or vertical rotations in this lesson.
Sketch or describe the object you built.

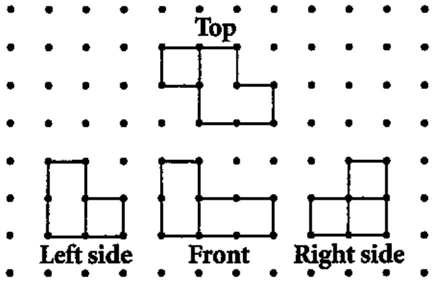
Sample Answer: A cube (of any size) with 6 congruent faces has the same front, top, and side views after each of the rotations in this lesson.

8.3 Building Objects from Their Views

Quick Review

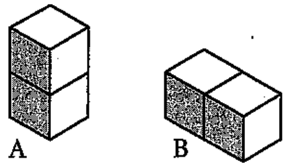
➤ You can often build an object given the front, top, and side views. Note that internal line segments in these views show changes in depth.

The views of an object are shown.

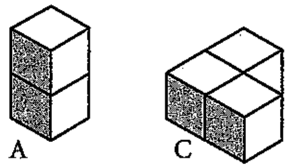


The object can be built using linking cubes:

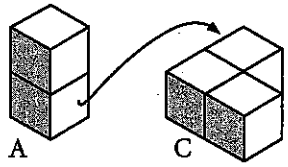
The front view shows that you need 2 cubes in a vertical column and 2 cubes in a horizontal row.



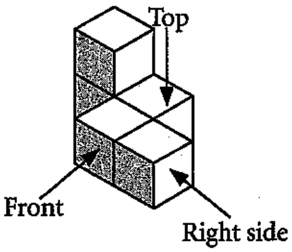
The top view shows that you need to add a cube onto B to make an L-shape C.



The top view also shows that A must be attached to C with a change in depth.



The resulting object looks like this:

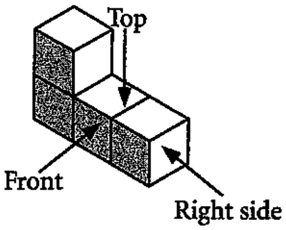


Then check the side views of the object to see if they match the given views. If they do, the object is correct.

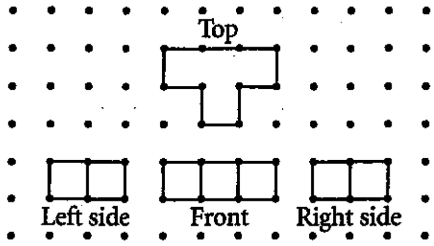
/12.

1. Match each set of views i) to iii) to the object (A to C).

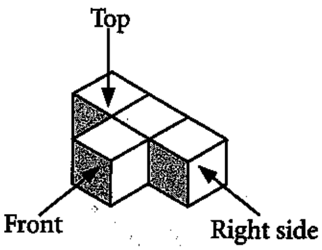
A: iii)



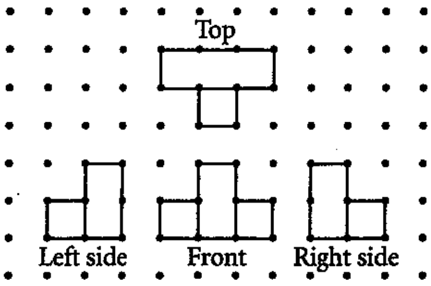
i)



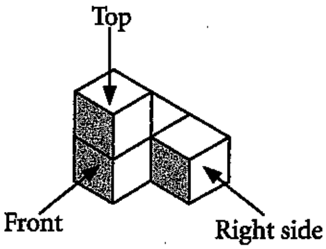
B: i)



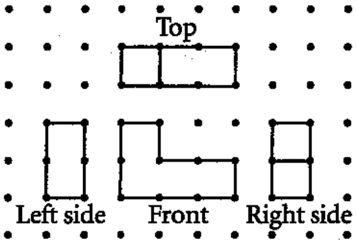
ii)



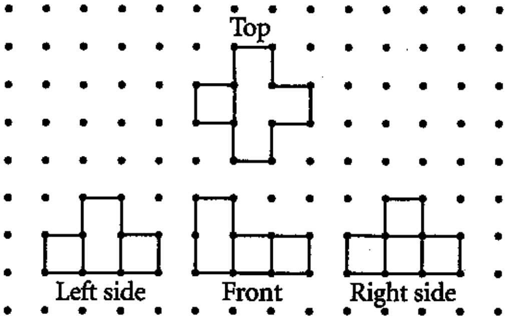
C: ii)

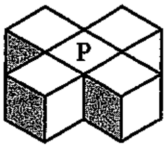


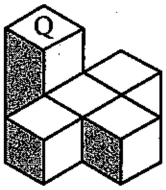
iii)

