UNIT 1

Square Roots and the Pythagorean Theorem

Just for Fun

What Do You Notice?

Follow the steps. An example is given.

		Example
1.	Pick a 4-digit number with different digits.	3078
2.	Find the greatest number that can be made with these digits.	8730
3.	Find the least number that can be made with these digits.	0378
4.	Subtract the least from the greatest.	8730 – 0378 = 8352
5.	Repeat steps 2, 3, and 4 with the result.	8532 – 2358 = 6174
6.	Continue to repeat steps 2, 3, and 4 until you notice something interesting.	7641 – 1476 = 6174

What do you notice?

____

Try these steps with the number 2395. What do you notice? Pick any 4-digit number. What do you notice?

____

Letter Symmetry

A letter has mirror symmetry if a straight line can be drawn through the letter so that one half of the letter is a mirror image of the other half. The straight lines can be vertical, horizontal, or slanted.
For example, the letter A has mirror symmetry, but the letter F does not.

Which letters have mirror symmetry?

Which letters have more than one line of symmetry?

____

Activating Prior Knowledge

Areas of Rectangles and Triangles

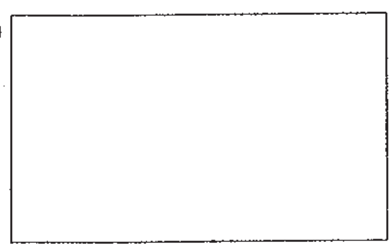
Area is the amount of surface a figure covers. It is measured in square units.

- To find the area of a rectangle, use the formula $A = bh$, where b is the base length and h the height of the rectangle.
- To find the area of a triangle use the formula $A = \frac{1}{2}bh$, where b is the base length and h is the height of the triangle.

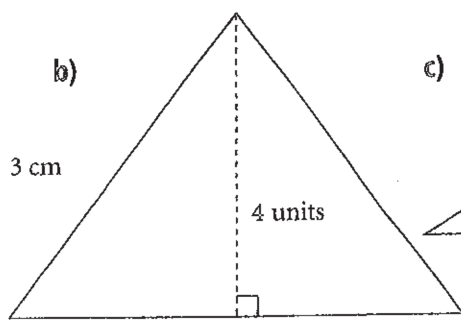
Example 1

Find the area of each figure.

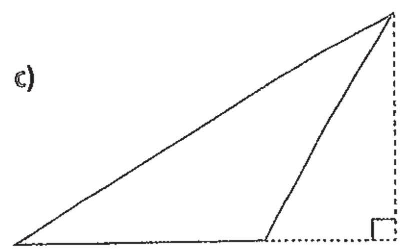
a)



b)



c)



Solution

a) The figure is a rectangle with base 5 cm and height 3 cm.
Substitute $b = 5$ cm and $h = 3$ cm into $A = bh$.

$A = 5 \text{ cm} \times 3 \text{ cm}$

$= 15 \text{ cm}^2$

The area is 15 cm^2. The abbreviation cm^2 stands for "square centimetres."

b) The figure is a triangle with base 6 units and height 4 units.
Substitute $b = 6$ units and $h = 4$ units into $A = \frac{1}{2}bh$.

$A = \frac{1}{2}(6 \text{ units} \times 4 \text{ units})$

$= 12$ square units

The area is 12 square units.

c) The figure is a triangle with base 5 cm and height 3 cm.
Substitute $b = 5$ cm and $h = 3$ cm into $A = \frac{1}{2}bh$.

$A = \frac{1}{2}(5 \text{ cm} \times 3 \text{ cm})$

$= \frac{1}{2}(15 \text{ cm}^2)$

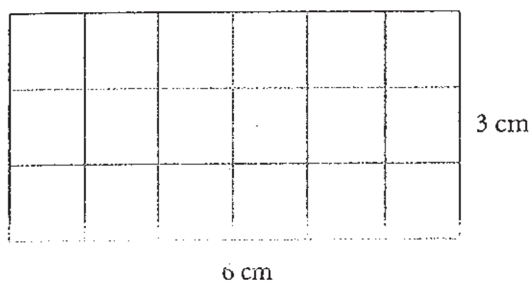
$= 7.5 \text{ cm}^2$

The area is 7.5 cm^2.

Check

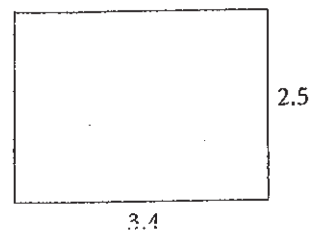
1. Find the area of each figure.

a)



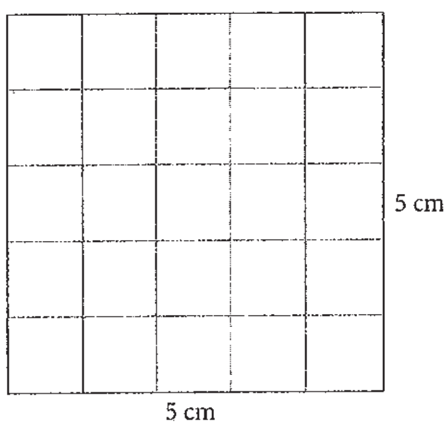
The area is _____ cm × _____ cm = _____

b)



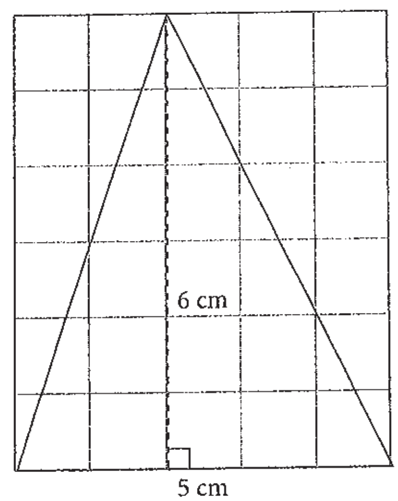
The area is ________________

c)



