UNIT 6

Linear Equations and Graphing

Just for Fun

Date Palindrome

A number palindrome is a number that reads the same backward as forward. 13631 is a number palindrome.

In this century, February 20, 2002 is a date palindrome when it is written in the day/month/year short form without slashes (DDMMYYYY). Write this date palindrome.

_____

Write two other date palindromes for this century.

_____

_____

Will you have a birthday that is a date palindrome? If so, what is it?

_____

Word Scramble

Unscramble the letters in each row to form a word in mathematics.

ILLTUMPY _____

BRATTCUS _____

RAILBAVE _____

NERPECT _____

COFTRAIN _____

LOVES _____

GREENTI _____

Make up your own scrambled words in mathematics for your friends to unscramble.

Four Fours

Use exactly four 4s and any mathematical symbols you know to make up as many expressions as you can with whole-number values between 1 and 20.
You may use symbols such as (), +, –, ×, ÷, and the decimal point. For example: $44 \div 44 = 1$

_____

Variation: Work with a friend. Make this activity more challenging by trying whole number values between 1 and 100.

Activating Prior Knowledge

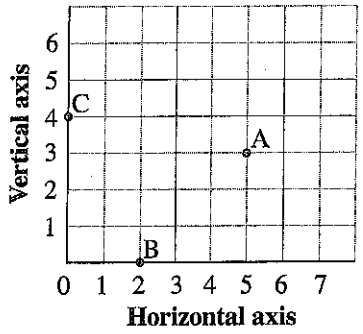
Graphing Ordered Pairs

An ordered pair, such as (5, 3), tells you the position of a point on a grid.
The first number is the horizontal distance from the origin, O.
The second number is the vertical distance from the origin, O.
The numbers of an ordered pair are also called the **coordinates** of a point.

➢ To graph the points A(5, 3), B(2, 0), and C(0, 4) on a grid:
To plot point A, start at 5 on the horizontal axis, then move up 3.

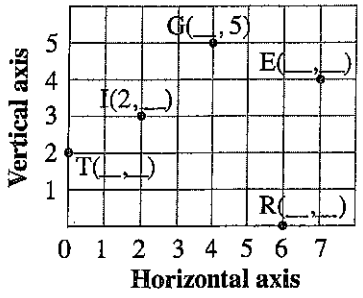
To plot point B, start at 2 on the horizontal axis, then move up 0. Point B is on the horizontal axis.

To plot point C, start at 0 on the horizontal axis, then move up 4. Point C is on the vertical axis.

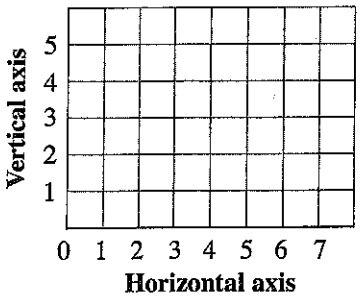


Checking

1. Write the ordered pair for each point on the grid.



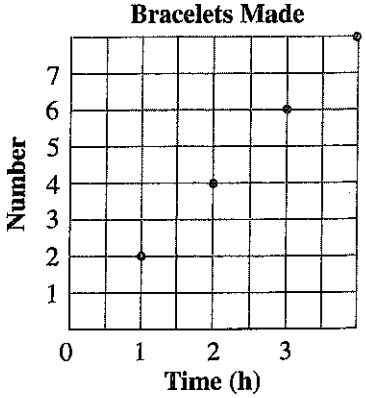
2. Plot and label these points:
A(0, 5), B(2, 4), E(4, 3), R(5, 0)



3. The graph shows the number of bracelets Jan can make over time.

a) How many bracelets can Jan make in 3 h? _____

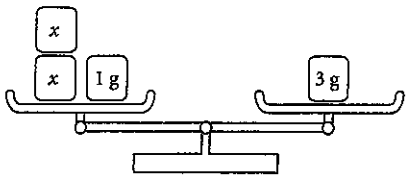
b) How long will it take to make 10 bracelets? _____

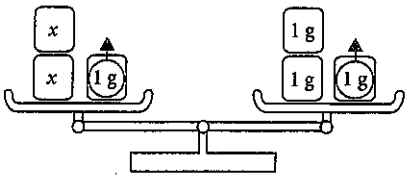
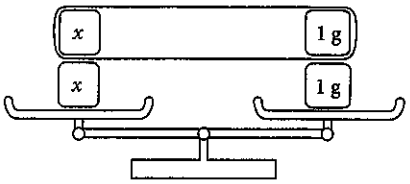


Preserving Equality

When you perform the same operation on both sides of an equation, the solution to the equation does not change. This is how the algebraic method of solving equations works.