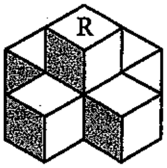


2. Which object, P, Q, or R, has these views? **Object Q**

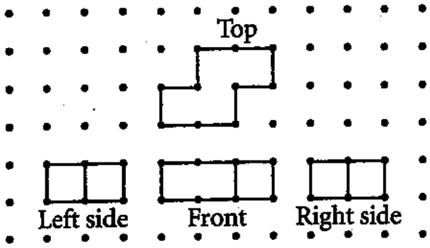


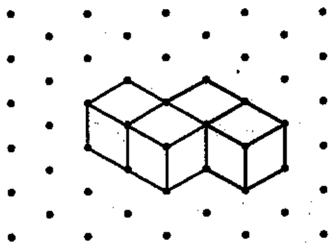




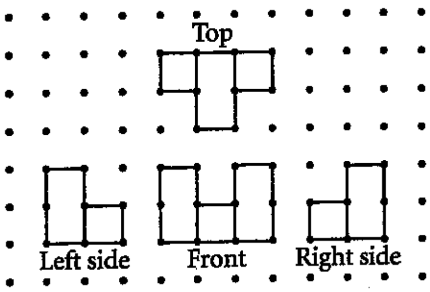


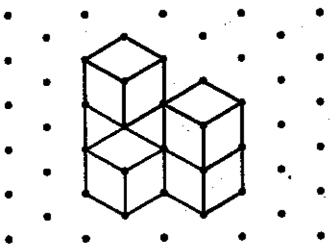
3. Use these views to build an object.



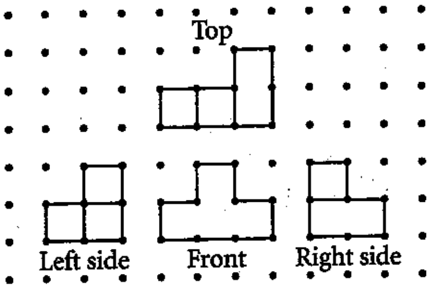


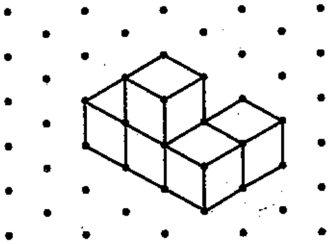
4. Use these views to build an object.



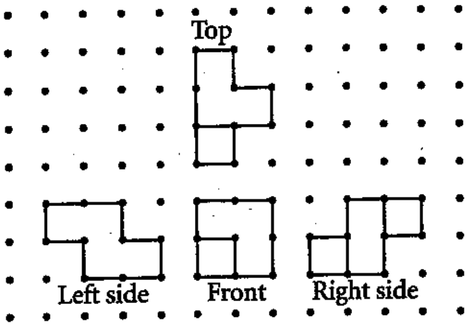


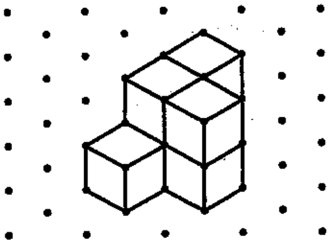
5. Use these views to build an object.





6. Use these views to build an object.





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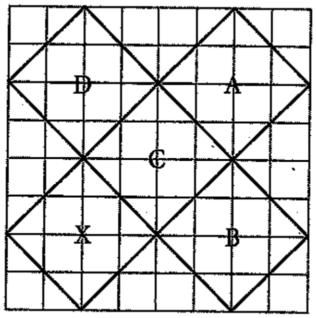
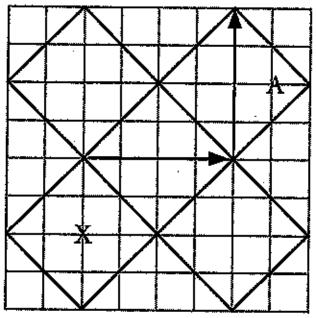
Quick Review

- The 3 different transformations—translation, reflection, and rotation—have been used to create a design.

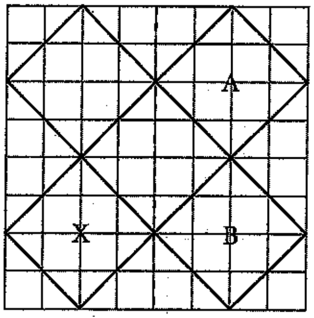
  Here are some transformations that can be identified in this design.



Square A is the image of Square X after a translation 4 units right and 4 units up.



Square B is the image of Square X after a reflection in the broken line.

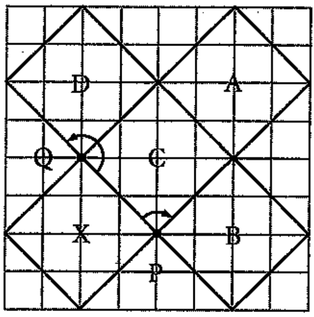


Square C is the image of Square X after a rotation of 90° clockwise about point P.