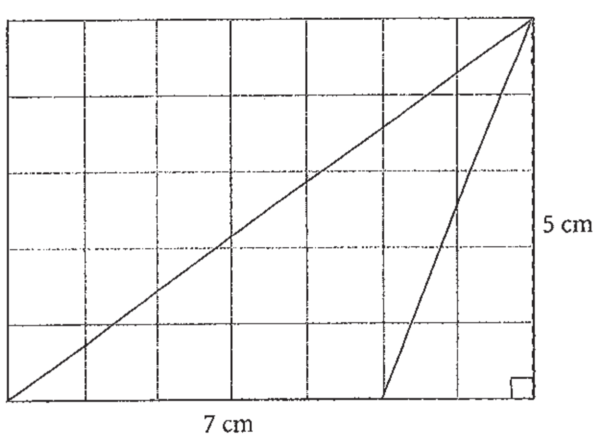
The area is ________________

d)



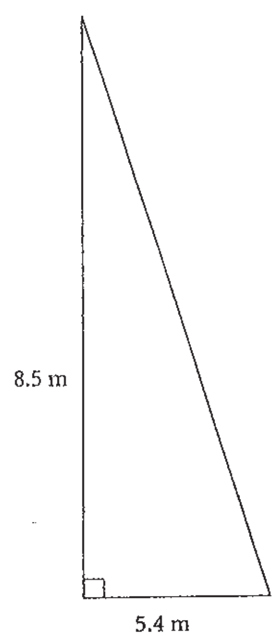
The area is $\frac{1}{2}$(_____ cm × _____ cm) = _____

e)



The area is ________________

f)



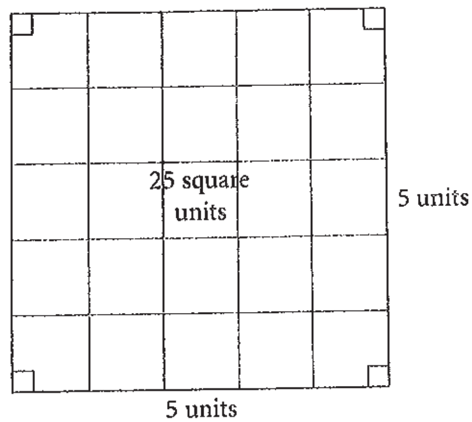
The area is

____

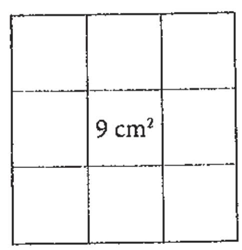
Quick Review

- When you multiply a number by itself, you *square* the number.
  The square of 5 is $5 \times 5 = 25$
  We write: $5^2 = 5 \times 5 = 25$
  We say: Five squared is twenty five.
  25 is a square number, or a perfect square.

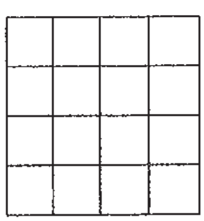
- You can model a square number by drawing a square whose area is equal to the square number.
  You can model 25 using a square with side length 5 units.



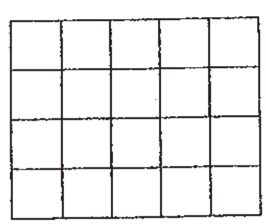
Find the perimeter of a square with area 9 cm^2.
First, find the side length of the square.
Since $3 \times 3 = 9$, the side length is 3 cm. So, the perimeter is 3 cm + 3 cm + 3 cm + 3 cm = 12 cm



16 is a perfect square because you can create a square with area 16 square units using square tiles.



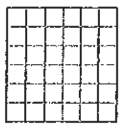
20 is not a perfect square because you cannot create a square with area 20 square units using square tiles. The 4×5 rectangle is the closest to a square that you can get.

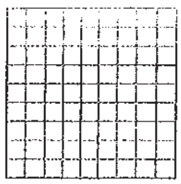


Practice

1. Match each diagram to the correct square number.

a) 

b) 

c) 

i) 36

ii) 81

iii) 16

2. Complete the statement for each square number.

a) 64 is a square number because 64 = ______ × ______ = ______

b) 49 is a square number because 49 = ______ × ______ = ______

3. Complete the table. The first row has been done for you.

a)	4^2	4 × 4	16
b)	3^2	______ × ______	
c)	______^{2}	7 × 7	
d)	11^2	______ × ______	

4. Match the area of the square with the correct side length.

a) 25 cm^2 i) 2 cm
b) 64 cm^2 ii) 10 cm
c) 4 cm^2 iii) 5 cm
d) 100 cm^2 iv) 8 cm

5. Use square tiles to decide whether 32 is a square number.

____

6. Use graph paper to decide whether 64 is a square number.

____

7. Which of the numbers are perfect squares? How do you know?

a) 81 — 81 is a perfect square because 81 = ______ × ______ = ______

b) 18 — 18 ______ a perfect square because I ______ draw a square with area 18 square units on grid paper.

c) 20 ____

____

d) 25 ____

8. Find the side length of the square with each area. Give the unit.

a) 49 cm^2 — _____ × _____ = 49, so the length of the side is 7 cm.

b) 900 mm^2 ______________

c) 121 cm^2 ______________

d) 169 m^2 ______________

9. Find the perimeter of each square.

a) side length 6 cm — Perimeter = _____ cm + _____ cm + _____ cm + _____ cm = _____ cm

b) area 25 m^2 — Side length is _____ m, because _____ × _____ = 25. So,

Perimeter = ________________ = ________________

c) area 144 m^2 ______________

10. If you multiply a perfect square by a different perfect square, is the answer also a perfect square? Give examples to explain your answer.