Consider the equation $2x + 1 = 3$.



Subtract 1 from both sides.	$2x + 1 - 1 = 3 - 1$ $2x = 2$	
Divide both sides by 2.	$2x \div 2 = 2 \div 2$ $x = 1$	

To show that the solution did not change, check it in the original equation.
Substitute $x = 1$ into $2x + 1 = 3$.

Left side $= 2x + 1$ Right side $= 3$
$= 2(1) + 1$
$= 2 + 1$
$= 3$

Since the left side equals the right side, $x = 1$ is the correct solution to $2x + 1 = 3$.

Checking

4. Write the operations you can perform, in the correct order, so that the solution to the equation does not change.

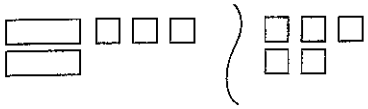
a) $3a - 2 = 4$

b) $\frac{c}{2} + 3 = -2$

6.1 Solving Equations Using Models

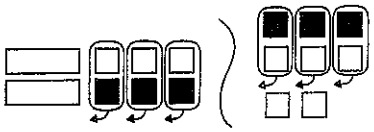
Quick Review

Algebra tiles and balance scales can both be used to model and solve equations.
To solve the equation $2x + 3 = 5$:

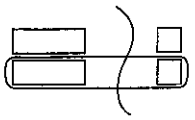


What you do to one side of the equation, you also do to the other side.

Isolate the x-tiles by adding 3 black tiles to make zero pairs. Then remove the zero pairs.



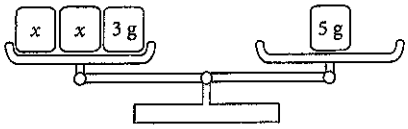
Arrange the tiles on each side into 2 equal groups. Compare groups.



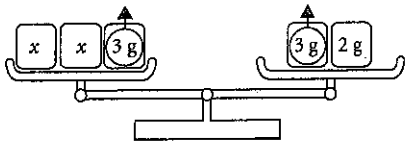
One x-tile equals 1 white tile.
So, $x = 1$.

Tip

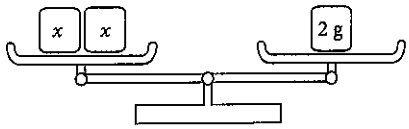
A white square tile models +1 and a black square tile models −1. These are called unit tiles. White rectangular tiles model variable tiles, or x-tiles. One white unit tile and one black unit tile form a ***zero pair.***



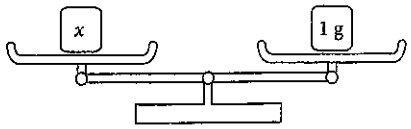
Replace 5 g in the right pan with 3 g and 2 g. Then remove 3 g from each pan.



The unknown masses are isolated in the left pan, and 2 g is left in the right pan.



The two unknown masses balance 2 g. So, each unknown mass is 1 g.

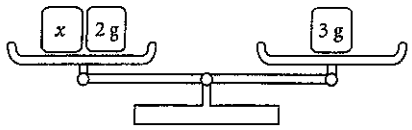


So, $x = 1$.

Practice

1. Write the equation modelled by each of the following.

a)

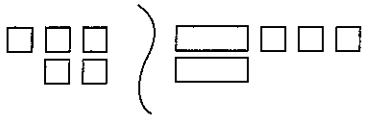


Tip

To isolate the x-tile or mass, make zero pairs.