Square C is also the image of Square X after a rotation of 90° counterclockwise about point Q.

Square D is the image of Square X after a rotation of 180° about point Q.

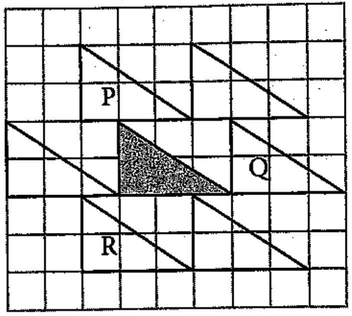
You can make a tracing of square X and rotate it about points P and Q to check these results.



- A rotation of 180° clockwise about a point gives the same image as a rotation of 180° counterclockwise about the same point.
- Under any transformation, the original shape and its image are always congruent.

Practice

1. Match each translation of the shaded triangle to its image.

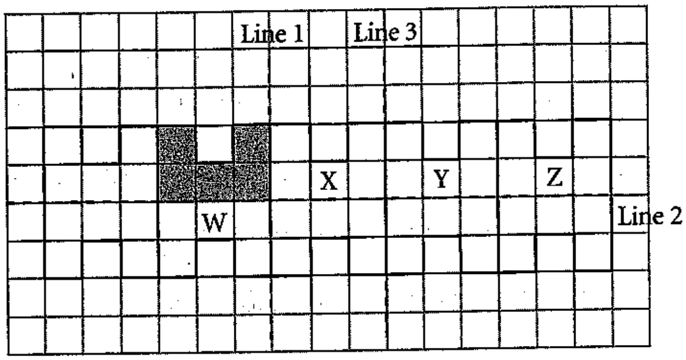


a) 3 units right **Triangle Q**

b) 1 unit left and 2 units down **Triangle R**

c) 1 unit left and 2 units up **Triangle P**

2. Match each reflection of the shaded octagon to its image.



a) reflection in Line 1

Shape X

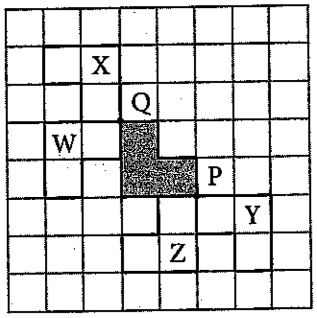
b) reflection in Line 2

Shape W

c) reflection in Line 3

Shape Z

3. Match each rotation of the shaded hexagon to its image.



a) 90° counterclockwise about point P

Shape Z

b) 180° about point P

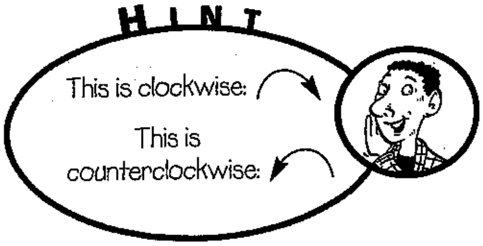
Shape Y

c) 90° clockwise about point Q

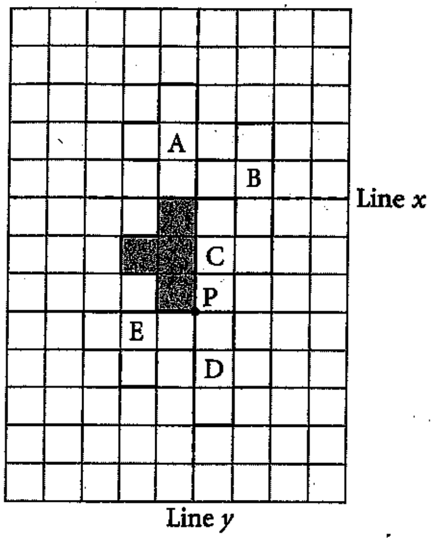
Shape W

d) 180° about point Q

Shape X



4. Match each transformation of the shaded polygon with its image.



a) a rotation of 180° about point P **Shape D**

b) a translation 3 units up **Shape A**

c) a reflection in Line x **Shape A**

d) a reflection in Line y **Shape C**

e) a rotation of 90° counterclockwise about point P
Shape E

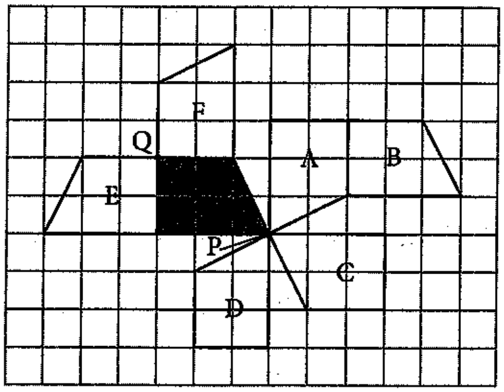
f) a translation 2 units right and 2 units up
Shape B