____

1.2

Quick Review

- When a number is multiplied by itself, the result is a square number.
  For example, 9 is a square number because $3 \times 3 = 9$.

- A number is a square number if it has an *odd* number of factors.
  For example, to check if 36 is a square number, first create a list of the factors of 36 in pairs as shown:
  1×36
  2×18
  3×12
  4×9
  6×6

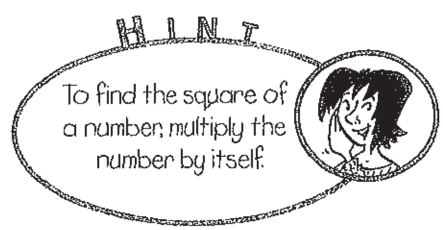
  Write these factors in ascending order, starting at 1:

  1, 2, 3, 4, (6), 9, 12, 18, 36

  There are nine factors of 36. This is an odd number, so 36 is a square number.

  Tip A number with an even number of factors is not a square number.

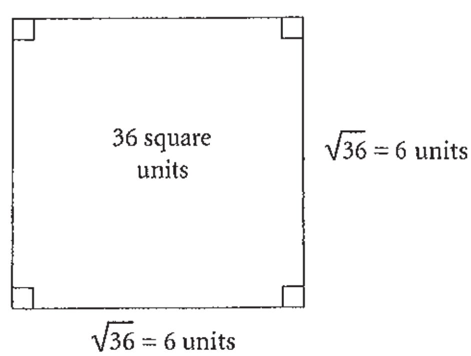
  In the ordered list of factors, notice that 6 is the middle number, and that $6 \times 6 = 36$.
  6 is called the **square root** of 36.
  We write the square root of 36 as $\sqrt{36}$

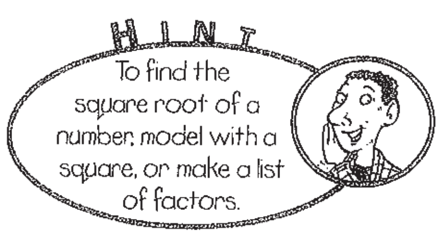


- Squaring and taking the square root are inverse operations.
  So, $\sqrt{36} = 6$ because $6^2 = 6 \times 6 = 36$.
  This means $\sqrt{6^2} = 6$

- You can find a square root using a diagram of square. The area is the square number.

- The side length of the square is the square root of the area.





Practice

1. List the factors of each number in ascending order. Which numbers are square numbers? For each of the square numbers, find the square root.

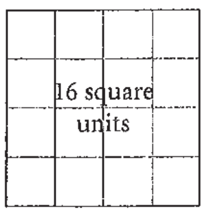
a) 196: ______________________ ______________________

b) 200: ______________________ ______________________

c) 441: ______________________ ______________________

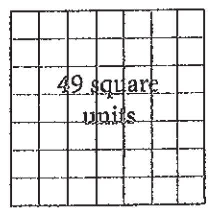
2. For each square, state the square number and the square root.

a)



square number ____________
square root ____________

b)



square number ____________
square root ____________

3. Complete the sentence for each square root. The first one has been done for you.

a) $\sqrt{25} = 5$ because $5^2 = 25$

b) $\sqrt{49} =$ ______ because ______ = ______

c) $\sqrt{100} =$ ______ because ______ = ______

d) $\sqrt{144} =$ ______ because ______ = ______

4. Complete each sentence. The first one has been done for you.

a) $\sqrt{16} = 4$ because $4^2 = 16$

b) ______ = 8 because $8^2 =$ ______

c) ______ = 9 because ______ = ______

d) ______ = 11 because ______ = ______

5. Match each number in column 1 to the number that is equal to it in column 2.

a) $\sqrt{9}$	i) 9
b) 81	ii) 9^2
c) 3^2	iii) $\sqrt{81}$
d) 9	iv) 3

6. Find each square root.

a) $\sqrt{64} =$ ________ b) $\sqrt{400} =$ ________ c) $\sqrt{225} =$ ________ d) $\sqrt{324} =$ ________

7. Find the square root of each number:

a) $5^2 =$ ________ b) $8^2 =$ ________ c) $16^2 =$ ________ d) $54^2 =$ ________

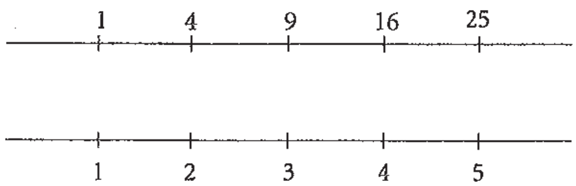
8. Find the number whose square root is

a) 17 ________ = ________ b) 22 ________ c) 30 ________

Quick Review

At Home At School

➢ To estimate the square root of a number that is not a perfect square, you can use a number line.

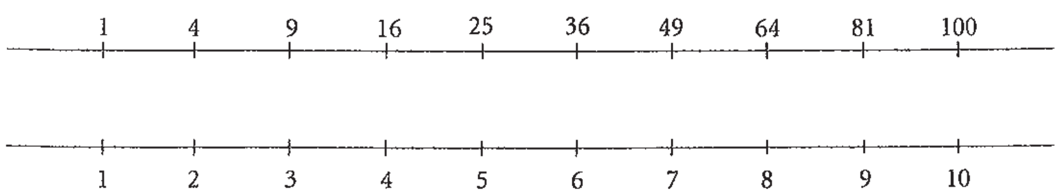


To estimate $\sqrt{10}$: Note that $\sqrt{10}$ lies between $\sqrt{9}$ and $\sqrt{16}$. So, $\sqrt{10}$ must have a value between 3 and 4, but closer to 3. Use trial and error and a calculator to get a closer approximation. Round to 2 decimal places.