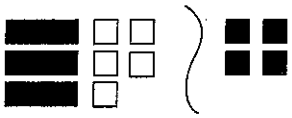
_____

b)



_____

c)



_____

2. Construct a model to represent each equation. Then solve the equation using your model. Verify the solution.

a) $x + 3 = 9$ _____

b) $3 = 2x - 5$ _____

c) $4x + 3 = 11$ _____

d) $14 = 5x + 4$ _____

3. Draw a model for each equation and the steps of its solution. Verify the solution.

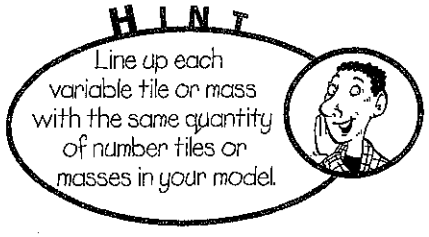
a) $a + 4 = 5$

b) $6 = c - 4$

c) $y - 2 = 4$

d) $5 = x + 3$

4. Draw a model for each equation and the steps of its solution. Verify the solution.



a) $2v = 6$

b) $4n = -8$

c) $5 = 5y$

d) $-6 = 3r$

5. Draw a model to represent the steps you took to solve each equation. Verify the solution.

a) $3x + 2 = 11$

b) $-5 = 5 + 2y$

6. Five more than twice a number is seven. Let n represent the number.

a) Write an equation you can use to solve for n.

b) Represent the equation for this problem with a model. Use the model to solve the equation.

c) Verify the solution and write a concluding statement.

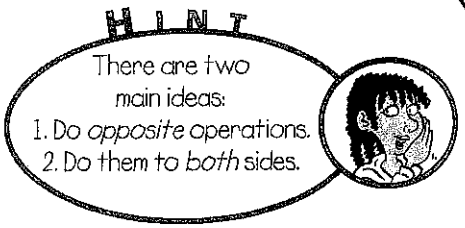
7. One less than three times a number is eleven. Write an equation and use a model to solve the problem. Verify the solution and write a concluding statement.

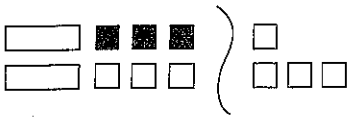
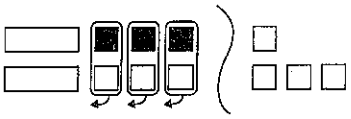
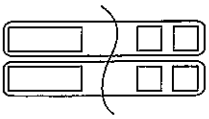
6.2 Solving Equations Using Algebra

At Home
At School

Quick Review

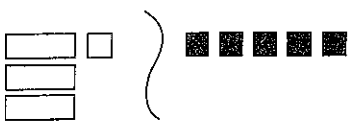
In Section 6.1, you solved the equation $2x - 3 = 1$ using algebra tiles. You are going to solve the same equation using algebra and compare it to the algebra tile model.

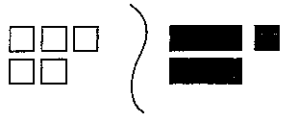


Algebra tile model	Algebra steps
	$2x - 3 = 1$
Isolate the x-tiles by adding +3 to both sides 	$2x - 3 + 3 = 1 + 3$
Remove zero pairs. 	$2x = 4$
Arrange the tiles on each side into 2 equal groups. 	Divide both sides by 2 to isolate the x-variable: $\frac{2x}{2} = \frac{4}{2}$
$x = 2$	$x = 2$

Practice

1. Write the equation modelled by each set of algebra tiles. Then solve the equation using both the algebra tile method and the algebra method.

a) 

b) 

2. Sketch algebra tiles to represent each equation. Then solve the equation using both the algebra tile method and the algebra method.

a) $2y - 1 = 7$

_____

b) $-4 = 2 + 3a$

_____

3. Use algebra to solve each equation. Verify the solution.

a) $6m + 5 = -7$

The solution is _____.

b) $3c - 2 = 2$

The solution is _____.

c) $2 + 5y = 2$

The solution is _____.

d) $4 - 3x = -5$

The solution is _____.

4. Each solution has an error. Check the solution and show that it is incorrect. Then show a correct solution.

a)
$$3y - 4 = 8$$
$$3y - 4 + 4 = 8 + 4$$
$$3y = 12$$
$$3y - 3 = 12 - 3$$
$$y = 9$$

The solution is _____.

b)
$$9 = 6 - 2x$$
$$9 + -6 = 6 - 6 - 2x$$
$$15 = -2x$$
$$\frac{15}{-2} = \frac{-2x}{-2}$$
$$-7\frac{1}{2} = x$$

The solution is _____.

5. For each part below, let the number be n. Write an equation and solve it algebraically, verify the solution, and then write a concluding statement.

a) Four less than three times a number is fourteen.

_____

b) The sum of twelve and twice a number is forty-four.