5. Identify each transformation of the shaded Shape X.
Describe each transformation in as many ways as you can.

a) Shape A is an image of Shape X. **Sample Answers:**

a rotation of 90° clockwise, or

270° counterclockwise, about point P



b) Shape B is an image of Shape X.

a translation 5 units right and 1 unit up

c) Shape C is an image of Shape X.

a rotation of 180° about point P

d) Shape D is an image of Shape X.

a rotation of 270° clockwise, or 90° counterclockwise, about point P

e) Shape E is an image of Shape X.

a reflection in the vertical line passing through point Q

f) Shape F is an image of Shape X.

a rotation of 90° counterclockwise, or 270° clockwise,

about point Q

Tip

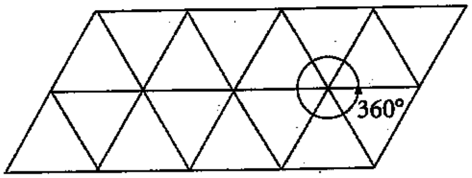
Make a tracing of Shape X. Translate, reflect, or rotate the shape to check your results.

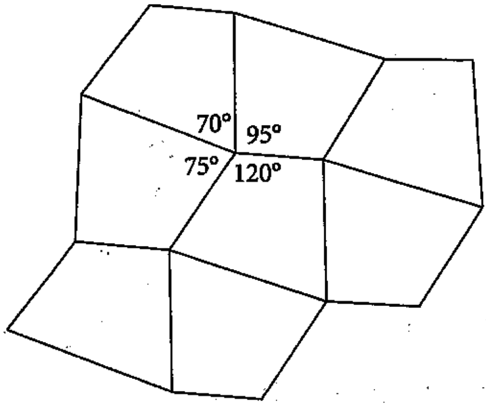
8.5 Constructing Tessellations

Quick Review

- When you can cover a page using congruent copies of a shape with no overlaps and gaps, the shape **tessellates,** creating a design called a **tessellation.**

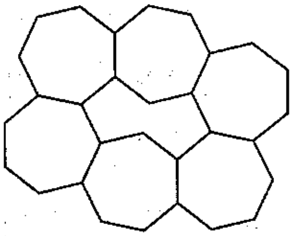
  All triangles and all quadrilaterals tessellate.





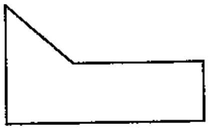
At any point where the vertices meet, the angles add up to 360°.

- There are some shapes that do not tessellate because they cover a page with overlap or gaps. For example, this heptagon does not tessellate.

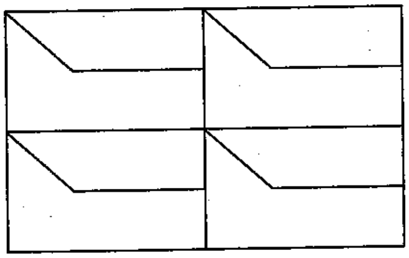


- You can combine shapes to tessellate. These combined shapes are called **composite shapes.**

  For example, Shape A combines with Shape B to form a quadrilateral that tessellates.


Shape A


Shape B



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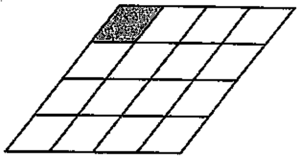
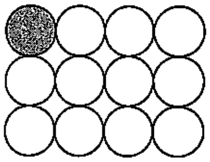
1. Which of these designs are tessellations? Justify your answer.

a)

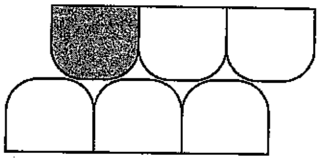


b)

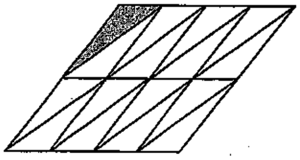


/3

c)



d)



Sample Answer: The designs in parts a) and d) are tessellations since they do not have overlaps or gaps.