Try 3.3: $3.3 \times 3.3 = 10.89$ too big
Try 3.2: $3.2 \times 3.2 = 10.24$ too big
Try 3.1: $3.1 \times 3.1 = 9.61$ too small
Try 3.16: $3.16 \times 3.16 = 9.99$ very close
$\sqrt{10}$ is approximately 3.16.

Practice

1. Use the number lines to complete each statement with whole numbers. The first one is done for you.



a) $\sqrt{5}$ lies between ______ and ______

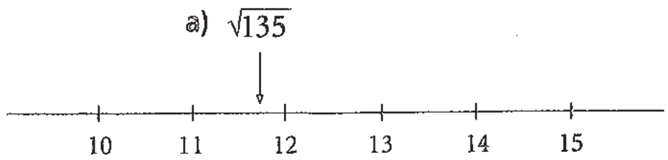
b) $\sqrt{20}$ lies between ______ and ______

c) $\sqrt{55}$ lies between ______ and ______

d) $\sqrt{2}$ lies between ______ and ______

HINT
Find perfect squares close to the number inside the square root symbol.

2. Place the letter of the question on the number line below. The first one is done for you.



a) $\sqrt{135}$ b) $\sqrt{201}$ c) $\sqrt{108}$ d) $\sqrt{167}$ e) $\sqrt{188}$

3. Which statements are true, and which are false?

a) $\sqrt{20}$ is between 19 and 21. ________

b) $\sqrt{20}$ is between 4 and 5. ________

c) $\sqrt{20}$ is closer to 4 than 5. ________

d) $\sqrt{20}$ is between $\sqrt{19}$ and $\sqrt{21}$. ________

4. Which are good estimates of the square roots?

a) $\sqrt{19} = 4.75$ ________________

b) $\sqrt{220} = 14.83$ ________________

5. Use a calculator and the trial and error method to approximate each square root to 1 decimal place. Record each trial.

a) $\sqrt{20} =$ ______ b) $\sqrt{57} =$ ______ c) $\sqrt{115} =$ ______ d) $\sqrt{175} =$ ______

6. Find the approximate side length of the square with each area. Answer to 1 decimal place.

a) $A = 50\ \text{cm}^2$ $s =$ ________

b) $A = 125\ \text{cm}^2$ $s =$ ________

c) $A = 18\ \text{cm}^2$ $s =$ ________

7. Which is the closest estimate of $\sqrt{99}$: 9.94 or 9.95 or 9.96? How did you find out?

8. What length of fencing is required to surround a square field with area 250 m^2? Answer to 2 decimal places.

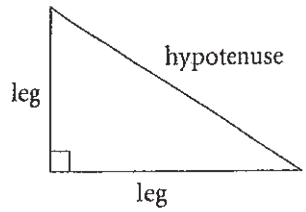
Side length = $\sqrt{\_\_\_\_\_}$ = ______

Perimeter = ______ + ______ + ______ + ______ = ______

____

Quick Review

- A right triangle has two **legs** that form the right angle. The side opposite the right angle is called the **hypotenuse**.

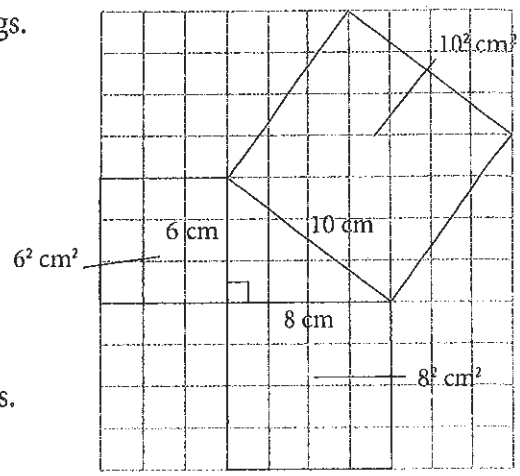


- The three sides of a right triangle form a relationship known as the **Pythagorean Theorem.**

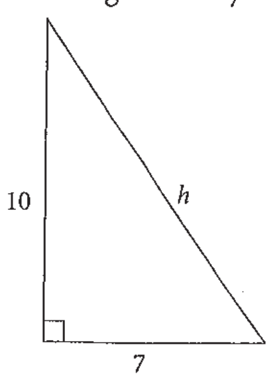
Pythagorean Theorem: The area of the square on the hypotenuse is equal to the sum of the areas of the squares on the legs.

- In the diagram:
  Area of square on hypotenuse:
  $10^2\ cm^2 = 100\ cm^2$
  Areas of squares on legs:
  $6^2\ cm^2 + 8^2\ cm^2 = 36\ cm^2 + 64\ cm^2$
  $= 100\ cm^2$
  Notice that $10^2 = 6^2 + 8^2$.

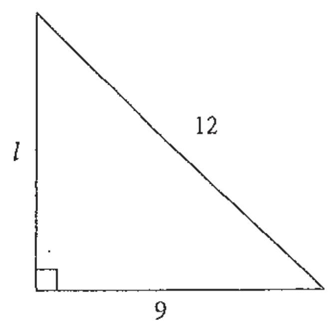
  This theorem is true for all right triangles.



- You can use the Pythagorean Theorem to find the length of any side of a right triangle when you know the lengths of the other two sides.