_____

6.3 Solving Equations Involving Fractions

Quick Review

Remember the two basic concepts in solving an equation:

1. Isolate the variable by using *opposite operations.*
2. Do operations to *both sides* to keep the equations in balance.

The opposite operation of addition is subtraction.
The opposite operation of multiplication is division.
The opposite operation of division is multiplication.

You can solve $2x = 10$ by dividing both sides by 2 because dividing is the opposite operation of multiplication.

The solution looks like this:

$2x = 10$

$\frac{2x}{2} = \frac{10}{2}$

$x = 5$

You can solve $\frac{a}{3} = 6$ by multiplying both sides by 3 because multiplication is the opposite operation of division.

The solution looks like this:

$\frac{a}{3} = 6$

$\frac{a}{3} \times 3 = 6 \times 3$

$a = 18$

Solve the equation $-10 + \frac{m}{5} = -14$

Remember, first you isolate the variable by doing opposite operations.

$-10 + \frac{m}{5} = -14$

$-10 + \frac{m}{5} + 10 = -14 + 10$

$\frac{m}{5} = -4$

$\frac{m}{5} \times 5 = -4 \times 5$

$m = -20$

To verify the solution, substitute $m = -20$ into $10 + \frac{m}{5} = -14$.

Left side $= -10 + \frac{m}{5}$

$= -10 + \frac{(-20)}{5}$

$= -10 + (-4)$

$= -14$

Right side $= -14$

Since the left side equals the right side, $m = -20$ is correct.

Practice

1. For each equation, describe the opposite operation required to isolate the variable.

a) $2x = 14$

b) $\frac{y}{3} = 20$

c) $m - 3 = 15$

d) $21 = \frac{a}{7}$

2. Solve each equation and verify the results.

a) $\frac{c}{6} = -3$

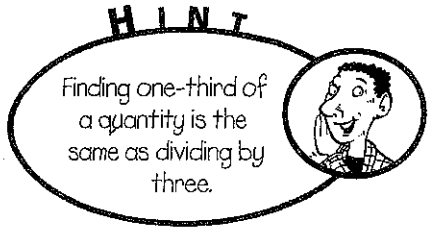
b) $\frac{n}{2} = 14$

c) $16 = -5y$

d) $9 = \frac{a}{4}$

3. The senior girls basketball team took one-third of the basketballs to their game. They took 7 balls to their game. How many basketballs were there altogether?

a) Let b be the total number of balls. Write an equation you can use to solve this problem.



b) Solve the equation.

c) Verify the solution and write a concluding statement.

4. Bob ate 22 jellybeans. His mom says that he ate one-quarter of the bag. How many jellybeans were in the bag to start with?

a) Set up an equation to solve this problem. Let j be the number of jellybeans in the bag.

b) Solve the equation and verify the result.

c) Write a concluding statement.

5. Solve each equation. Verify the results.

a) $\frac{w}{3} + 6 = 2$

b) $-1 = \frac{y}{4} + 3$

c) $\frac{x}{5} - 2 = -10$

d) $4 + \frac{c}{10} = 8$

6.4

Quick Review

At Home / At School

To multiply $3(x + 4)$:

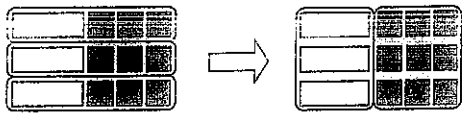
Draw a diagram.

	x	4
3	$3x$	12

$3(x + 4) = 3x + 12$

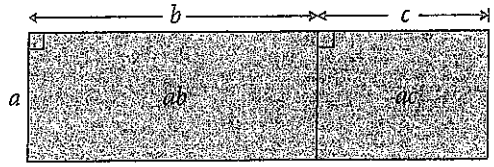
To multiply $2(x - 3)$:

Use algebra tiles.