2. Which of these shapes tessellate? Use a drawing to justify your answer.

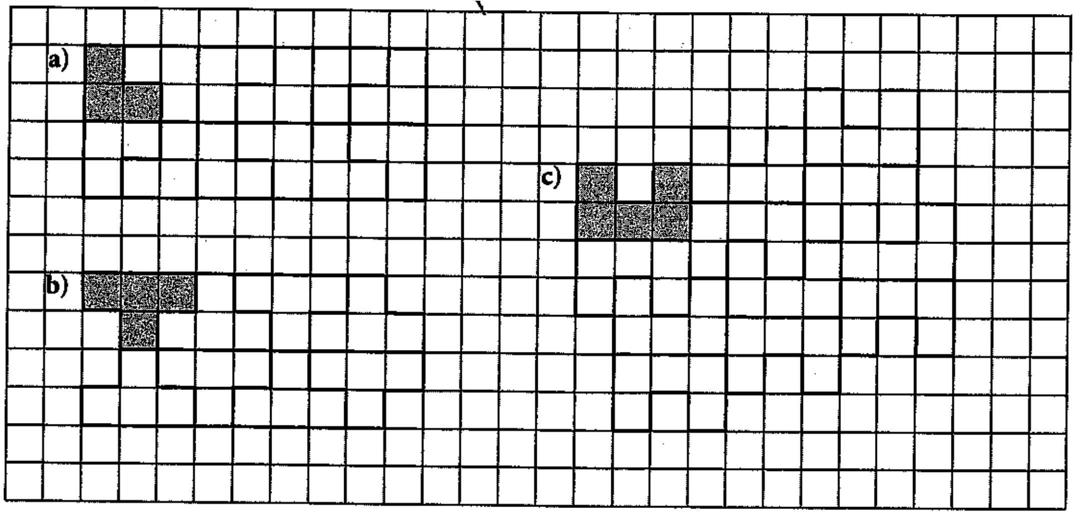
a) L-shape **This shape tessellates. See diagram for part a).**

b) T-shape **This shape tessellates. See diagram for part b).**

c) U-shape **This shape tessellates. See diagram for part c).**

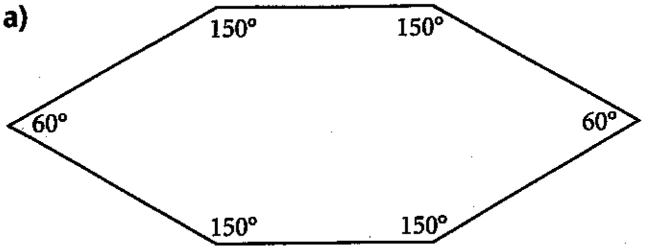
/6

Sample Answer:

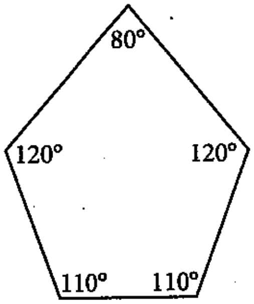


3. Which of the polygons can be used to create a tessellation?
Justify your answer by checking if copies of the polygon can surround a point.

a)



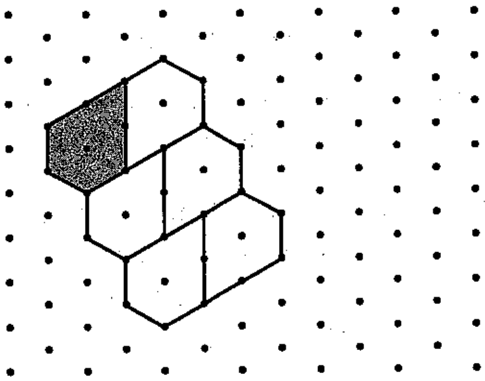
b)



/3

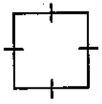
Sample answer: The hexagon tessellates because 150° + 150° + 60° = 360°.

4. Create a composite shape that tessellates using a regular hexagon and one or more equilateral triangles. Show your tessellation on the isometric dot paper.
Sample answer:

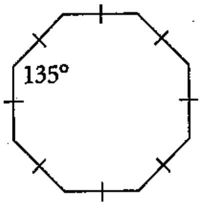


/3

5. Arlene is planning to create a tessellating quilt pattern using one of these shapes.



square



regular octagon

a) Which shape can Arlene use? Why?

Sample answer: Arlene can use the square because quadrilaterals tessellate.

/2

b) Can Arlene use a combination of these shapes to create a tessellating quilt pattern? Explain.

Sample answer: Yes, a combination of squares and regular octagons can surround a point: 135° + 135° + 90° = 360°

/2

8.6 Identifying Transformations in Tessellations

Quick Review

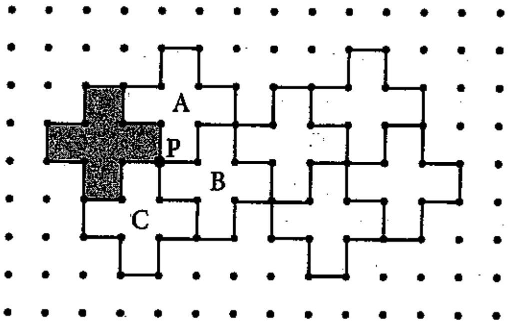
- ➤ A tessellation can be described using transformations of shapes.
- ➤ Under each transformation, the area of the shape does not change. This is known as **conservation of area.**
- ➤ A tessellation may be described by one or more than one type of transformation.

This tessellation can be described by translations or by rotations.
Start with the shaded shape.

To get Shape A, translate the shaded shape 2 units right and 1 unit up.

To get Shape B, translate the shaded shape 3 units right and 1 unit down.

To get Shape C, translate the shaded shape 1 unit right and 2 units down.



Alternatively:

To get Shape A, rotate the shaded shape 90° clockwise about point P.