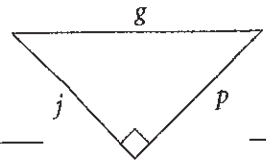
To calculate the hypotenuse h, solve for h in this equation:
$h^2 = 7^2 + 10^2$
$h^2 = 49 + 100$
$h^2 = 149$
$h = \sqrt{149}$
Use a calculator: $h \doteq 12.2$

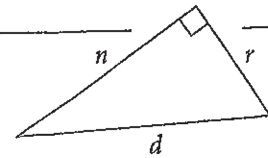


To calculate the leg with length l, solve for l in this equation:
$12^2 = l^2 + 9^2$
$144 = l^2 + 81$
$144 - 81 = l^2 + 81 - 81$
$63 = l^2$
$\sqrt{63} = l$
Use a calculator: $l \doteq 7.9$ cm

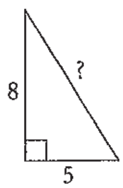
Practice

1. Identify the legs and the hypotenuse of each right triangle.

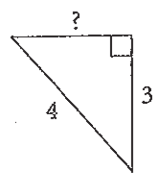
a) 

b) 

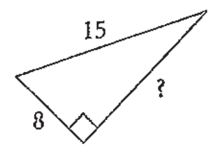
2. Circle the length of the unknown side in each right triangle.

a) 

$\sqrt{13}$ $\sqrt{89}$

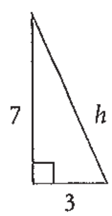
b) 

$\sqrt{7}$ 5

c) 

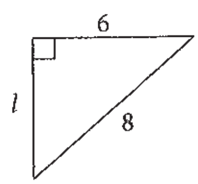
17 $\sqrt{161}$

3. Find the length of the unknown side in each right triangle. Use a calculator to approximate each length to 2 decimal places, if necessary.

a) 

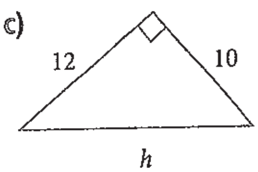
h^2 = _____ + _____

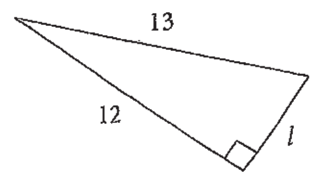
=

b) 

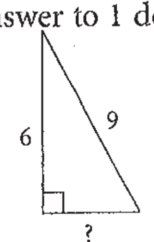
_____ = _____ + _____

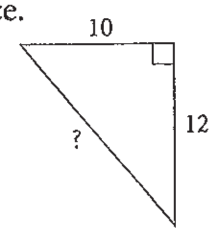
=

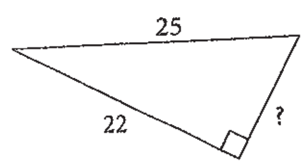
c) 

d) 

4. Find the length of the unknown side in each triangle. Use a calculator to approximate each answer to 1 decimal place.

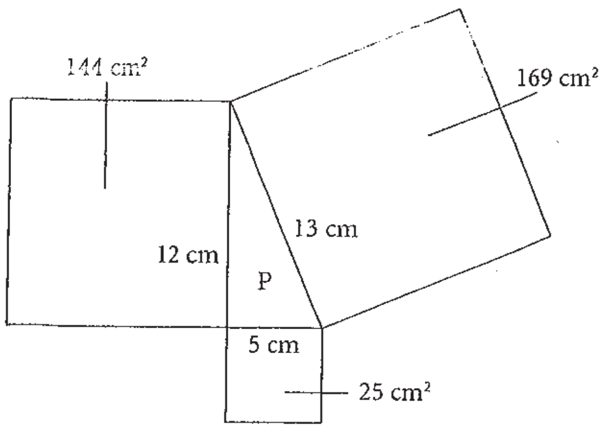
a) 

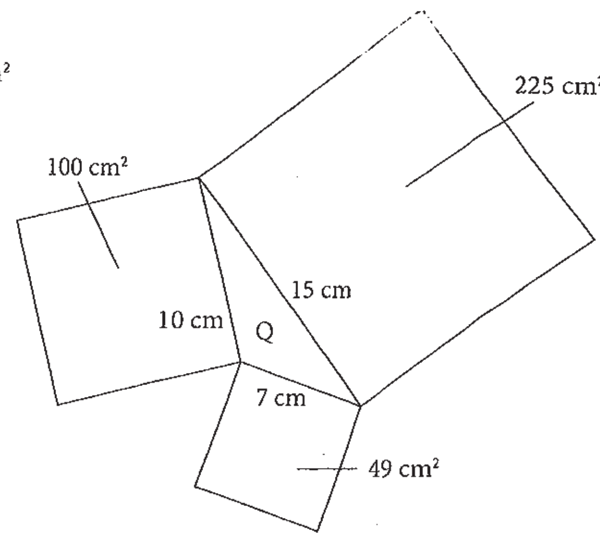
b) 

c) 

Quick Review

- The Pythagorean Theorem is true for right triangles only.
- To see which triangle is a right triangle, check to see if the area of the square on the longest side is equal to the sum of the areas of the squares on the other two sides.

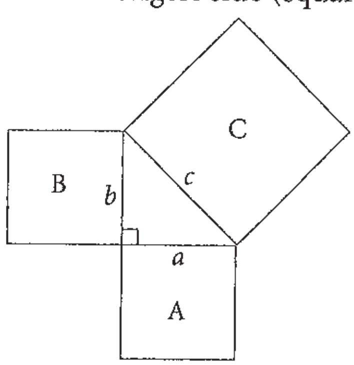




$25\text{ cm}^2 + 144\text{ cm}^2 = 169\text{ cm}^2$ $49\text{ cm}^2 + 100\text{ cm}^2 \neq 225\text{ cm}^2$

The Pythagorean Theorem applies to triangle P, but not to triangle Q.