$2(x - 3) = 2x - 6$

The distributive property says that $a(b + c) = ab + ac$



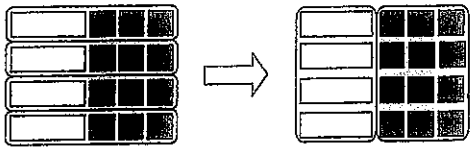
Practice

1. Write the equation represented by each model.

a)

	c	5
4	$4c$	20

b)



_____ _____

2. Use algebra tiles to model each equation.

a) $2(y + 5) = 2y + 10$

b) $3(w - 1) = 3w - 3$

3. Use the distributive property to expand each expression.

a) $3(u - 6) =$ _____

b) $2(5 + q) =$ _____

c) $5(r + 1) =$ _____

d) $7(3 - p) =$ _____

4. Expand.

a) $-6(a - 7) =$ _____

b) $4(-5 - w) =$ _____

c) $-2(x - 20) =$ _____

d) $-1(b + 8) =$ _____

5. Vanessa expanded $3(y - 2)$ below. Did she make an error? YES/NO

$3(y - 2) = 3y - 5$

If so, write the correct solution.

_____

6. Hazar is having 4 friends over to play video games. Each person will spend $6 on game rental and $4 on drinks and snacks.

a) Write two expressions for the total cost for Hazar and his friends.

_____

b) Evaluate each of the expressions.

_____

c) Show how the distributive property is being illustrated in this question.

_____

6.5 Solving Equations Involving the Distributive Property

Quick Review

Francis thought of his favourite number.
He subtracted 9.
Then he multiplied the difference by –2.
The product was 10.
What is Francis's favourite number?

Let n represent Francis's favourite number. Write an equation to solve for n.

Start with n.	n
Subtract 9.	$n-9$
Multiply the difference by –2.	$-2(n-9)$
The product is 10.	$-2(n-9)=10$

Solve the equation.

$$-2(n-9)=10$$
$$-2n+18=10$$
$$-2n+18-18=10-18$$
$$-2n=-8$$
$$\frac{-2n}{-2}=\frac{-8}{-2}$$
$$n=4$$

Francis's favourite number is 4.

Practice

1. Solve each equation using the distributive property. Verify the results.

a) $4(r+3)=-8$

b) $15=3(p-7)$

c) $-3(m-2)=21$

d) $3=5(x+7)$

e) $-6(7+r)=30$

f) $0=2(-2+h)$

2. Brittany has some cookies. She gave four of them to friends. If she doubles the number that she has left, she will have 12 cookies.

a) Choose a variable to represent the number of cookies Brittany had at the start.

b) Write an algebraic expression to represent how many she would have if she gave four of them away to friends.

c) Now write an expression to double what you wrote in part b).

d) Write an equation for this problem and solve it.

e) Verify your answer and write a concluding statement.

Quick Review

At Home / At School

If a relation is represented by the equation $y = 2x + 1$, you can write a table of values as:

x	1	2	3	4	5	6	7
y	3	5	7	9	11	13	15

A related pair of x and y values is called an ordered pair.
Some ordered pairs for this relation are:
(1, 3), (2, 5), (3, 7), (4, 9), (5, 11), (6, 13), (7, 15), (x, y)

Tip

$0.50 is the same as $\frac{1}{2}$ of $1, so $\frac{t}{2}$ represents the cost of the toppings.

A one-scoop ice-cream cone costs $3.00 plus $0.50 for each topping. An equation for this relation is $c = 3 + \frac{t}{2}$, where t represents the number of toppings and c represents the cost of the ice-cream cone in dollars.

Use different values of t to complete a table of values.

$t = 0$	$t = 1$	$t = 2$	$t = 3$
$c = 3 + \frac{t}{2}$	$c = 3 + \frac{t}{2}$	$c = 3 + \frac{t}{2}$	$c = 3 + \frac{t}{2}$
$= 3 + \frac{0}{2}$	$= 3 + \frac{1}{2}$	$= 3 + \frac{2}{2}$	$= 3 + \frac{3}{2}$
$= 3 + 0$	$= 3 + 0.5$	$= 3 + 1$	$= 3 + 1.5$
$= 3$	$= 3.5$	$= 4$	$= 4.5$

A table of values is:

t	c
0	3
1	3.5
2	4
3	4.5

To find the cost of an ice-cream cone with 5 toppings, substitute $t = 5$ into the equation.

$c = 3 + \frac{t}{2}$

$= 3 + \frac{5}{2}$

$= 3 + 2.5$

$= 5.5$

An ice-cream cone with 5 toppings costs $5.50.

To find how many toppings are on a crazy ice-cream cone that costs \$7.50, substitute $c = 7.5$ into the equation.

$$7.5 = 3 + \frac{t}{2}$$

$$7.5 - 3 = 3 + \frac{t}{2} - 3$$

$$4.5 = \frac{t}{2}$$

$$4.5 \times 2 = \frac{t}{2} \times 2$$

$$9 = t$$

A crazy ice-cream cone that costs \$7.50 has 9 toppings!