To get Shape B, rotate the shaded shape 180° about point P.

To get Shape C, rotate the shaded shape 90° counterclockwise about point P.

You can make a tracing of the shaded shape and translate it or rotate it about point P to check these results.

To complete the tessellation, repeat these translations or rotations on the shaded shape.

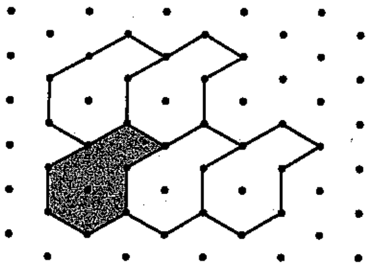
Practice

1. Identify the transformation in this tessellation. Circle your answer.

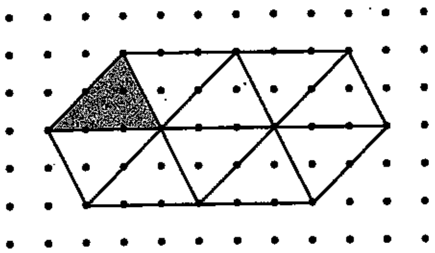
[✓ translation]

reflection

rotation



2. Identify the two transformations in this tessellation. Circle your answer.

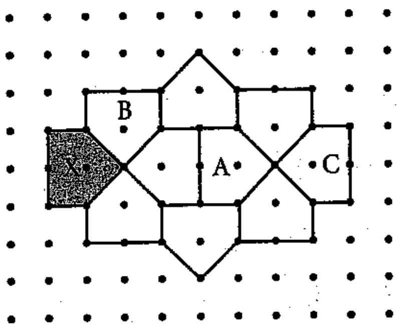


translation and reflection

[✓ translation and rotation]

rotation and reflection

3. Identify the transformations in this tessellation.



Use these words or phrases to complete each sentence.

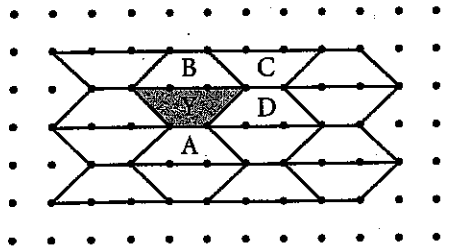
translation, reflection, rotation, vertical line, horizontal line, 4 units up, 4 units right, 90° , 180°, clockwise, counterclockwise

a) Shape A is a **translation** of Shape X **4 units right**.

b) Shape B is a **rotation** of Shape X **90° clockwise** about a point.

c) Shape C is a **reflection** of Shape X in a **vertical line**.

4. In the tessellation, Shape Y is the starting shape.



Describe the transformation needed to get to each of the lettered shapes.

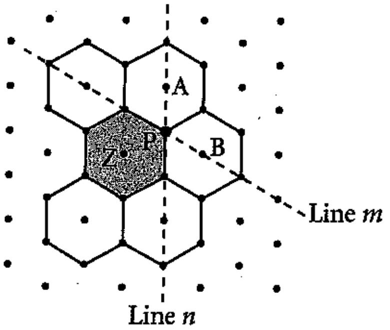
A: **Sample Answer: Reflect (in the side shared by Shape A and Shape Y.)** horizontal line.

B: **Sample Answer: (Reflect in the side shared by Shape B and Shape Y.)** "

C: **Sample Answer: Translate 2 units right and 1 unit up.** (2,1)

D: **Sample Answer: Rotate about the midpoint of the side shared by Shape D and Shape Y.** 180°.

5. In the tessellation, Shape Z is the starting shape.



Describe as many different transformations as you can to get to each lettered shape.

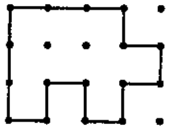
A: **Sample Answer: Rotate Shape Z 120° clockwise about point P.**

Or, reflect Shape Z in Line *m*.

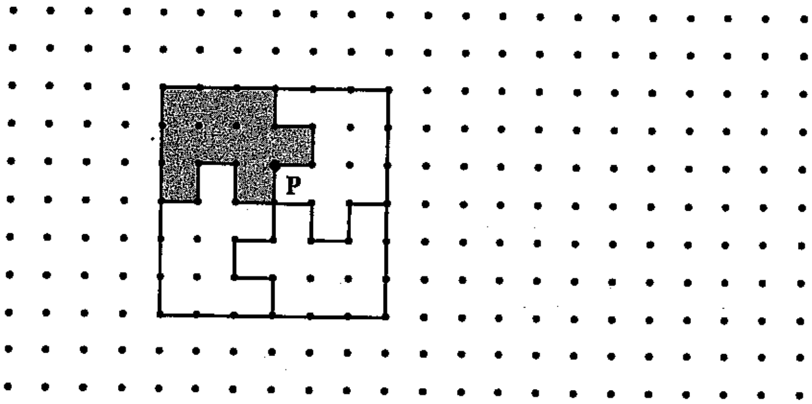
B: **Sample Answer: Rotate Shape Z 120° counterclockwise about point P.**

Or, reflect Shape Z in Line *n*.