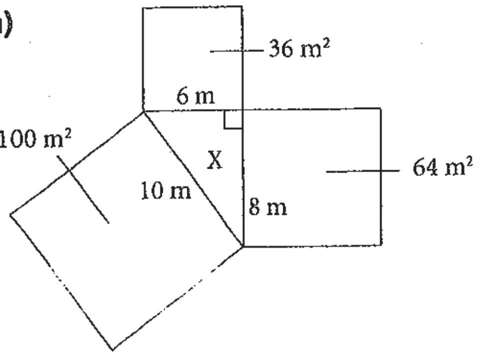
- A set of three whole numbers that satisfy the Pythagorean Theorem is called a Pythagorean triple. For example, 5-12-13 is a Pythagorean triple because $5^2 + 12^2 = 13^2$
- For a right triangle:
  area of square on the longest side (square C) = area of square A + area of square B



- For a Pythagorean triple *a*-*b*-*c*:
  $c^2 = a^2 + b^2$

1. Fill in the blanks from the list of choices to make the sentence true.

a)

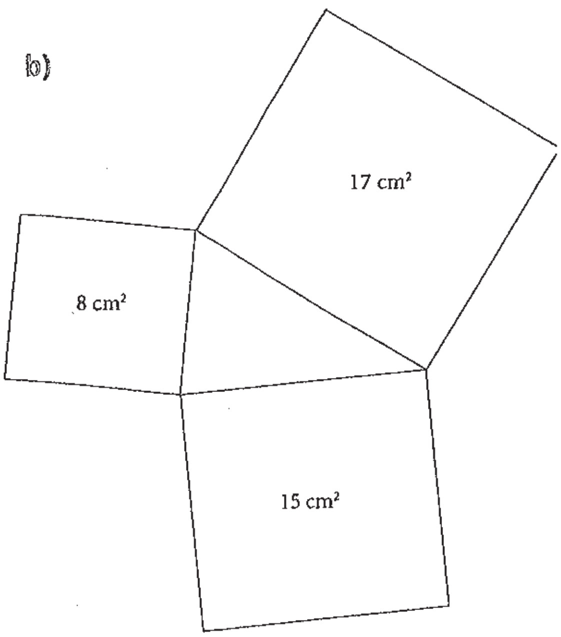


Triangle X ________ a right triangle because

____

is   is not $6 + 8 \neq 10$ $100 = 64 + 36$

b)



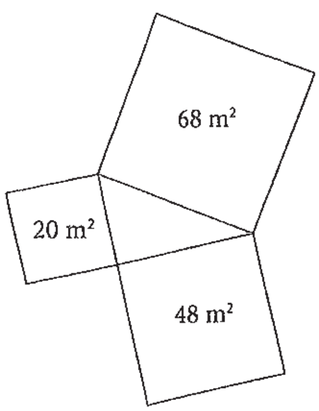
Triangle Y ________ a right triangle because

____

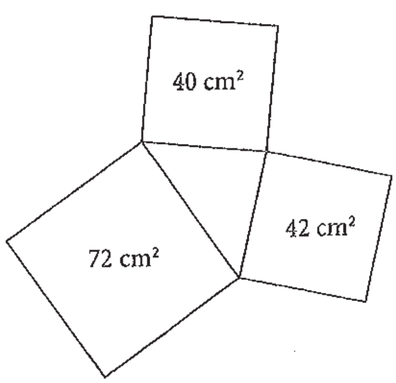
is   is not $8^2 + 15^2 = 17^2$ $8 + 15 \neq 17$

2. Which of the following triangles are right triangles? Explain.

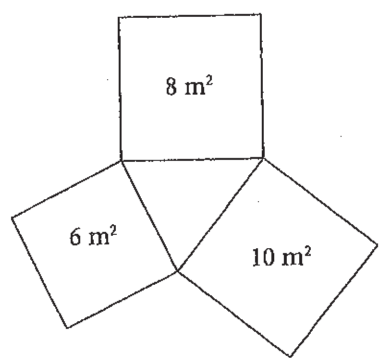
a)



b)



c)



______________ ______________ ______________

______________ ______________ ______________

3. Determine whether the triangle with each set of side lengths is a right triangle. Justify your answer.

a) 20 cm, 30 cm, 40 cm
_____ cm^2 + _____ cm^2 = _______ cm^2
_____ cm^2 = _______ cm^2
The triangle ________ a right triangle
because ____________________________

b) 30 mm, 40 mm, 50 mm
____
____

c) 20 m, 21 m, 29 m
____
____

d) 60 cm, 11 cm, 62 cm
____
____

4. Fill in the blanks to make the sentence true.

The set of numbers 7, 24, 25 is a Pythagorean triple because _______ + _______ = _______

5. Which of these sets of numbers are Pythagorean triples? Explain.

a) 10, 50, 60
This ___________ a Pythagorean triple
because $10^2 + 50^2$ _______ 60^2

b) 12, 35, 37
____
____

6. Two numbers of a Pythagorean triple are given. Find the missing number. The numbers are listed in ascending order.

a) 7, 24, ________
The missing number is the ___________ of the sum of the _________ of the first two numbers.

____

= ______

= ______

b) 16, 30, ________

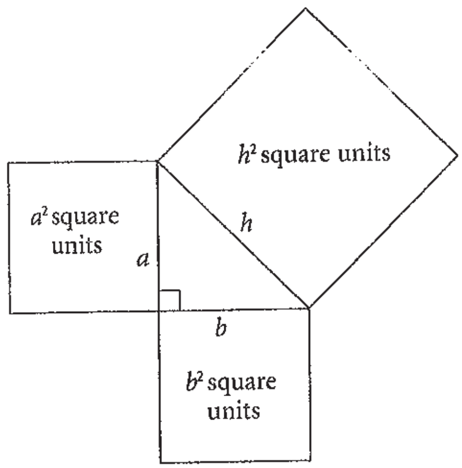
c) 10, _________, 26

7. Doug wants to check that a lawn he is planting is a rectangle. He measures the width of the lawn to be 10 m, the length to be 24 m, and the diagonal to be 26 m. Is the lawn a rectangle? Explain.
If the lawn is a rectangle, then the width, length, and diagonal form a ___________ triangle.