Practice

1. Copy and complete each table of values.

a) $y = x - 7$

x	y
−3	
−2	
−1	
0	
1	
2	
3	

b) $y = -x + 14$

x	y
−3	
−2	
−1	
0	
1	
2	
3	

c) $y = -3x$

x	y
−3	
−2	
−1	
0	
1	
2	
3	

2. Make a table of values for each relation.

a) $y = x + 4$ **b)** $y = -2x + 2$ **c)** $y = 5 - x$

3. The equation of a linear relation is: $w = 6r + 3$

a) Substitute 33 for w in the equation.

_____

b) Solve the equation to complete the ordered pair (_____, 33) for this relation.

4. Repeat the steps of question 3 to complete the following ordered pairs for the relation $w = 6r + 3$.

a) (_____, 15) b) (_____, –21)

5. The equation of a linear relation is: $d = 4t + 6$
Find the missing number in each ordered pair.

a) (2, _____) b) (_____, 18)

c) (12, _____) d) (–4, _____)

6. Bergy's Hamburger Emporium sells its famous double-cheese mushroom burger for $4. The relation $c = 4n$ represents the cost, c, of n hamburgers.

a) Use the relation to complete the table of values.

n	1	2	3	4	5
c					

b) How many hamburgers would have to be sold to have a cost of $28?

_____ hamburgers would have to be sold.

6.7 Graphing Linear Relations

Quick Review

At Home / At School

Daniel works at the local gas bar. He is paid \$5 per shift plus \$10 per hour for each hour that he works. David is only paid for whole hours. An equation that relates his earnings to the number of hours he works is $e = 5 + 10n$, where e represents his earnings for a shift that lasts n hours.

Substitute values for n to find corresponding values of e.

When $n = 0$, $e = 5 + 10(0)$
$= 5 + 0$
$= 5$

When $n = 1$, $e = 5 + 10(1)$
$= 5 + 10$
$= 15$

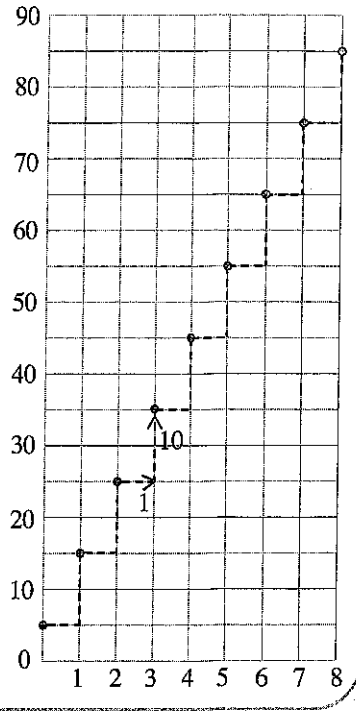
A table of values is:

n	0	1	2	3	4	5	6	7	8
e	5	14	25	35	45	55	65	75	85

To graph the relation, plot n along the horizontal axis and e along the vertical axis.
Label the axes and write the equation of the relation on the graph.
The points lie on a straight line, so the relation is linear.
Since Daniel only gets paid for whole numbers of hours, do not join the points. These data are **discrete**. This means that there are numbers between those given that are not meaningful in the context of the problem.

The graph shows that for every hour Daniel works, his pay increases by \$10. As the number of hours increases, so does his pay.

Graph of $e = 5 + 10n$

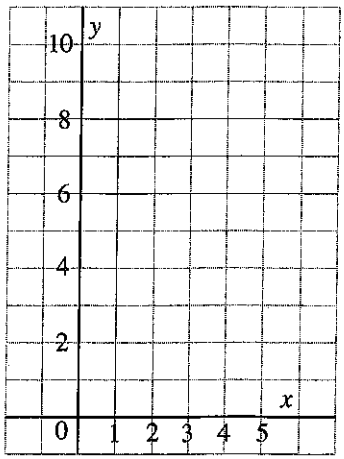


Practice

You will need grid paper.