6. Use this shape, or one of your own shapes, to create a tessellation on square dot paper. Identify the transformations you used.



Sample Answer:

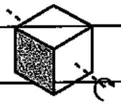


Sample Answer: Start with the shaded shape. Rotate it 90°, 180°, and 270° clockwise about point P to get to the other shapes. Then translate the large square to cover the page.

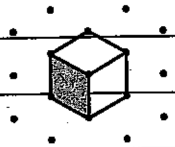
In Your Words

Here are some of the important mathematical words of this unit.
Build your own glossary by recording definitions and examples here. The first one is done for you.

axis of rotation the straight line about which an object or a shape is rotated
For example, the broken line is an axis of rotation.

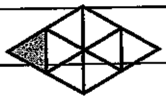


isometric drawing drawing on isometric (triangular) dot paper that shows the three dimensions of an object
For example, this is an isometric drawing of a cube.



transformation any movement (translation, reflection, or rotation) of a shape that results in an image congruent to the original shape
For example, reflection in a mirror is a transformation.

tessellation a design created using congruent copies of a shape that covers a plane with no overlaps or gaps
For example, this is a tessellation of an equilateral triangle.



composite shape a new shape formed by the combination of two or more shapes
For example, the combination of an equilateral triangle and a regular hexagon with the same side length is a composite shape.

conservation of area the fact that the area of a shape does not change under a transformation

List other mathematical words you need to know.

Sample Answer: translation, reflection, rotation, image, translation arrow, line of reflection, point of rotation

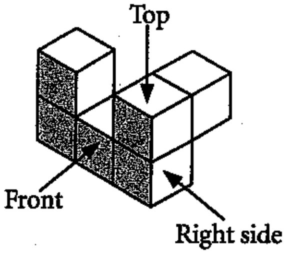
Unit Review

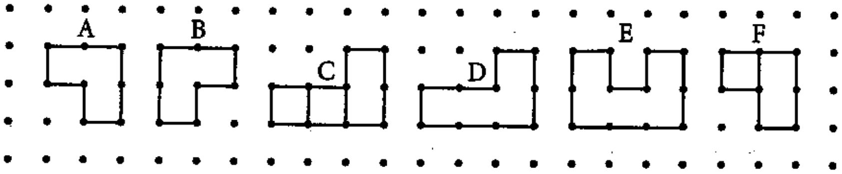
LESSON

8.1 **1.** Match each of the front, top, and side views of this object to the correct figure.

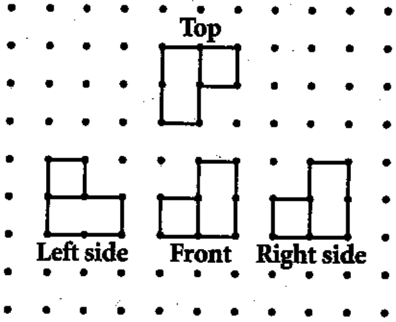
Front: E Top: C

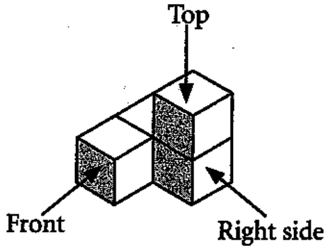
Left side: F Right side: B



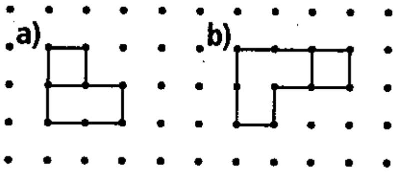


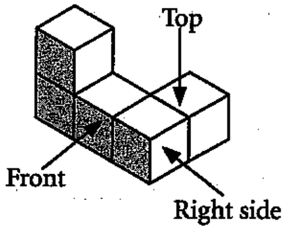
2. Sketch the front, top, and side views of this object.





8.2 **3.** This object is rotated horizontally. The two new views are shown.





Describe the rotation that produced each view.

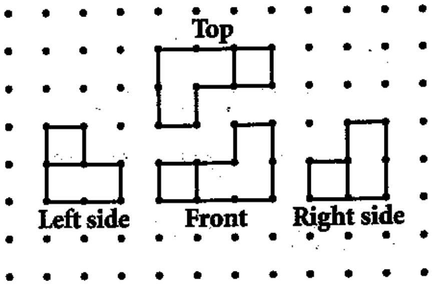
a) This is the front view after a rotation of 90° clockwise