____
____
____

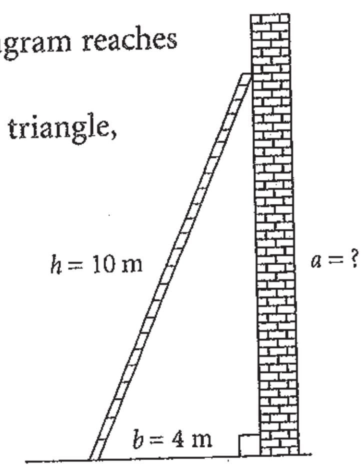
Quick Review

- The Pythagorean Theorem applies to right triangles.
- An algebraic equation for the Pythagorean Theorem is $h^2 = a^2 + b^2$, where h is the length of the hypotenuse and a and b are the lengths of the legs.



- You can apply the Pythagorean Theorem to problems involving right triangles.

You can calculate how high up the wall the ladder in the diagram reaches using the formula $h^2 = a^2 + b^2$
Since the length of the ladder is the hypotenuse of the right triangle, we label it h. The lengths of the two legs of this triangle are labelled a and b.



Substitute $b = 4$ and $h = 10$ into $h^2 = a^2 + b^2$

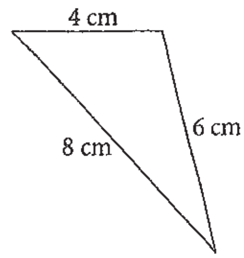
$10^2 = a^2 + 4^2$
$100 = a^2 + 16$
$100 - 16 = a^2 + 16 - 16$
$84 = a^2$
$\sqrt{84} = a$
$9.2 \doteq a$

a is approximately 9.2 m. The ladder reaches approximately 9.2 m up the wall.

Tip
*It does not matter which leg is labelled a and which leg is labelled b, so long as a and b label the **legs** and h labels the **hypotenuse**.*

Practice

1. Use the Pythagorean Theorem to check if this is a right triangle.



Substitute $a =$ ____, $b =$ ____, and $h =$ ____ into the formula $h^2 = a^2 + b^2$

$h^2 =$ ______ $a^2 + b^2 =$ ______

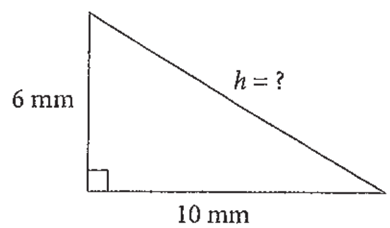
Circle the choices that make the sentence true.

Since h^2 *equals / does not equal* $a^2 + b^2$, the triangle *is / is not* a right triangle.

For questions 2 to 5, give each length to 1 decimal place.

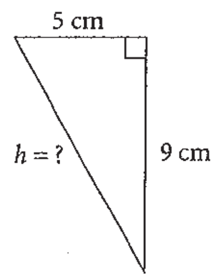
2. Use the equation $h^2 = a^2 + b^2$ to find the length of the unknown side.

a)



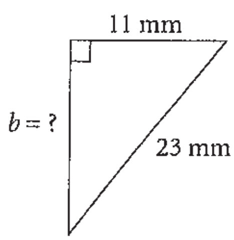
$h \doteq$ ________

b)



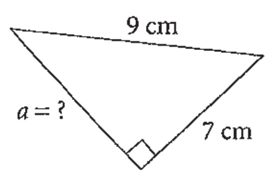
$h \doteq$ ________

c)



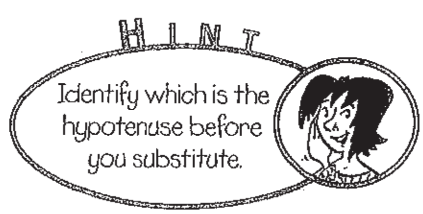
$b \doteq$ ________

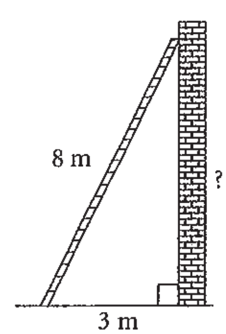
d)



$a \doteq$ ________

3. An 8-m ladder leans against a wall. How far up the wall does the ladder reach if the foot of the ladder is 3 m from the base of the wall? Show your work.

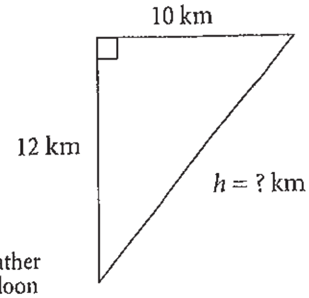




$b \doteq$ ________

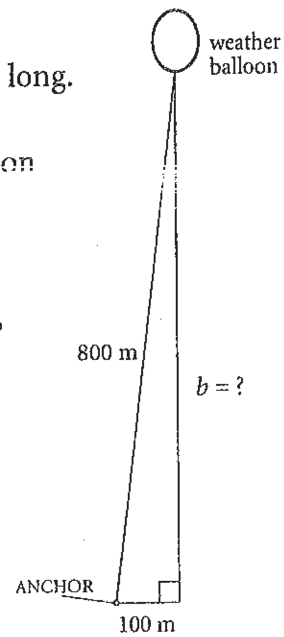
The ladder can reach a height of ______, to 1 decimal place.

4. A ship leaves port and travels 12 km due north. It then changes direction and travels due east for 10 km. How far must it travel to go directly back to port? Sketch a diagram to explain.



The ship must travel _________, to 1 decimal place, to go directly back to port.

5. A weather balloon is anchored by a cable 800 m long. The balloon is flying directly above a point that is 100 m from the anchor. How high is the balloon flying? Give your answer to the nearest metre.