1. **a)** Graph the table of values.

x	y
0	2
1	4
2	6
3	8
4	10

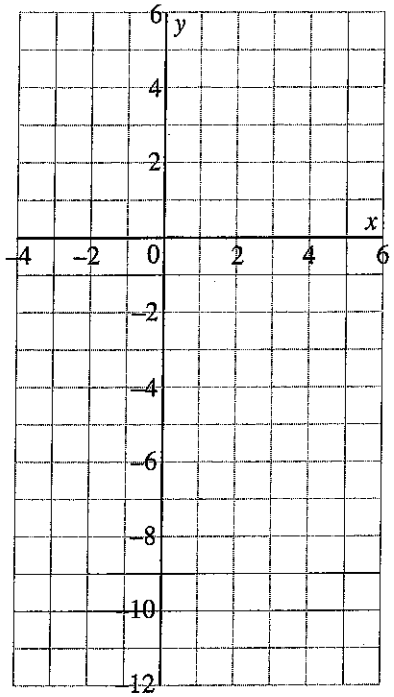


b) Describe the relationship between the variables in the graph.

---

2. a) Graph the table of values.

x	y
–3	5
–1	1
1	–3
3	–7
5	–11



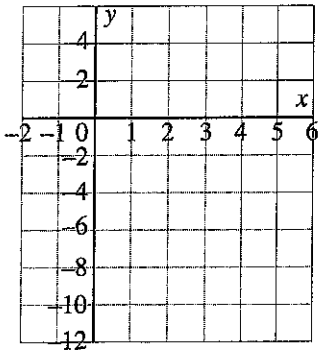
b) Describe the relationship between the variables in the graph.

---

3. a) Complete the table of values for the relation with equation $y = -3x + 2$.

x	y
–1	5
0	2
1	
2	–4
	–7
4	

b) Graph the ordered pairs.

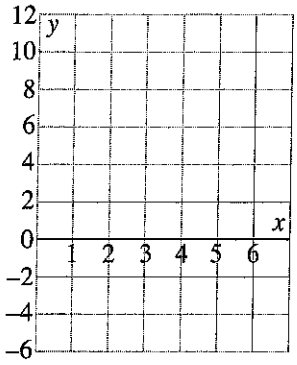


4. For $y = 3x - 4$:

a) Make a table of values using values of x from 0 to 5.

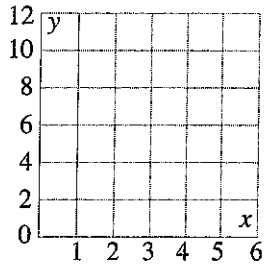
x						
y						

b) Graph the relation.

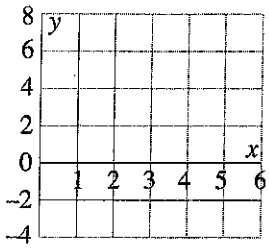


5. Graph each relation for integer values from 0 to 5.

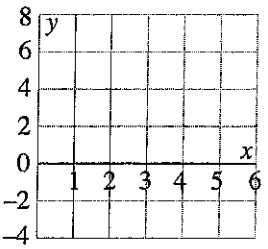
a) $y = x + 6$



b) $y = 2x - 3$



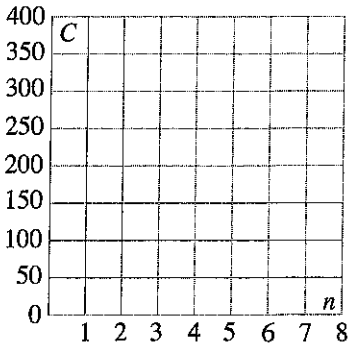
c) $y = -4 + 2x$



6. The snowboard club is planning a trip to a local hill. A bus company will charge them using the formula $C = 50 + 40n$, where C is the total cost for n people.

a) Make a table of values and draw a graph for the cost for 1 to 7 people.

n							
C							



b) A parent group is willing to give the club $410. How many people could go on the trip with that amount of money?

Substitute _____ for C in the equation and solve.

_____ people could go on the trip with $410.

In Your Words

Here are some of the important mathematical words of this unit.
Build your own glossary by recording definitions and examples here. The first one is done for you.

distributive property *multiplying a number by a sum of two numbers is the same as multiplying the first number by each number in the sum and then finding the sum of the products*

For example, $5(a + b) = 5a + 5b$

opposite operation

algebra tiles

ordered pair

table of values

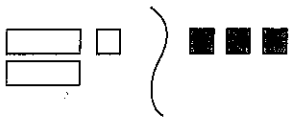
linear relation

List other mathematical words you need to know.