b) This is the top view after a rotation of 180°

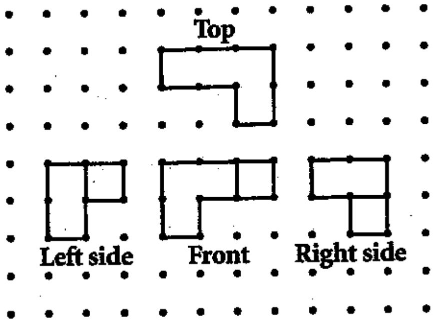
LESSON

4. Use linking cubes to build the object in question 3. Draw the views of the object after each rotation.

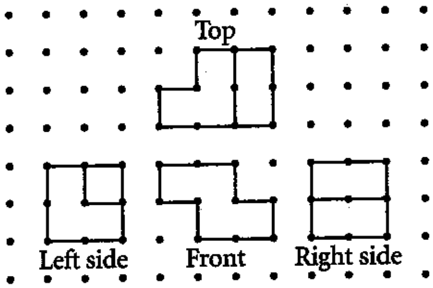
a) a vertical rotation of 90° toward you

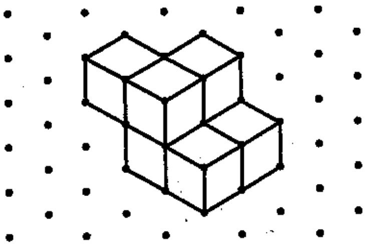


b) a vertical rotation of 180°



8.3 **5.** Use these views to build an object. How can you check that the object is correct?





8.4 **6.** Match each transformation of the shaded shape to its image.

a) a translation 1 unit right and 3 units up B

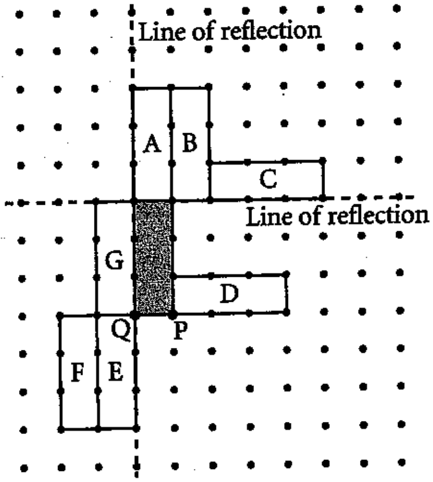
b) a translation 2 units left and 3 units down F

c) a reflection in the vertical line G

d) a reflection in the horizontal line A

e) a rotation of 180° about point Q E

f) a rotation of 90° clockwise about point P D



LESSON

8.5 **7.** Which of these shapes tessellate? Justify your answer.

a) a triangle with angles 58°, 102°, and 20°

Sample Answer: The triangle tessellates since all triangles tessellate.

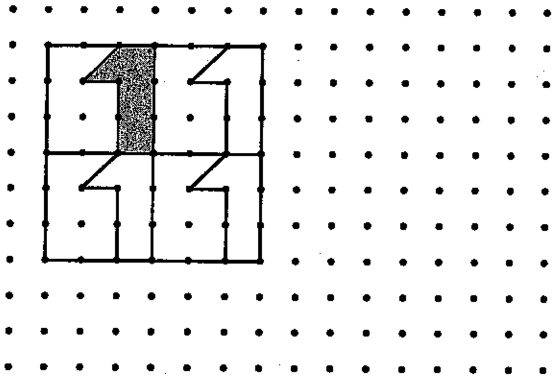
b) a square

Sample Answer: The square tessellates since all squares tessellate.

c) a regular 12-sided polygon with each angle 150° **Sample Answer: The 12-sided polygon does not tessellate since the sum of 150° angles can never be 360°.**

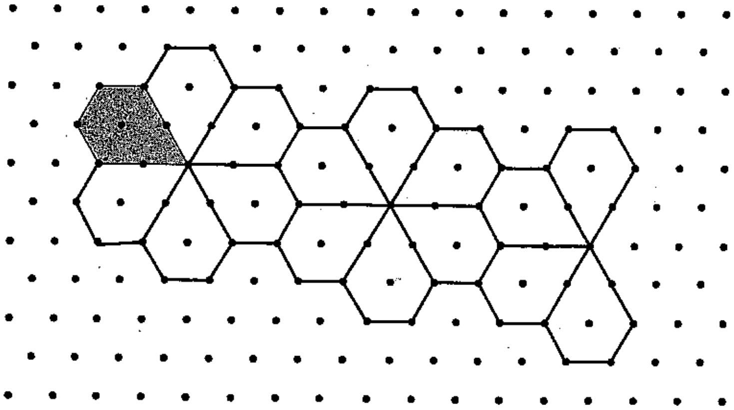
8. The shaded shape does not tessellate. Combine it with another shape to form a composite shape that tessellates. Show your tessellation.

Sample Answer:



8.6 **9.** **a)** Use the shaded shape to create a tessellation on isometric dot paper.

Sample Answer:



b) Use as many different transformations as you can, and describe the transformations you used.

Sample Answer: Possible transformations are rotations of 60°, reflections in common sides, and translations.