The balloon is flying at a height of __________, to the nearest metre.

6. A rectangular field is 40 m long and 30 m wide. Carl walks from one corner of the field to the opposite corner, along the edge of the field. Jade walks across the field diagonally to arrive at the same corner. How much shorter is Jade's shortcut?

Tip
Sketch a diagram first.

The diagonal of the field measures ______.
Jade walks ______.
Carl walks ______ + ______ = ______
Jade's shortcut is ______ – ______ = ______ shorter.

7. What is the length of a diagonal of a square with area 100 cm^2? Give your answer to 1 decimal place.

The side length of the square is the square root of _______, or ______ cm.

The diagonal of the square is the __________ of the right triangle with sides ______ and ______.

The length of the diagonal of the square is _______, to 1 decimal place.

In Your Words

Here are some of the important mathematical words of this unit.
Build your own glossary by recording definitions and examples here. The first one is done for you.

perfect square (square number)

the product of a whole number multiplied by itself

For example, 25 is 5×5, so 25 is a perfect square.

square root ______

legs of a right triangle ______

hypotenuse ______

Pythagorean Theorem ______

Pythagorean triple ______

List other mathematical words you need to know.

Unit Review

LESSON

1.1 1. Circle the perfect squares. Use a diagram to support your answer.

a) 36 b) 63 c) 121 d) 99

1.2 2. Simplify without using a calculator.

a) $8^2 =$ ____________ b) $\sqrt{49} =$ ____________

c) $12^2 =$ ____________ d) $\sqrt{121} =$ ____________

3. List the factors of each number in ascending order. Circle the numbers that are perfect squares.

a) 50 b) 196

____________________ ____________________

c) 84 d) 225

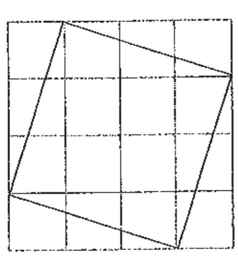
____________________ ____________________

1.3 4. The area of a square is given. Find its side length. Circle the side lengths that are whole numbers.

a) 18 cm^2 b) 169 cm^2 c) 200 cm^2

________ ________ ________

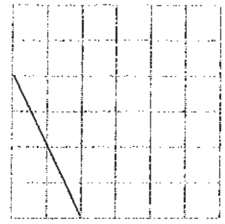
5. Find the area of the square. Then write the side length of the square.



Area = ____________

Side length = ____________

6. Construct a square on the line segment.
Find the length of the line segment.



Length = ____________

1.4 7. Evaluate.

a) $\sqrt{8 \times 8}$ = ____ b) $\sqrt{54 \times 54}$ = ____ c) $\sqrt{153 \times 153}$ = ____

8. Between which two whole numbers is each square root?

a) $\sqrt{45}$ ____ b) $\sqrt{18}$ ____ c) $\sqrt{55}$ ____ d) $\sqrt{135}$ ____

9. Estimate each root in question 8 to 1 decimal place.

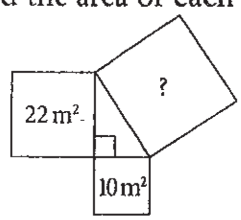
a) ________ b) ________ c) ________ d) ________

10. Circle the better estimate.

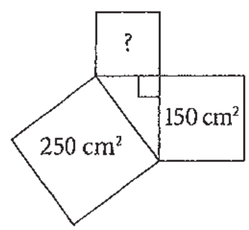
a) $\sqrt{75} \doteq$ 8.65 or 8.66? b) $\sqrt{90} \doteq$ 9.49 or 9.50?

1.5 11. Find the area of each indicated square.

a)



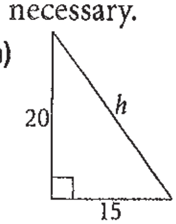
b)



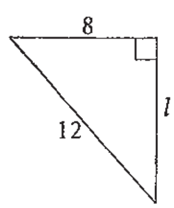
________________ ________________

12. Find the length of each side labelled with a variable. Give answers to 1 decimal place, if necessary.

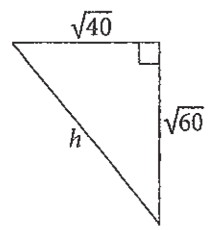
a)



b)



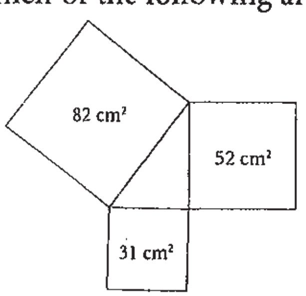
c)



________________ ________________ ________________

1.6 13. Which of the following are right triangles? Justify your answer.

a)



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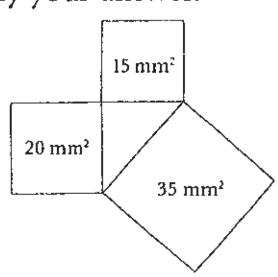

________________ ________________

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LESSON

14. Circle the sets of numbers that are Pythagorean triples.

a) 10, 24, 26

b) 12, 15, 20

c) 7, 24, 26

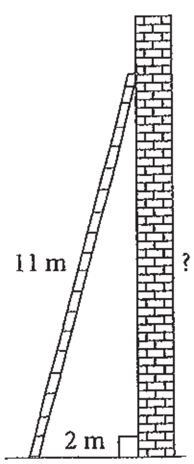
d) 11, 60, 61

1.7 15. A ship travels for 14 km toward the south. It then changes direction and travels for 9 km toward the east. How far does the ship have to travel to return directly to its starting point? Answer correct to 2 decimal places.

Tip: *Draw a diagram.*

The ship must travel ________

16. How high up the wall does the ladder reach? Answer correct to 2 decimal places.



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