Unit Review

LESSON

6.1 **1.** Write the equation represented by the model. Then solve the equation using the model, showing your steps.



2. Use a model to solve $4 + 3c = -5$.

6.2 **3.** Solve each equation algebraically and verify the result.

a) $4y - 7 = 13$

b) $-9 = 5 + 2m$

LESSON

4. Maria solved the equation $4 - 2p = 6$ using the steps below. Did Maria make an error? YES/NO

If Maria made an error, correct it.

$$4 - 2p = 6$$
$$4 - 4 - 2p = 6 - 4$$
$$-2p = 2$$
$$\frac{-2p}{2} = \frac{2}{2}$$
$$p = 1$$

5. Rajinder collects hockey cards. He currently has 75. He has a plan to collect 12 more each week. After how many weeks will he have a total of 147?

a) Write an equation that you can use to solve this problem.

Let w represent the _____.

_____

b) Solve the equation.

c) Verify your result and write a concluding statement.

_____

6.3 6. Solve each of the following equations and verify the results.

a) $\frac{t}{2} = 4$ b) $\frac{w}{3} + 4 = -2$ c) $6 = 3 + \frac{x}{5}$

_____ _____ _____

6.4 7. Expand using the distributive property.

a) $6(v - 3)$ b) $-9(3 + p)$ c) $-1(-2 + w)$

_____ _____ _____

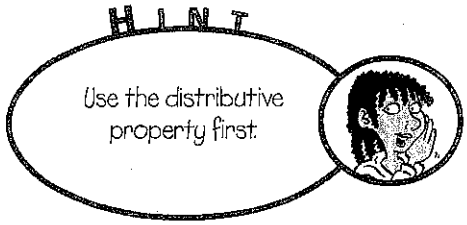
LESSON

8. Match each expression in Column 1 with an equivalent expression in Column 2.

Column 1	Column 2
a) $3(t-4)$	**i)** $3t+12$
b) $-3(t+4)$	**ii)** $-3t-12$
c) $3(t+4)$	**iii)** $-3t+12$
d) $-3(t-4)$	**iv)** $3t-12$

6.5 **9.** Solve each equation and verify the results.

a) $5(a-3)=20$ **b)** $-2(n+3)=-10$



_____ _____

c) $7=4(2+y)$ **d)** $-2(x+3)=-6$

_____ _____

6.6 **10.** Complete the table of values for each relation.

a) $y=x-4$

x	−2	−1	0	1	2
y					

b) $y=-2x+5$

x	−2	−1	0	1	2
y					

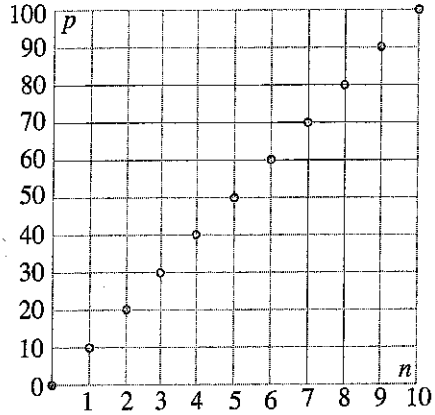
11. The equation of a linear relation is $y=4x-3$. Find the missing number in each ordered pair.

a) (2, _____) **b)** (_____, −11) **c)** (_____, 13)

LESSON

6.7

12. The graph below represents the relation of the percent score, p, on a math test and the number of questions, n, correct out of 10.
The equation for the relation is $p = 10n$.



a) State the ordered pair that represents the highest score.

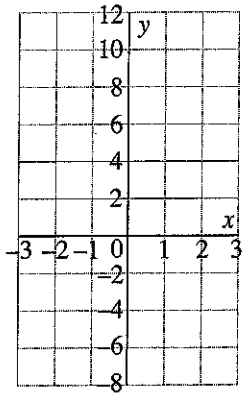
_____

b) Describe the relationship between the variables on the graph.

_____

13. a) Draw a graph of the relation represented by the table of values.

x	y
–2	10
–1	6
0	2
1	–2
2	–6



b) Describe how you know that this is a linear relation.

_____

14. On grid paper, draw the graph of each relation for integer x values from –2 to 2.

a) $y = 2x - 1$

b) $y = 10 - x$